Performance (Utilization) for different Data Link Protocols

(Adapted from Fred Halshall: Data Communications, Computer Networks and Open Systems, Third Edition
and
William Stallings, Data and Computer Communications, Fourth Edition
and
Select Lecture Notes CSEE 461 and other class material)

Consider the following equation for calculating the bandwidth utilization in its simplest form:

$$Utilization = U = \frac{T_f}{T_t} \qquad \text{——————} (1)$$

where T_f is the time it takes to transmit a frame, and T_t is the total time that line is engaged in the transmission of a single frame (i.e. including any propagation delays, waiting for acknowledgments, etc).

From homework 1-2, we have an equation for a Transmission time of a bit (or frame, or packet):

$$T_x = \frac{\text{Number of bits to be transmitted N}}{\text{Link bit rate R in bits per second}}$$, so specifically for a Data Link layer frame,

$$T_f = \frac{\text{Number of bits in a single frame}}{\text{Link bit rate R in bits per second}}$$

is the transmission time of a frame.

For error-free operation using Stop-and-Wait Protocol, we have

$$U = \frac{T_f}{T_f + 2T_p} \qquad \text{——————} (2)$$

Where T_p *is the propagation delay time.*

Dividing (2) by T_f, we have

$$U = \frac{1}{1 + \frac{2T_p}{T_f}} = \frac{1}{1 + 2a} \qquad (\textit{in HW1, } a = T_p / T_x = T_p / T_f)$$

Notice,

In lecture slide 9-5,

$$U = \frac{p/b}{p/b + 2d} = \frac{1}{1 + \frac{2db}{p}}$$

Where,

$T_f = p/b$, $T_p = d$

$$U = \frac{T_f}{T_f + 2T_p}$$

If errors occur, we must modify equation ① to take into account the fact that the link utilization will be decreased proportionally to the number of times it takes to retransmit a frame until an error-free frame arrives at the destination. U is reduced simply by the number of attempts to transmit each frame since only the corrupted frames are retransmitted. Let that number be N_r.
Therefore,

$$U = \frac{T_f}{N_r T_t}$$

For stop-and-wait protocol, we therefore have the following expression for calculating the link utilization given the number of retransmissions, transmit time for the frame, and the propagation delay:

$$U = \frac{1}{N_r(1+2a)}$$

A simple expression for N_r can be derived by considering the probability P that a single frame is in error. If we assume that ACKs (acknowledgments) and NAKs are never in error, the probability that it will take exactly i attempts to transmit a frame successfully is $P^{i-1}(1-P)$. Thus

$$N_r = \sum_{i=1}^{\infty} iP^{i-1}(1-P) = \frac{1}{1-P}$$

For stop-and-wait protocol, the above utilization of the link can then also be expressed in terms of the probability of a frame being corrupted during the transfer, and the transmit time for the frame and the propagation delay. In other words:

$$U = \frac{1}{N_r(1+2a)} = \frac{1-P}{1+2a} \quad \text{with } N_r = 1/(1-P)$$

Example:

If the error rate P for a single frame is 0.1, the probability of retransmit that frame is 0.1; In the same way, the probability for retransmit that retransmitted frame is 0.1 x 0.1 = 0.01 . For the next retransmission is 0.1 x 0.1 x 0.1 = 0.001 For P = 0.1 we have

$$N_r = 1 + 0.1 + 0.01 + 0.001 + 0.0001 \ldots and..so..on.. = \frac{1}{1-P} = 1.11111.....$$

For the sliding window protocol, the utilization is obviously directly proportional to the size of the window. Therefore, for the sliding window protocol in error free operation we have, given the window size of N:

$$U = \begin{cases} 1 & N > 2a+1 \\ \dfrac{N}{2a+1} & N < 2a+1 \end{cases}$$

The wait-and-stop protocol is a special case of the sliding window protocol where N is 1.

For selective-reject protocol, in case of a transmission link with errors, we can use the same reasoning as applied to stop-and-wait protocol to derive the utilization of a link with errors from the expression for the utilization for the error-free link. That is, the error-free equations must be divided by the number of retransmissions needed in order to transmit an error-free frame, that number denoted by N_r. Again, N_r =

1/(1-P), where P is the probability of a frame being corrupted on that link. *N* denotes, as before, the window size. So

Selective-reject:

$$U = \begin{cases} 1-P & N > 2a+1 \\ \dfrac{N(1-P)}{2a+1} & N < 2a+1 \end{cases}$$

The same reasoning will still apply for go-back-N protocol, but we must be more careful in approximating $N_{r.}$ Each error generates a requirement to retransmit K frames rather than just one frame. Thus

$$N_r = \sum_{i=1}^{\infty} f(i) P^{i-1}(1-P) = 1-K+\frac{K}{1-P} = \frac{1-P+KP}{1-P} \qquad \textit{where,} \quad f(i) = 1+(i-1)K = (1-K)+Ki$$

Therefore, the expression for the link utilization for the Go-back-N protocol, for a link with errors is:

$$U = \begin{cases} \dfrac{1-P}{1+P(N-1)} & N > 2a+1 \\ \dfrac{N(1-P)}{(2a+1)(1-P+NP)} & N < 2a+1 \end{cases}$$

Note that for N = 1, the selective-reject protocol reduces to stop-and-wait protocol.