Physical Modeling Synthesis of Sound

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One View of Sound

Sound is a waveform,
we can record it, store it,
and play it back accurately

PCM playback is all we need for
interactions, movies, games, etc.

But, take one visual analogy:

"If I take lots of polaroid images, I can flip through them real fast and make any image sequence"

Interaction? We manipulate lots of PCM

Views of Sound

- Time Domain x(t)
  (from physics, and time's arrow)
- Frequency Domain X(f)
  (from math, and perception)
- Production what caused it
- Perception our "image" of it

Views of Sound

- The Time Domain
  is most closely related to
  Production
- The Frequency Domain
  is most closely related to
  Perception

Views of Sound: Time Domain

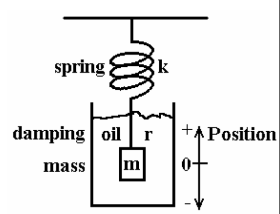
Sound is produced/modeled by physics,
described by quantities of

- Force force = mass * acceleration
- Position *x(t)* actually *[x(t), y(t), z(t)]*
- Velocity Rate of change of position *dx/dt*
- Acceleration Rate of change of velocity *dv/dt*
  (2nd derivative of position) d^2x/dt^2

Examples: Mass,Spring,Damper Wave Equation

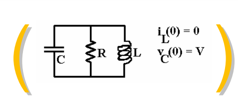
Mass/Spring/Damper

F = ma = - ky - rv - mg
ma = - ky - rv
(if gravity negligible)



Solution:

$$\frac{d^2y}{dt^2} + \frac{r}{m}\frac{dy}{dt} + \frac{k}{m}y = 0$$

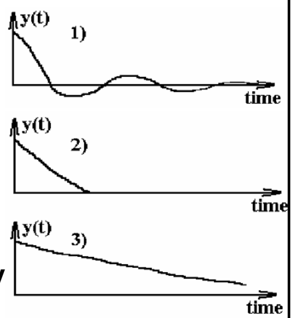


2nd Order Linear Diff Eq. Solution

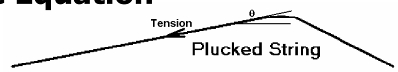
1) Underdamped:
$y(t) = Y_0 e^{-t/\tau} \cos(\omega t)$
exp. * oscillation

2) Critically damped:
fast exponential decay

3) Overdamped:
slow exponential decay



The Wave Equation



$df_y = (T \sin\theta)_{x+dx} - (T\sin\theta)_x$ **(for each dx of string)**

$f(x+dx) = f(x) + \delta f/\delta x \, dx + \ldots$ **(Taylor's series in space)**

assume $\sin\theta = \theta$ **(for small** θ**)**

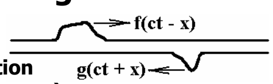
$F = ma = \rho \, dx \, d^2y/dt^2$ **(**ρ **= mass/length)**

Solution:
The wave equation
$(c^2 = T/\rho)$

$$\frac{d^2y}{dx^2} = \frac{1}{c^2}\frac{d^2y}{dt^2}$$

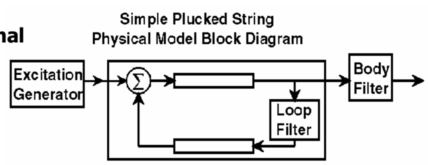
Traveling Wave String Solution

D'Alembert Solution of
2nd order wave equation
(left and right going waves)



"Digital Waveguide Filter" Model (Smith)

- **Bi-directional delay lines**
- **Filters for loss, radiation, other**



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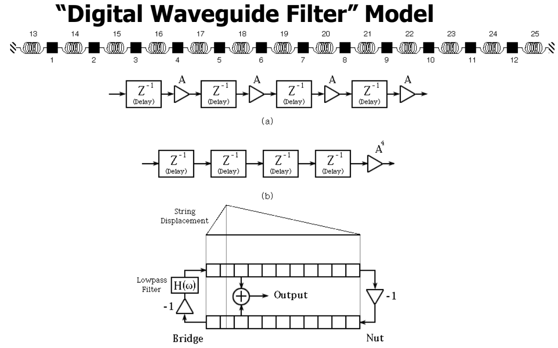

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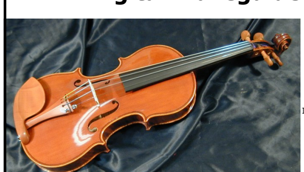

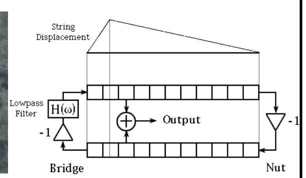

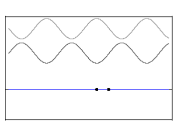

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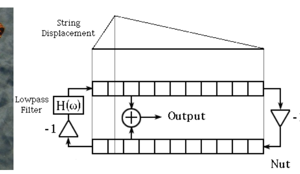
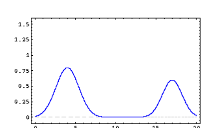
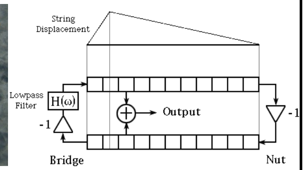
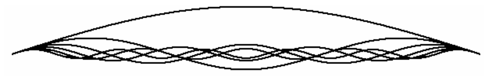




"Digital Waveguide Filter" Model



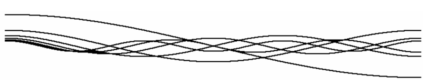
Modal String Solution



- **Superimposed <u>spatial</u> sine waves**
  (modes derive from spatial "boundary conditions")
- **Modes result in frequency "partials" (in time)**
- **Harmonic (*f, 2f, 3f*, etc.) relationship**
  (speed of sound *c* = constant)
- **Stiffness can cause minor stretching of harmonic frequencies (*c(f)*)**

Modal Solution for Bars

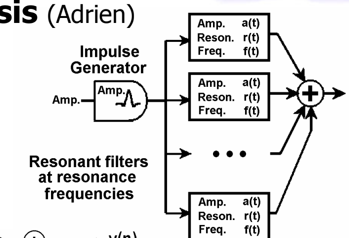
- **Bars are often free at one or both ends**

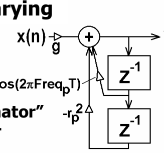


- **Spatial modal solution still holds**
- **Modes no longer harmonic. Stiffness of rigid bars "stretches" frequencies.**
- **Modes: *f, 2.765f, 5.404f, 8.933f*, etc.**

Modal Synthesis (Adrien)

- **Impulse generator excites filters**
- **Filters shape spectrum, model eigenmodes**
- **Filter parameters can be time-varying**



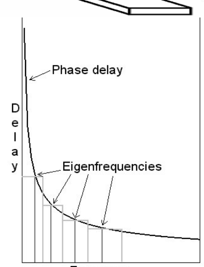


"2nd order resonator" digital filter

```
y[n]  = g*x[n];
y[n] += b1*y[n-1];
y[n] += b2*y[n-2];
n++;
```


Stiffness in Bars

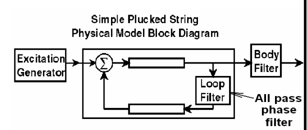
- **Stiffness makes wave propagation frequency dependent (*c(f)*)**
- **Models:**
  - **Modal partials**
  - **Use all-pass phase filter to "stretch" waveguide harmonics**
  - **Merge waveguide with modal by modeling each mode with filter and delay**



Stiffness

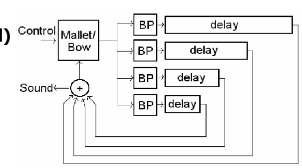
All-pass waveguide (Smith & Jaffe)

- *Acoustics View: Frequency dependent propagation*
- *Filter View: Stretch comb filter harmonics*



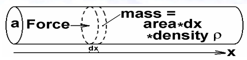
Banded waveguides (Essl)

- *Acoustics View: Wave train closures*
- *Filter View: Comb filters with one resonance each*



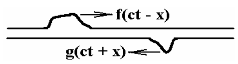
Or a purely modal model (lacks space and time)

Tubes

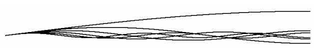


- Open or closed at either end
- Wave equation solution same as strings
- Modes always harmonic because speed of sound is constant with frequency
- Solutions:

  Waveguide

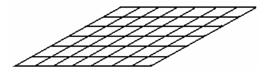


  or Modal



Open + Closed: odd 1/4 wavelengths

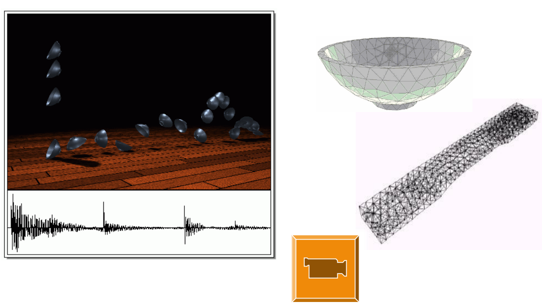
Two and Higher Dimensions



- 2 (N) Dimensional Waveguide Meshes
- or Finite Elements and Finite Differences
  - Discretize objects into cells (elements)
  - Express interactions between them
  - Express differential equation for system
  - Solve by discrete steps in space and time
- or Modal Solution

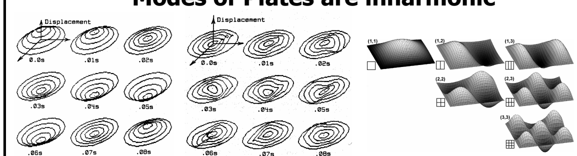
Finite Elements

(with O'Brien and Essl)



Hi-D Modal Solutions

Modes of Plates are inharmonic



Center strike Edge strike
round = Bessel function roots

Square Plate Modes
= sqrt(I) factors

Modes in higher dimensions are problematic
(impossible analytically except in very simple cases)

Where Are We So Far?

- Physical descriptions (equations)
- Give rise to solutions:
  1. Traveling Waves
  2. Spatial/Frequency Modes
- We can solve the equations directly using
  3. Finite Elements/Meshes
- How to choose? Are there more?

Waveguides

- Strengths:
  - Cheap in both computation and memory
  - Parametrically meaningful, extensible for more realism
- Weaknesses:
  - Little in the real world looks, behaves, or sounds exactly like a plucked string, flute, etc.
  - Each family needs a different model
  - No general blind signal model

Modal Modeling

- **Strengths:**
  - Generic, flexible, cheap if only a few modes
  - Great for modeling struck objects of metal, glass, wood
- **Weaknesses:**
  - No inherent spatial sampling
  - No (meaningful) phase delay
  - Hard to interact directly and continuously (rubbing, damping, etc).
  - No general blind signal model (closest)

Meshes, Finite Elements

- **Strengths**
  - (somewhat) arbitrary geometries
  - Less assumptions than parametric forms
  - Can strike, damp, rub, introduce non-linearities at arbitrary points
- **Weaknesses:**
  - Expensive
  - Don't know all the computational solutions
  - Sampling in space/time (high Q problems)
  - Dispersion is strange (diagonals vs. not)
  - No general blind signal model

Sound Views: Frequency Domain

- Many physical systems have <u>modes</u> (damped oscillations)
- Wave equation (2nd order) or Bar equation (4th order) need 2 or 4 "boundary conditions" for solution
- Once boundary conditions are set solutions are sums of exponentially damped sinusoidal modes

References and Resources

Synthesis ToolKit in C++ (STK)

- STK: a set of classes in C++ for rapid experimentation with sound synthesis. Available for free (source, multi-platform)
  - http://www.cs.princeton.edu/~prc
  - http://www-ccrma.stanford.edu/~gary
  - http://www-ccrma.stanford.edu/software/stk
- Based on "Unit Generators," the classical computer music/sound building blocks:
- Oscillators, Filters, Delay Lines, etc.
- Build your own algorithms from these

Book on interactive sound synthesis

Many examples and figures from these notes

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SMS Web site. URL:http://www.iua.upf.es/~sms.