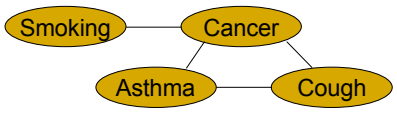
Markov Networks

Markov Networks

- **Undirected** graphical models



- Potential functions defined over cliques

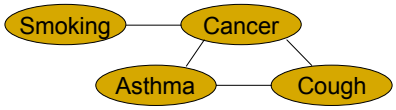
$$P(x) = \frac{1}{Z}\prod_c \Phi_c(x_c)$$

$$Z = \sum_x \prod_c \Phi_c(x_c)$$

Smoking	Cancer	Φ(S,C)
False	False	4.5
False	True	4.5
True	False	2.7
True	True	4.5

Markov Networks

- **Undirected** graphical models



- Log-linear model:

$$P(x) = \frac{1}{Z}\exp\left(\sum_i w_i f_i(x)\right)$$

Weight of Feature *i* | Feature *i*

$$f_1(\text{Smoking, Cancer}) = \begin{cases} 1 & \text{if } \neg \text{ Smoking } \vee \text{ Cancer} \\ 0 & \text{otherwise} \end{cases}$$

$$w_1 = 1.5$$

Hammersley-Clifford Theorem

If Distribution is strictly positive (P(x) > 0)

And Graph encodes conditional independences

Then Distribution is product of potentials over cliques of graph

Inverse is also true.

("Markov network = Gibbs distribution")

Markov Nets vs. Bayes Nets

Property	Markov Nets	Bayes Nets
Form	Prod. potentials	Prod. potentials
Potentials	Arbitrary	Cond. probabilities
Cycles	Allowed	Forbidden
Partition func.	Z = ?	Z = 1
Indep. check	Graph separation	D-separation
Indep. props.	Some	Some
Inference	MCMC, BP, etc.	Convert to Markov

Inference in Markov Networks

- **Goal**: compute marginals & conditionals of

$$P(X) = \frac{1}{Z}\exp\left(\sum_i w_i f_i(X)\right) \qquad Z = \sum_X \exp\left(\sum_i w_i f_i(X)\right)$$

- Exact inference is #P-complete
- Conditioning on Markov blanket is easy:

$$P(x \mid MB(x)) = \frac{\exp\left(\sum_i w_i f_i(x)\right)}{\exp\left(\sum_i w_i f_i(x=0)\right) + \exp\left(\sum_i w_i f_i(x=1)\right)}$$

- Gibbs sampling exploits this

MCMC: Gibbs Sampling

state ← random truth assignment
for *i* ← 1 **to** *num-samples* **do**
  for each variable *x*
    sample *x* according to P(*x*|*neighbors*(*x*))
    state ← *state* with new value of *x*
P(*F*) ← fraction of states in which *F* is true

Other Inference Methods

- Belief propagation (sum-product)
- Mean field / Variational approximations

MAP/MPE Inference

- **Goal:** Find most likely state of world given evidence

$$\max_y \; P(y \mid x)$$

Query Evidence

MAP Inference Algorithms

- Iterated conditional modes
- Simulated annealing
- Graph cuts
- Belief propagation (max-product)