CSE 473: Introduction to Artificial Intelligence

Hanna Hajishirzi

Search
(Un-informed, Informed Search)

slides adapted from
Dan Klein, Pieter Abbeel ai.berkeley.edu
And Dan Weld, Luke Zettelmoyer

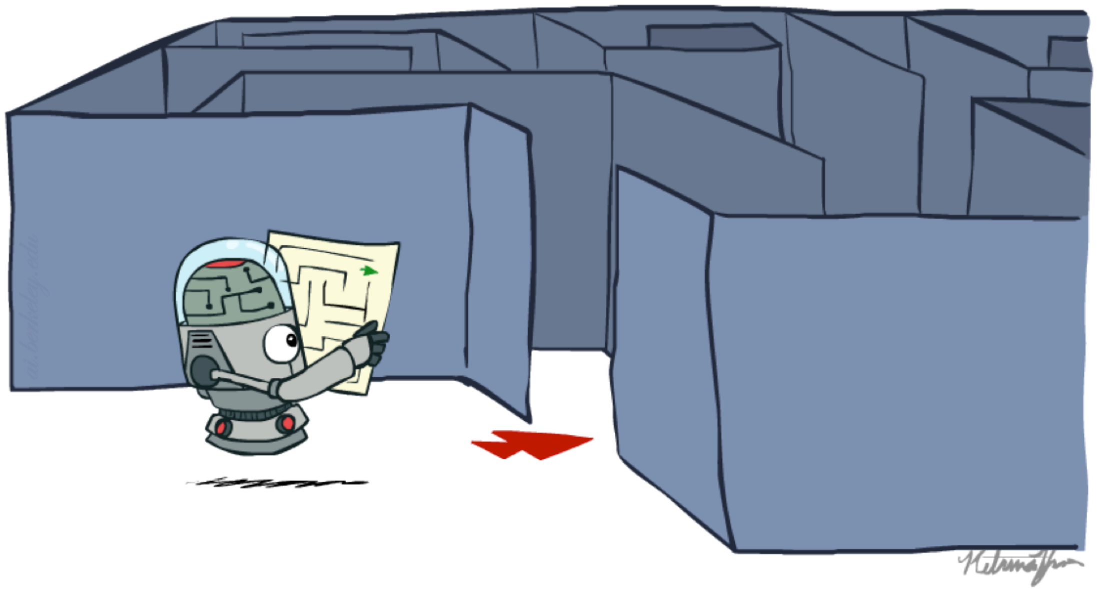
Announcements

- HW1 is released
  - Due: Friday 6pm

- PS1 is due: Next Wednesday (April 14th)

Recap: Search

- Search problem:
  - States (configurations of the world)
  - Actions and costs
  - Successor function (world dynamics)
  - Start state and goal test

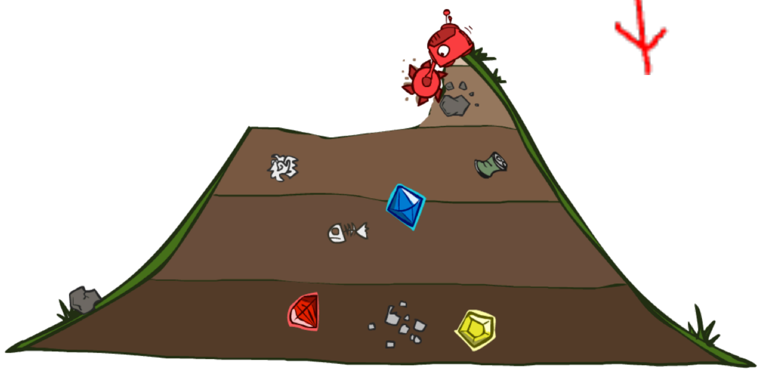


- Search tree:
  - Nodes: represent plans for reaching states
- Search algorithm:
  - Systematically builds a search tree
  - Chooses an ordering of the fringe (unexplored nodes)
  - Optimal: finds least-cost plans

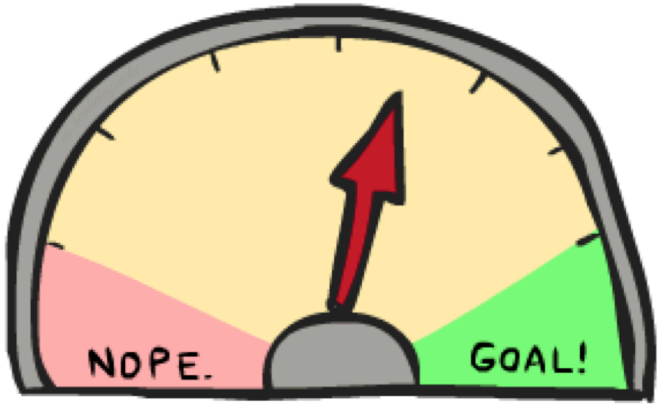
Informed Search

- Uninformed Search
  - DFS
  - BFS
  - UCS

?



- Informed Search
  - Heuristics
  - Greedy Search
  - A* Search
  - Graph Search



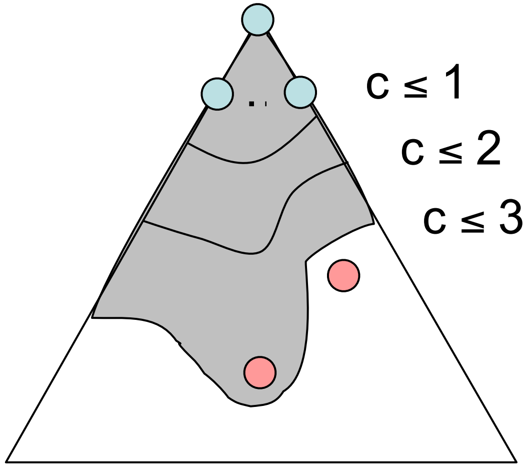
Uniform Cost Issues

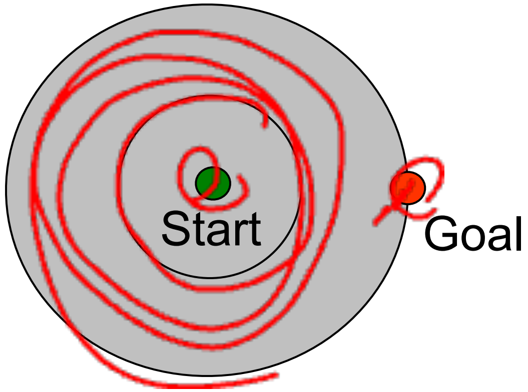
- Remember: UCS explores increasing cost contours

- The good: UCS is complete and optimal!

- The bad:
  - Explores options in every “direction”
  - No information about goal location

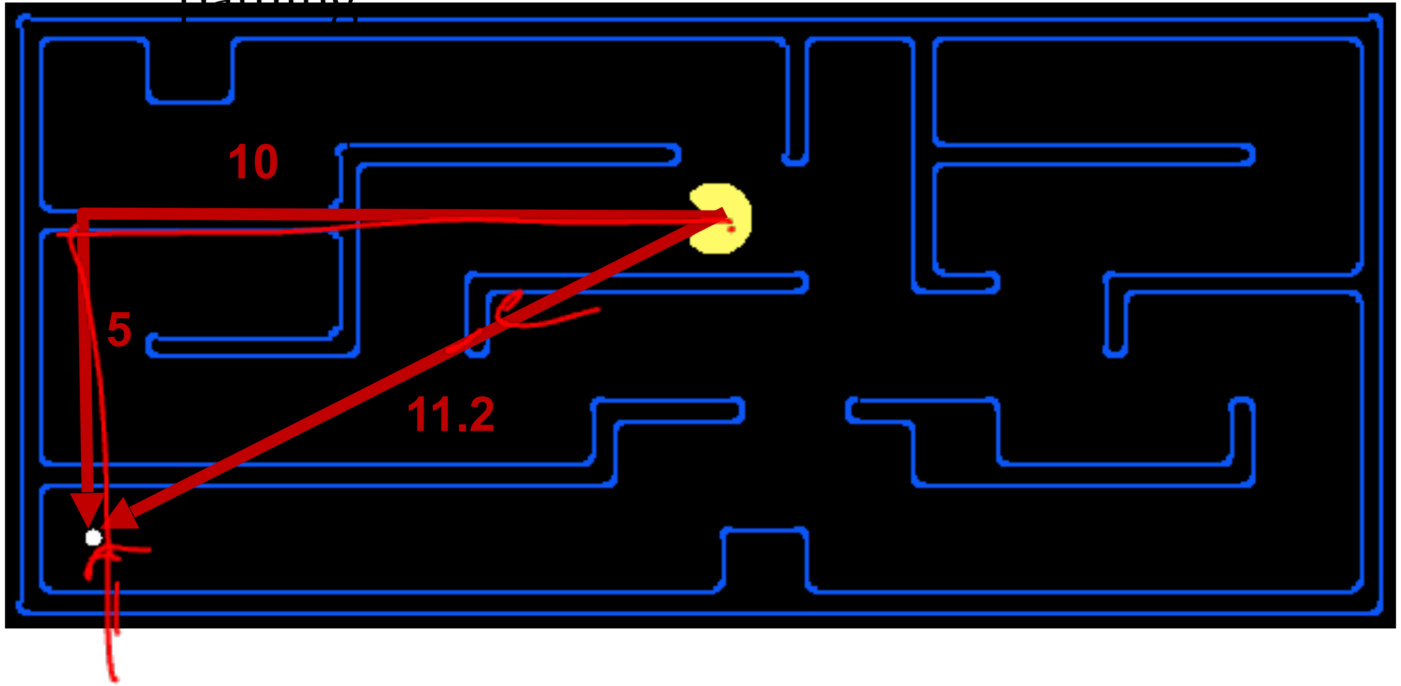
- We’ll fix that soon!

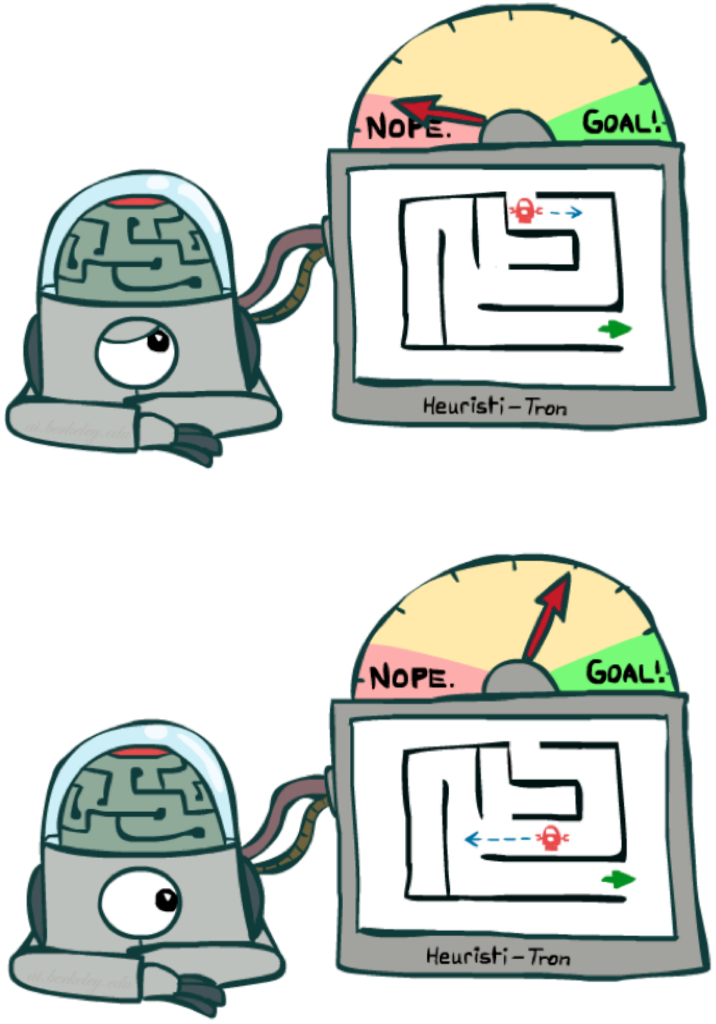




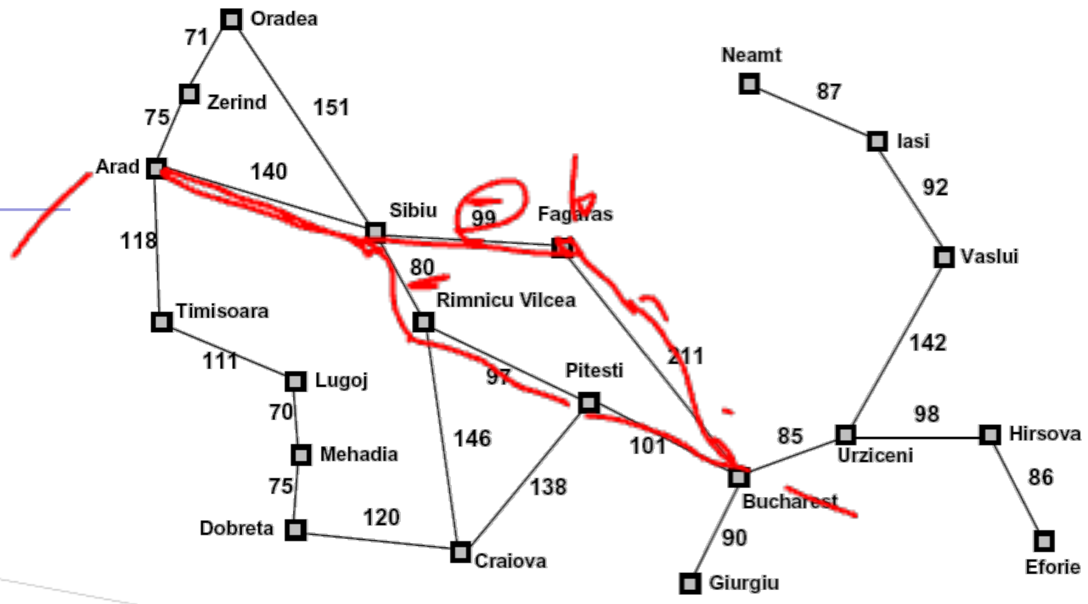
Search Heuristics

- A heuristic is:
  - A function that *estimates* how close a state is to a goal
  - Designed for a particular search problem
  - Pathing?
  - Examples: Manhattan distance, Euclidean distance for pathing

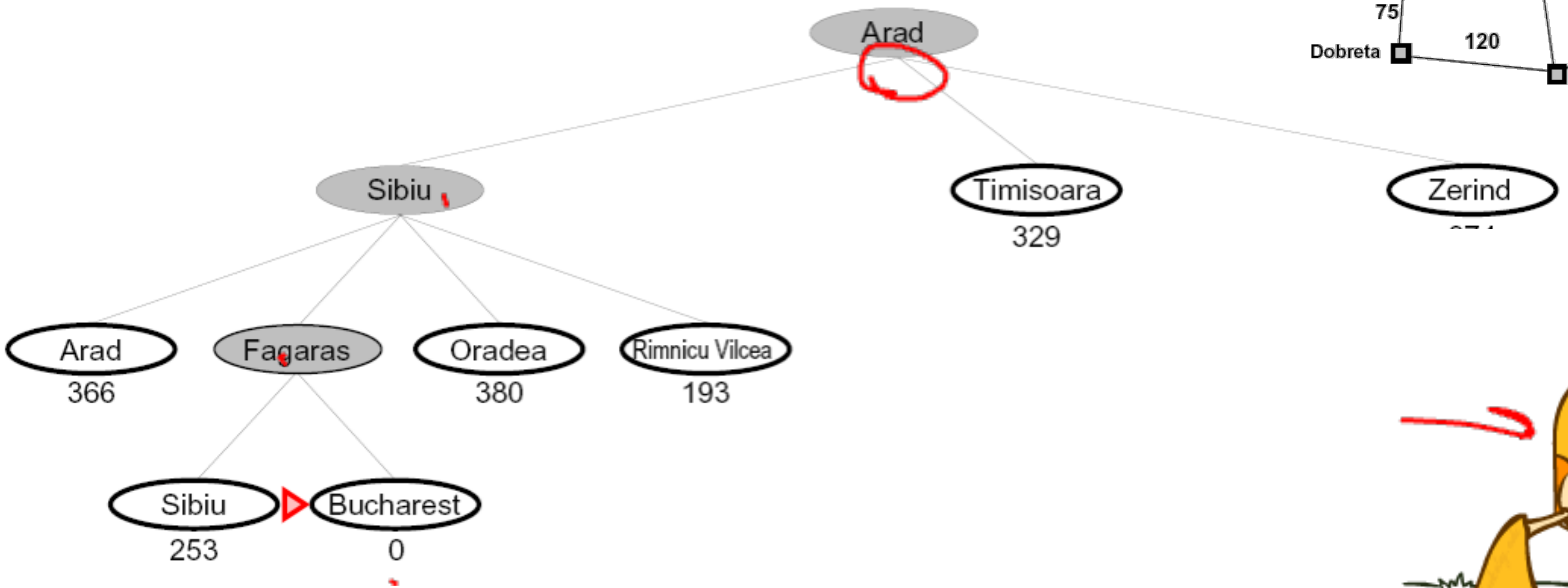


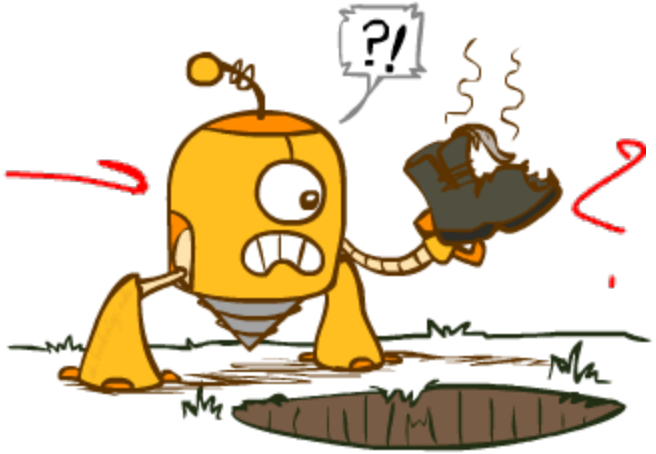


Greedy Search



- Expand the node that seems closest…

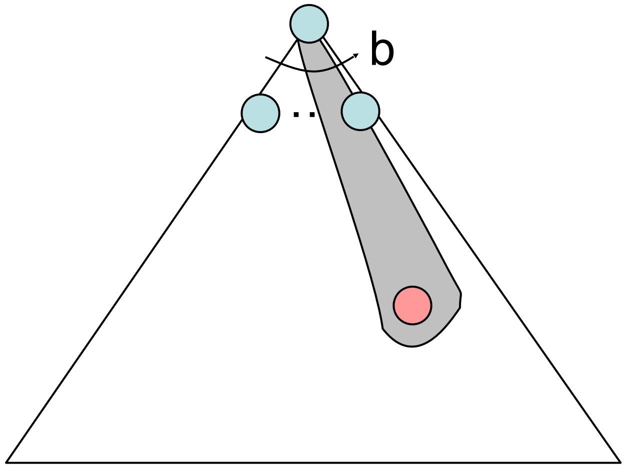




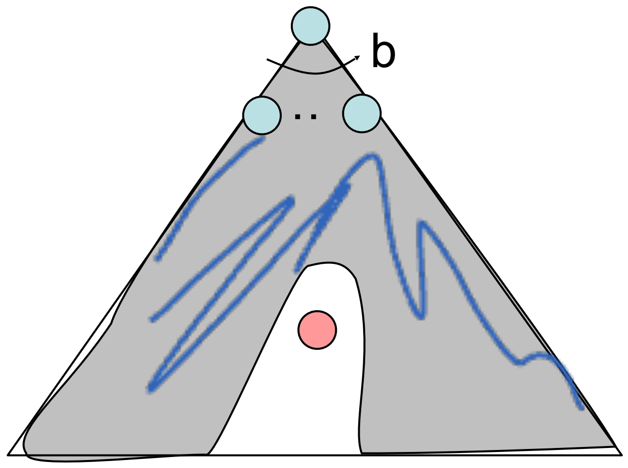
- Is it optimal?
  - No. Resulting path to Bucharest is not the shortest!

Greedy Search

- Strategy: expand a node that you think is closest to a goal state
  - Heuristic: estimate of distance to nearest goal for each state



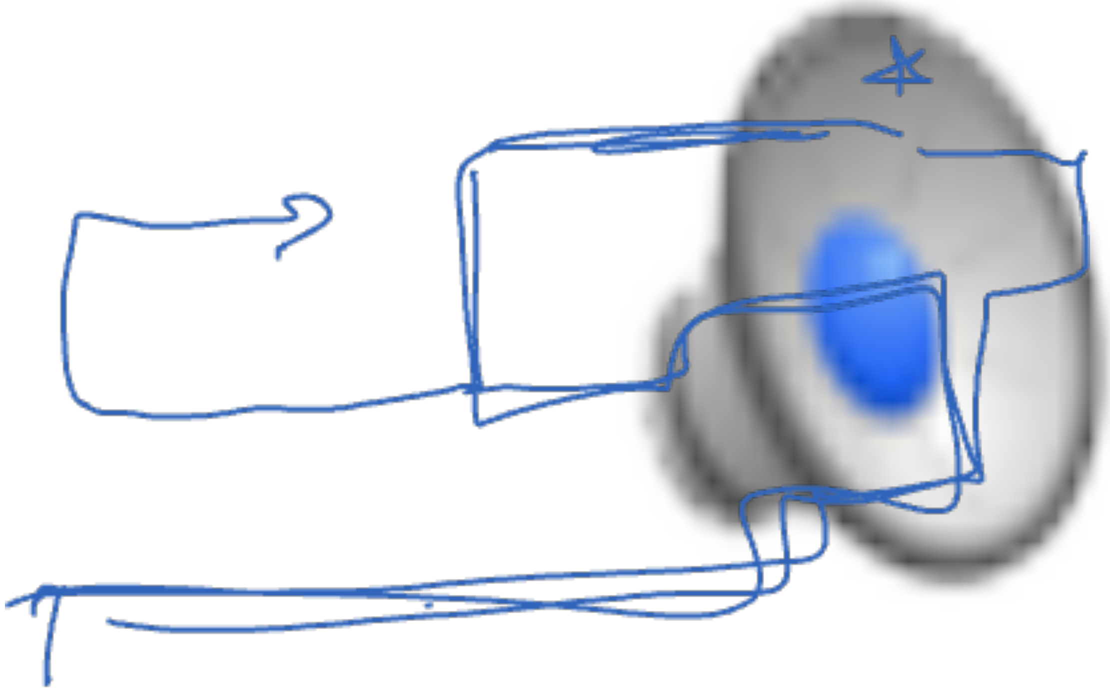
- A common case:
  - Best-first takes you straight to the (wrong) goal



- Worst-case: like a badly-guided DFS

Video of Demo Contours Greedy (Empty)

Video of Demo Contours Greedy (Pacman Small Maze)



A* Search

A* Search

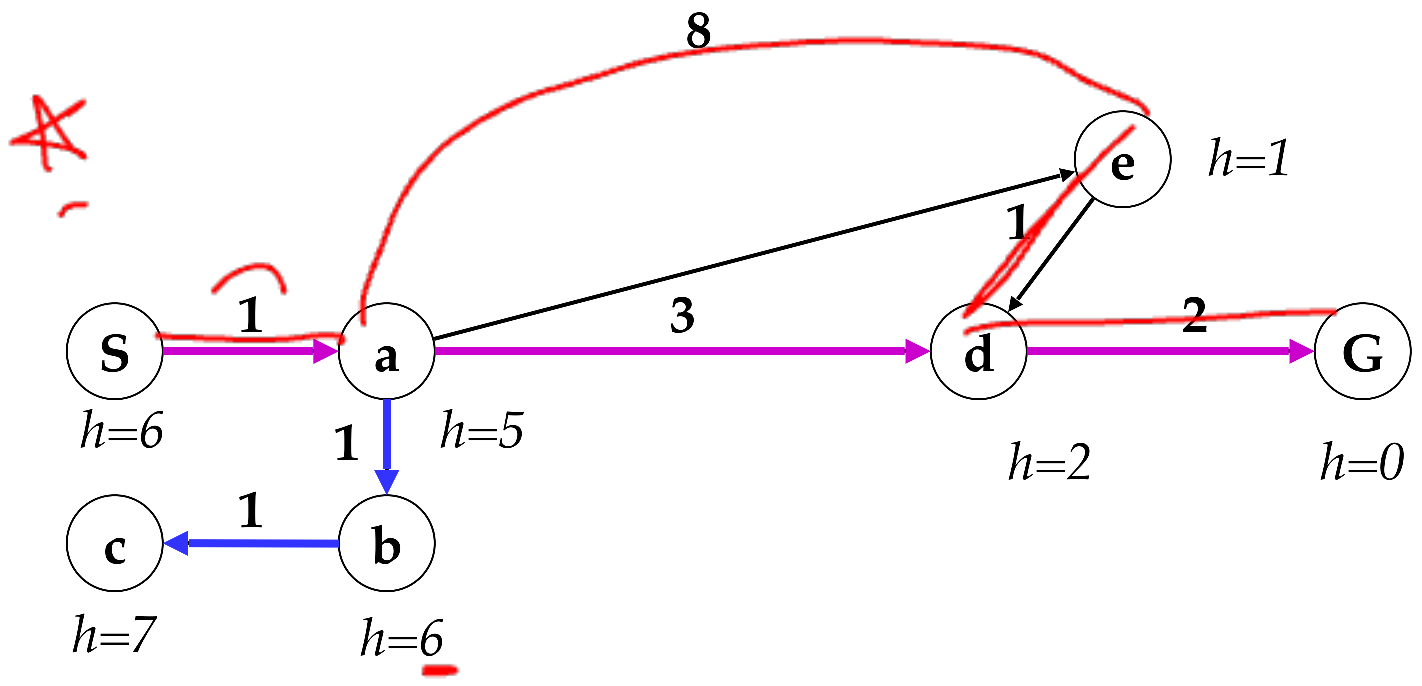
– UCS

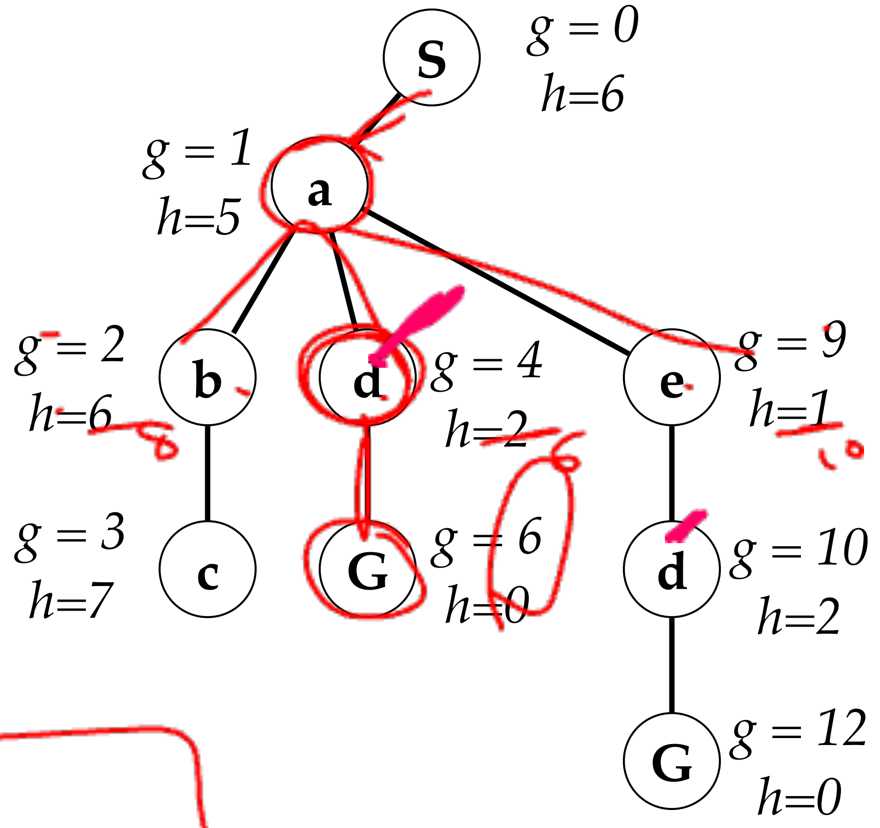
– greedy

Combining UCS and Greedy

g+h

- Uniform-cost orders by path cost, or *backward cost* g(n)
- Greedy orders by goal proximity, or *forward cost* h(n)



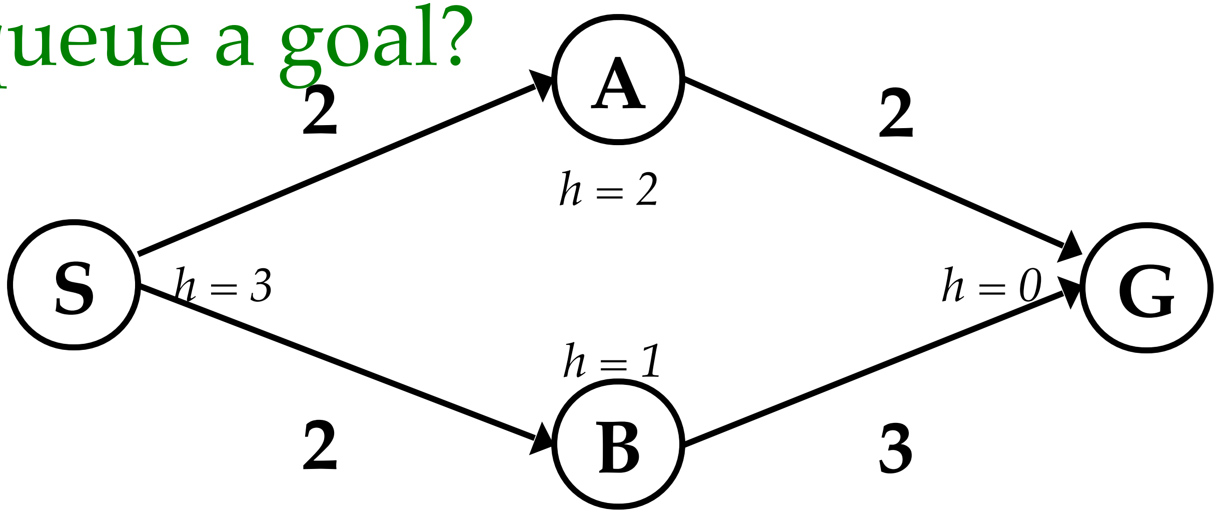


- A* Search orders by the sum: f(n) = g(n) + h(n)

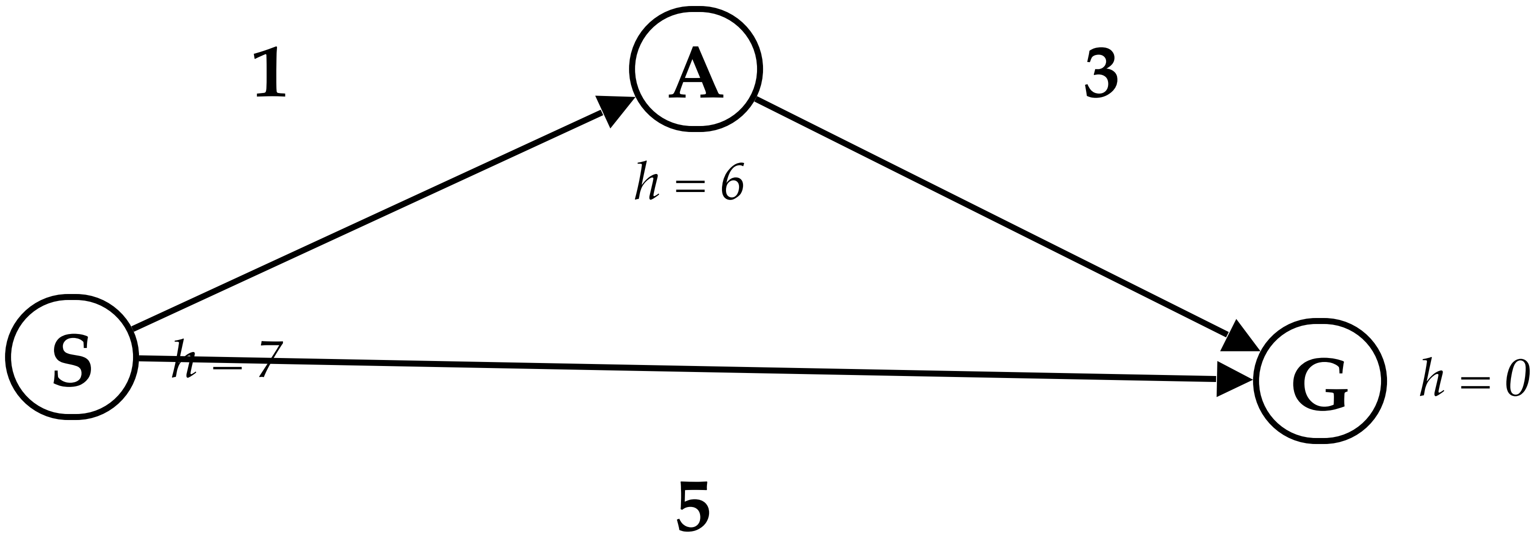
Example: Teg Grenager

Questions

- Should we stop when we enqueue a goal?

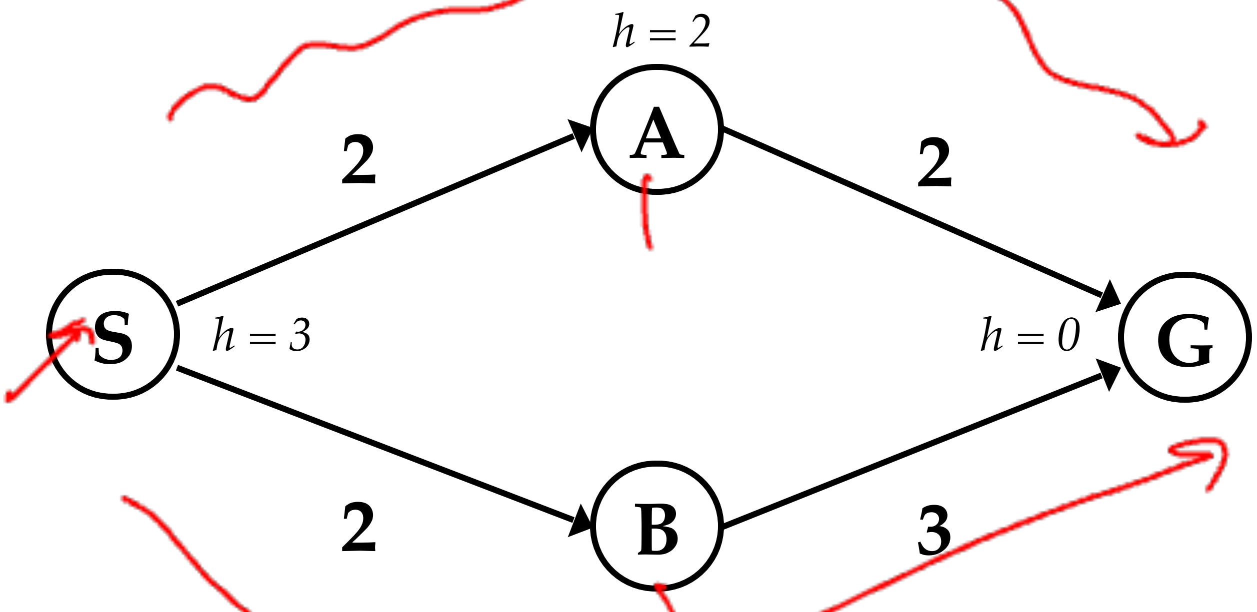


- Is A* optimal?



When should A* terminate?

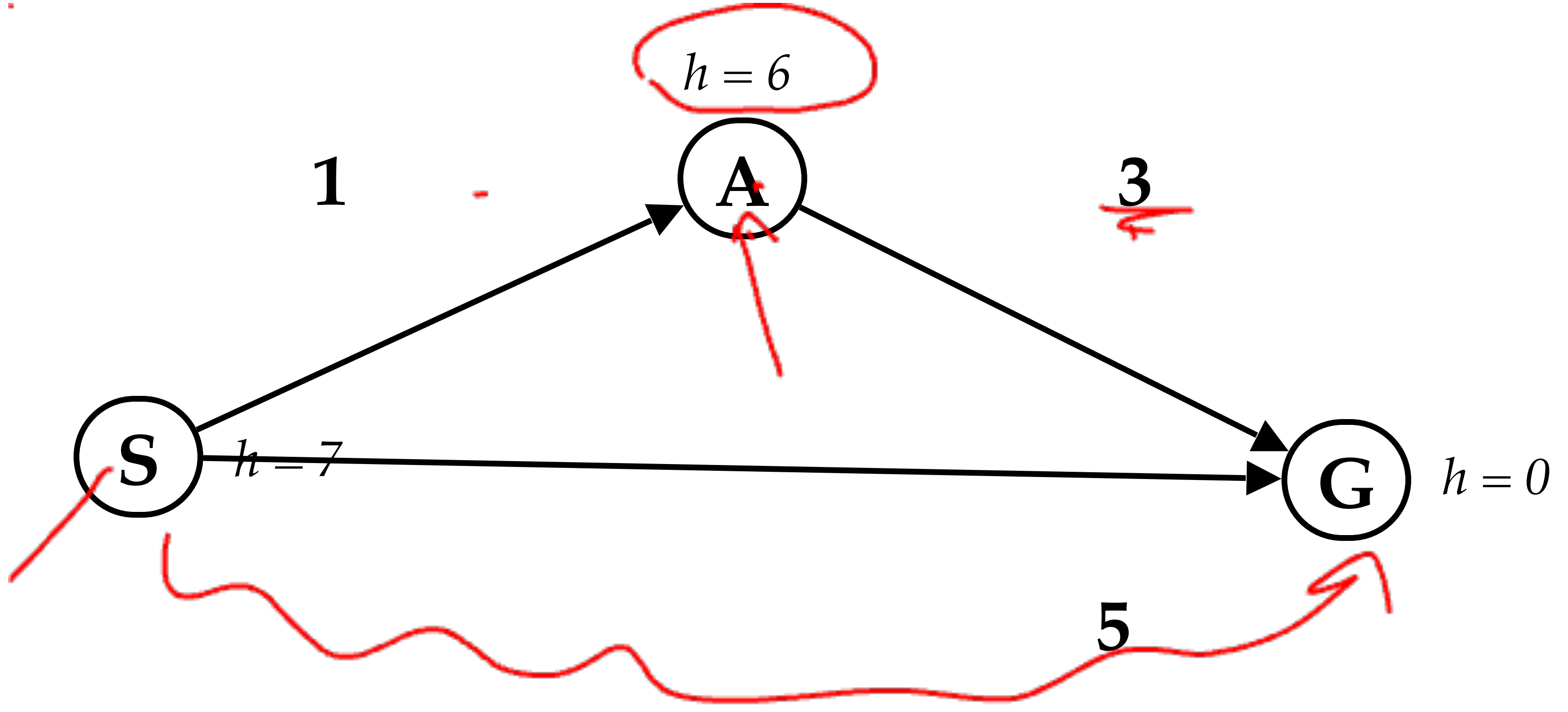
- Should we stop when we enqueue a goal?



- No: only stop when we dequeue a goal

	g	h	+
~~S~~	~~0~~	~~3~~	~~3~~
~~S->A~~	~~2~~	~~2~~	~~4~~
~~S->B~~	~~2~~	~~1~~	~~3~~
S->B->G	5	0	5
S->A->G	4	0	4

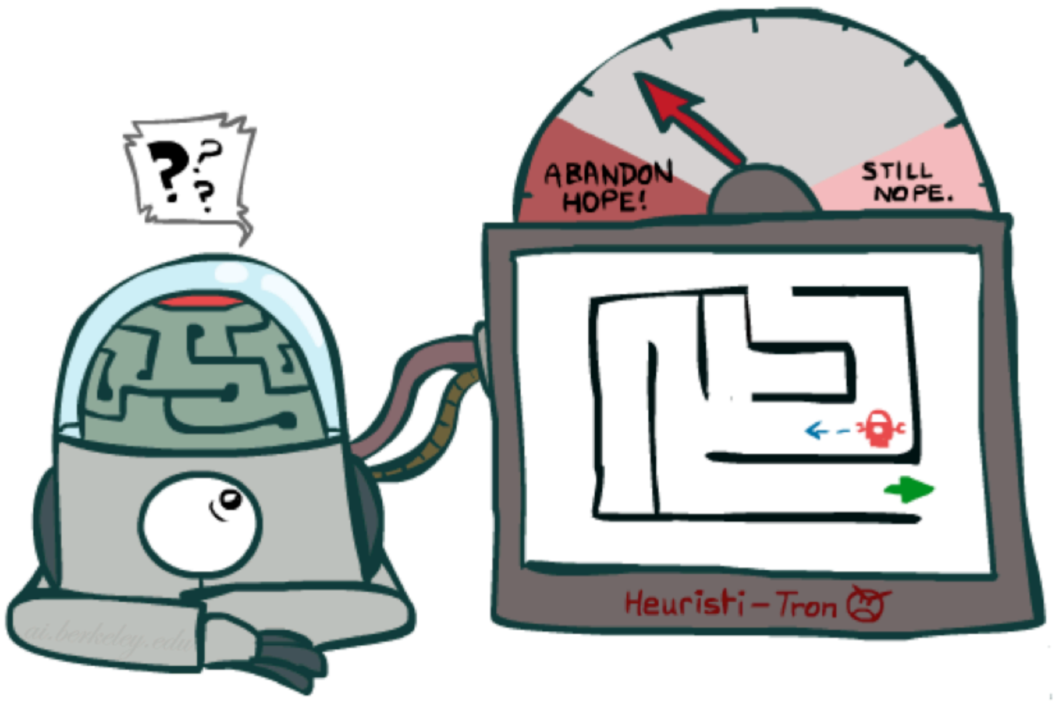
Is A* Optimal?



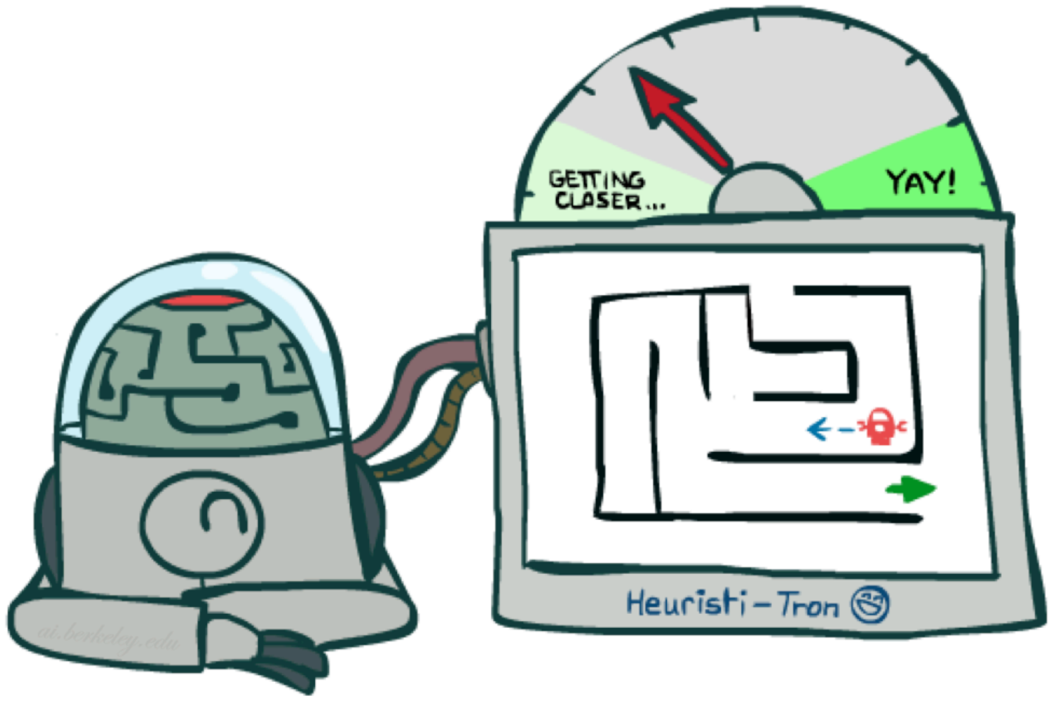
	g	h	+
~~S~~	~~0~~	~~7~~	~~7~~
S->A	1	6	7
S->G	5	0	5

- What went wrong?
- Actual bad goal cost < estimated good goal cost
- We need estimates to be less than actual costs!

Idea: Admissibility



Inadmissible (pessimistic) heuristics break optimality by trapping good plans on the fringe



Admissible (optimistic) heuristics slow down bad plans but never outweigh true costs

Admissible Heuristics

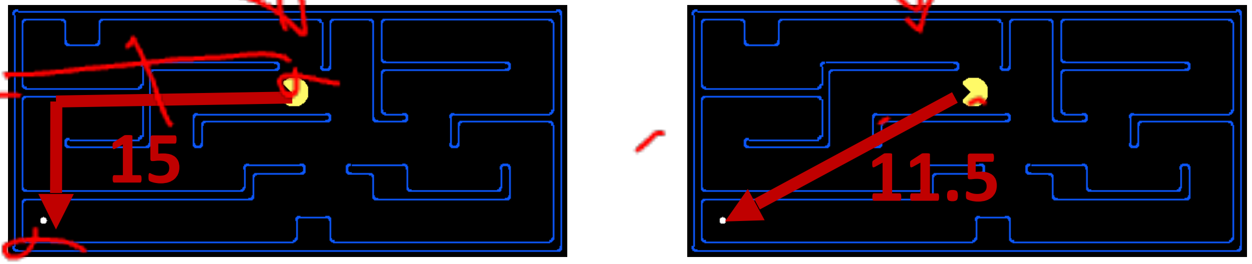
- A heuristic h is *admissible* (optimistic) if:

$$0 \le h(n) \le h^*(n)$$

$h(n) = 0$

where $h^*(n)$ is the true cost to a nearest goal

- Examples:

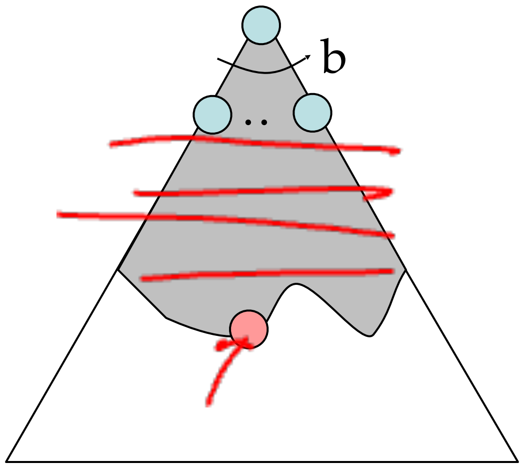


0.0

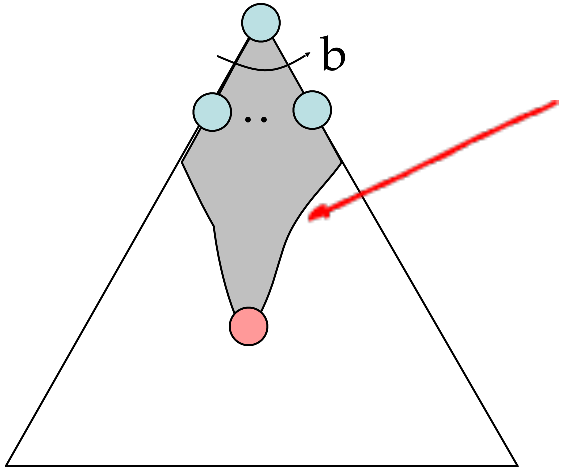
- Coming up with admissible heuristics is most of what's involved in using A* in practice.

Properties of A*

Uniform-Cost

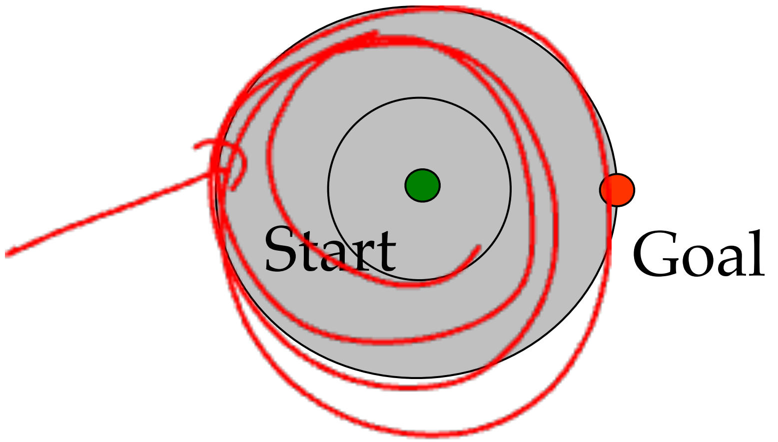


A*

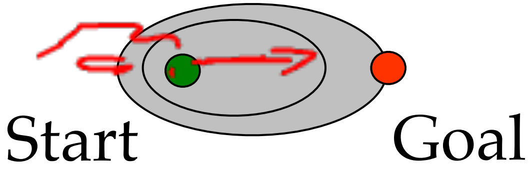


UCS vs A* Contours

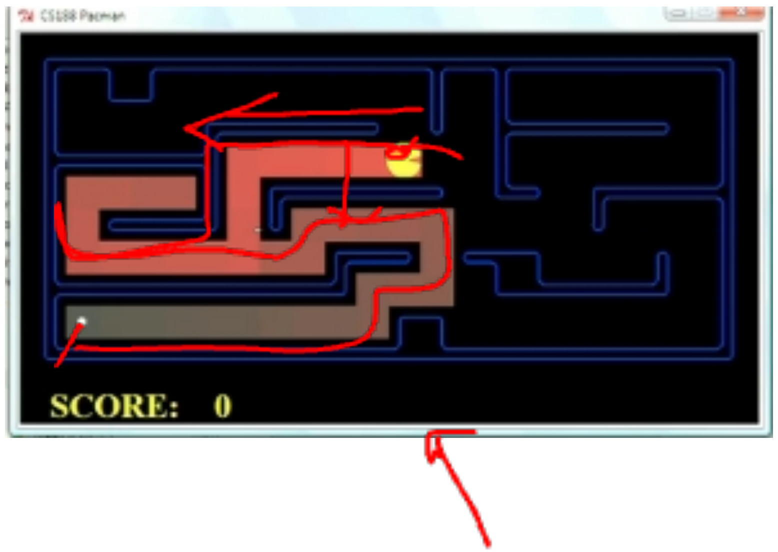
- Uniform-cost expands equally in all "directions"



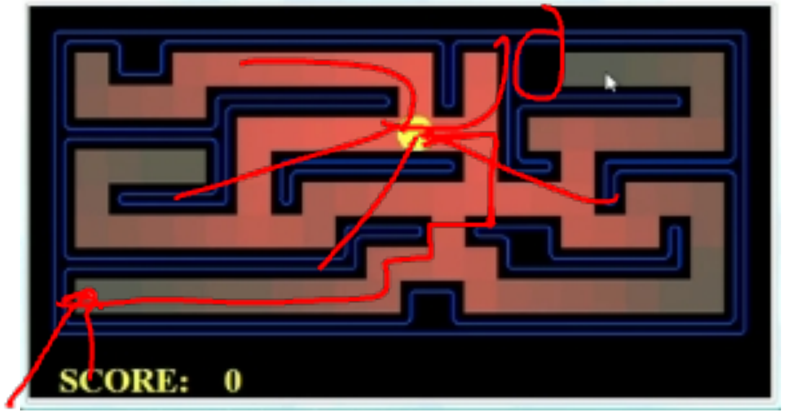
- A* expands mainly toward the goal, but does hedge its bets to ensure optimality



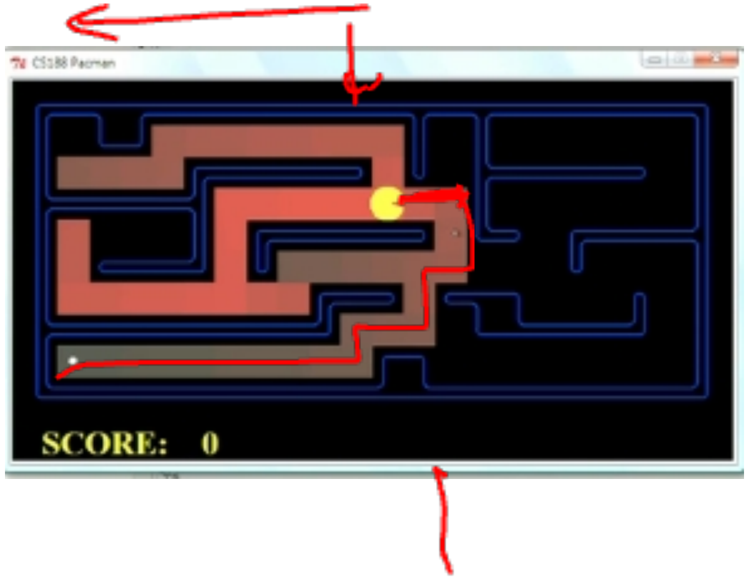
Comparison



Greedy



Uniform Cost



A*

Video of Demo Contours (Empty) -- UCS

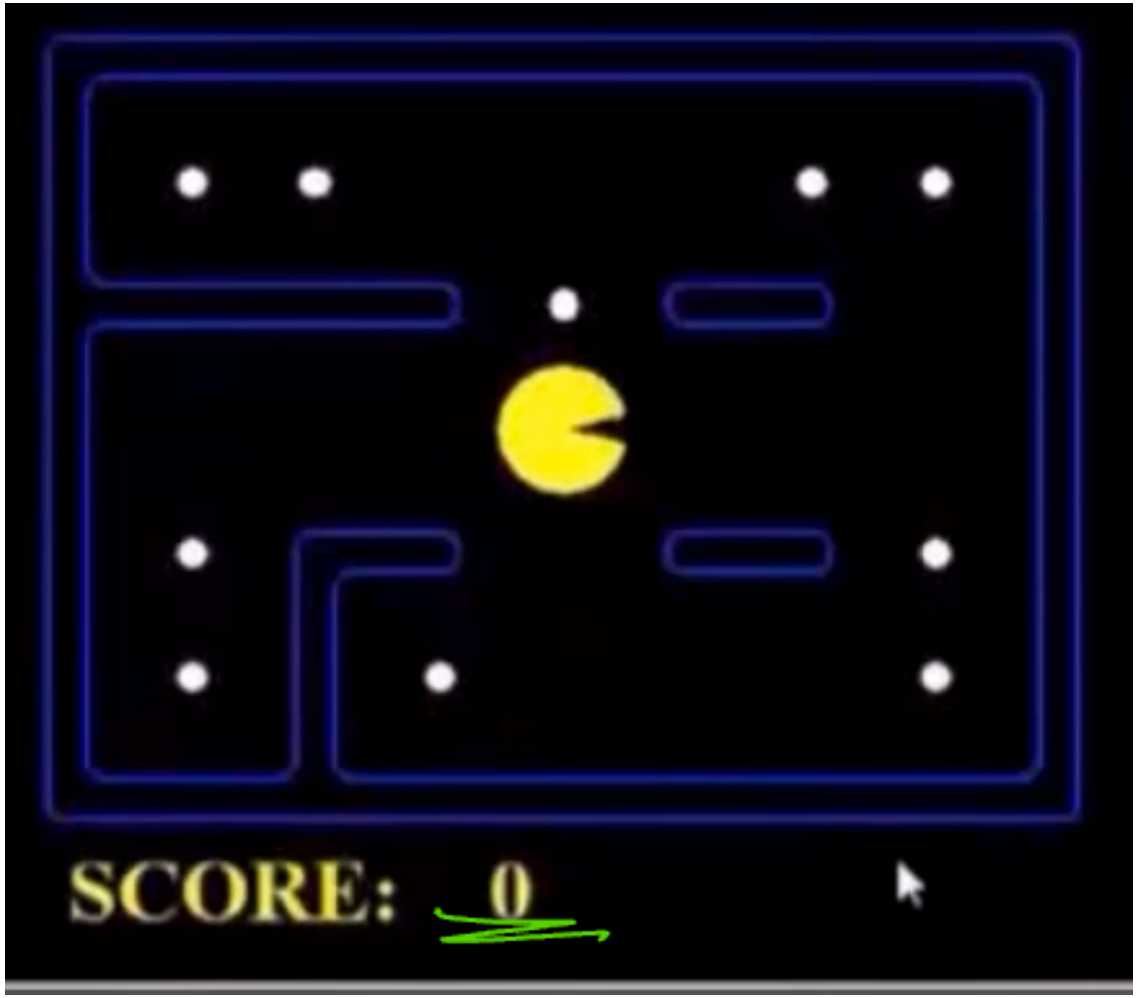
Video of Demo Contours (Empty) -- Greedy

Video of Demo Contours (Empty) – A*

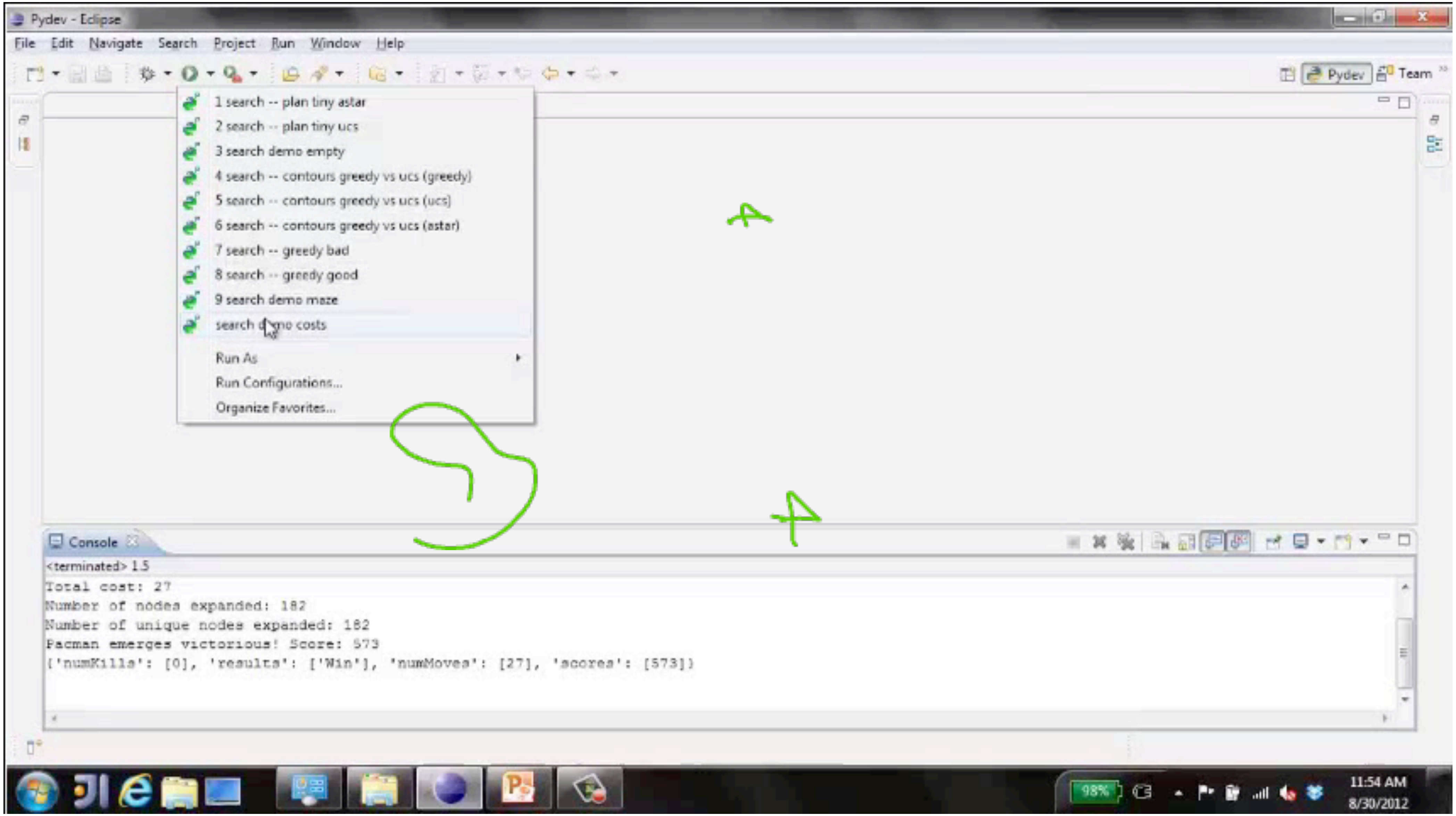
UCS vs. A*

9000

180



Video of Demo Empty Water Shallow/Deep – Guess Algorithm



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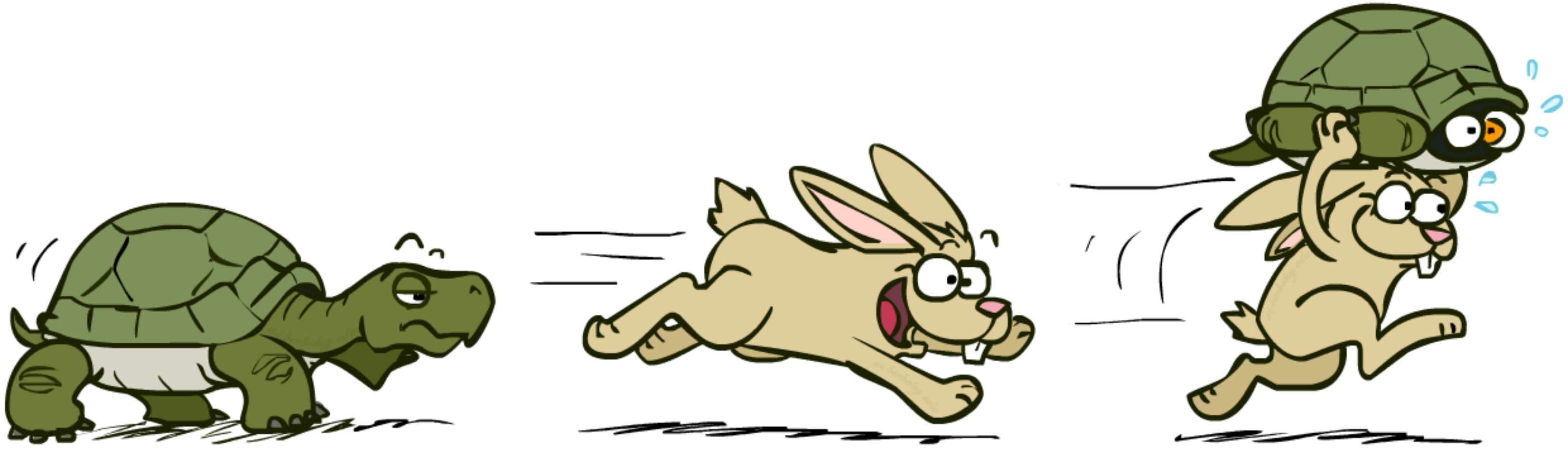
A*: Summary

A*: Summary

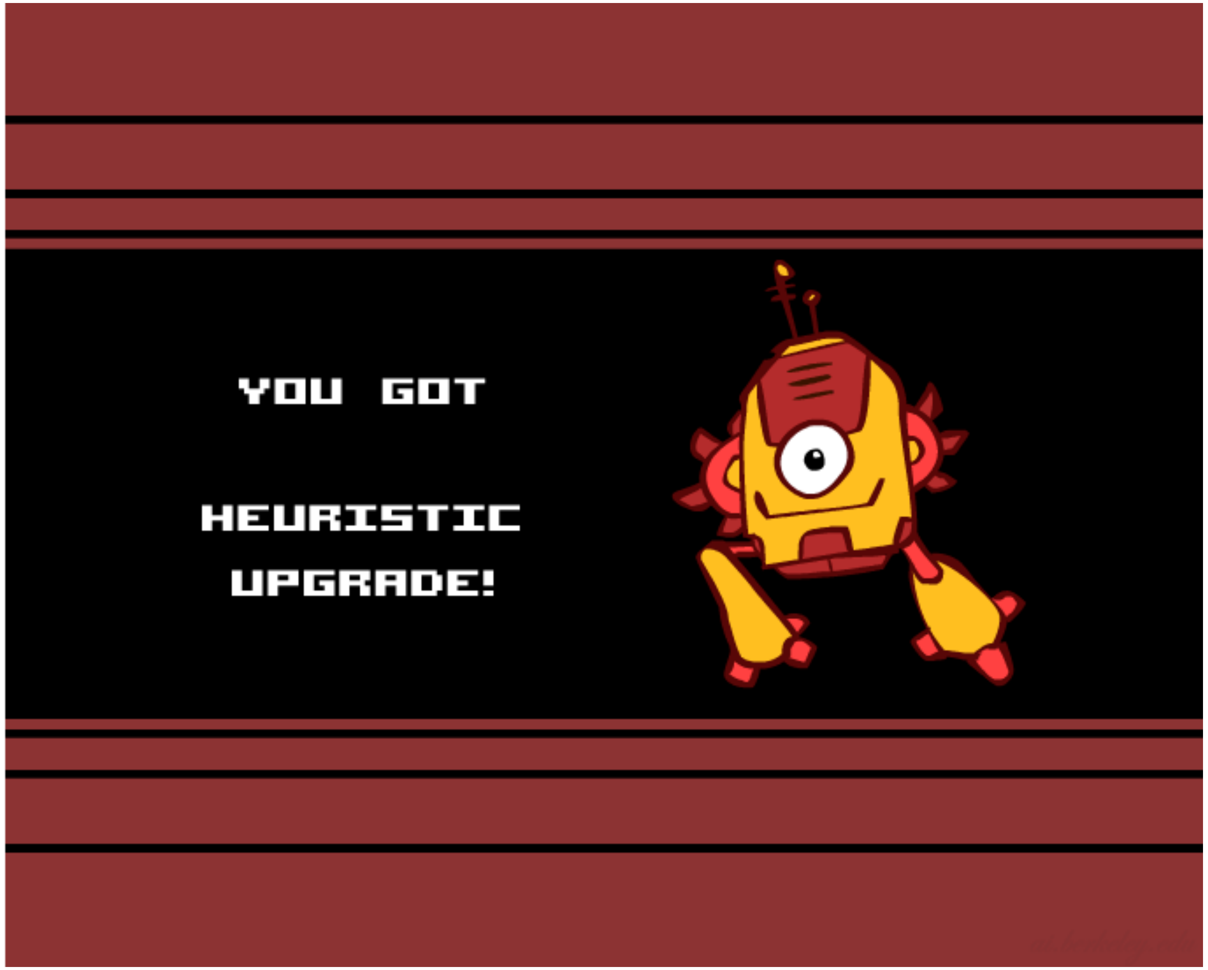
- A* uses both backward costs and (estimates of) forward costs

  g h

- ~~A* is optimal~~ with admissible (optimistic) heuristics
- Heuristic design is key: often use relaxed problems



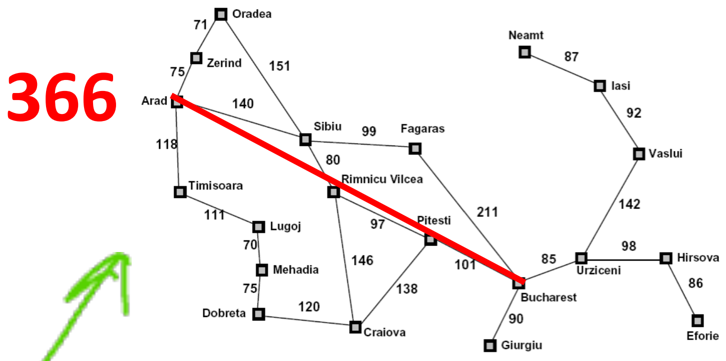
Creating Heuristics

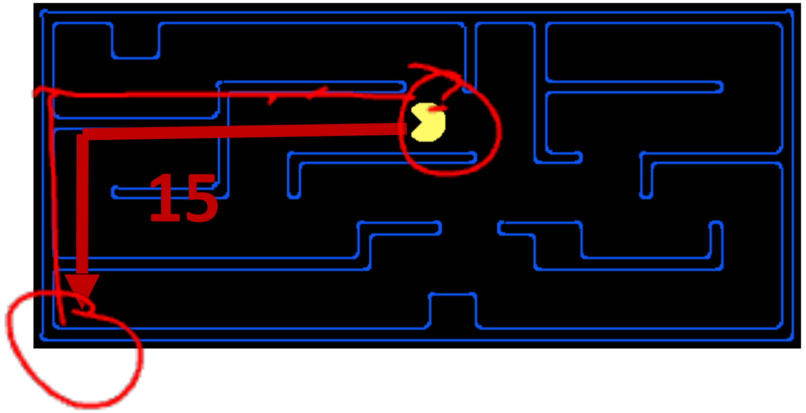


Creating Admissible Heuristics

- Most of the work in solving hard search problems optimally is in coming up with admissible heuristics

- Often, admissible heuristics are solutions to *relaxed problems*, where new actions are available



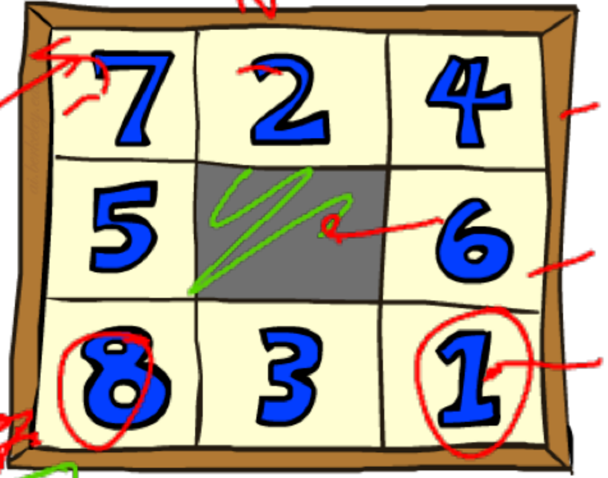


- Inadmissible heuristics are often useful too

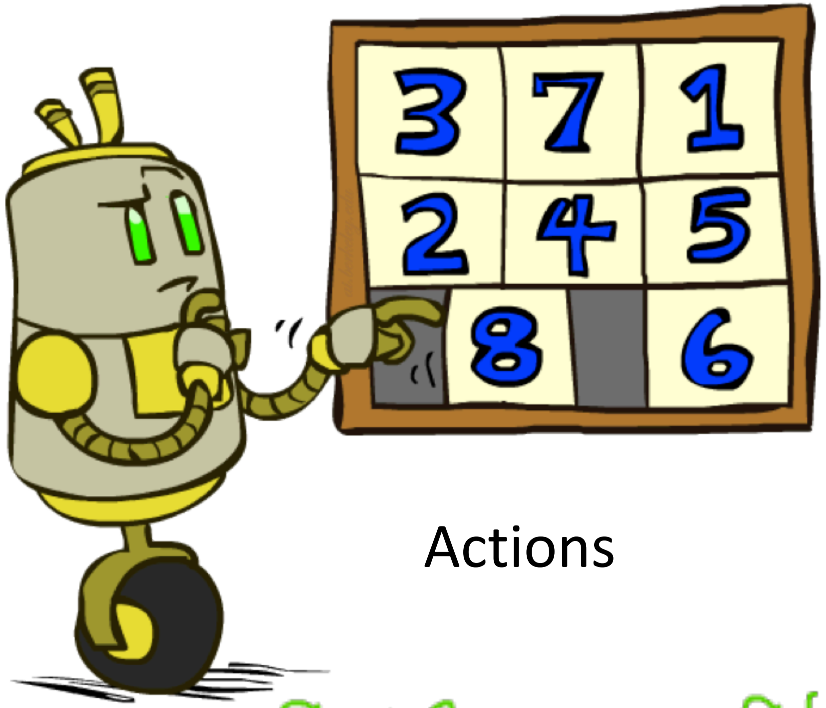
Example: 8 Puzzle

start state

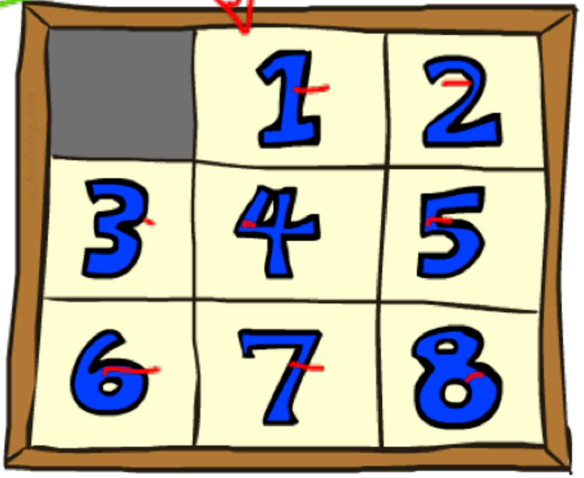
goal



Start State



Actions



Goal State

- What are the states?
- How many states?
- What are the actions?
- How many successors from the start state?
- What should the costs be?

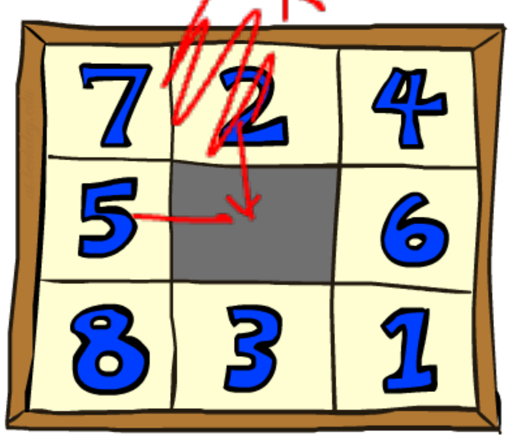
9 x 8 · · 9!

Admissible heuristics?

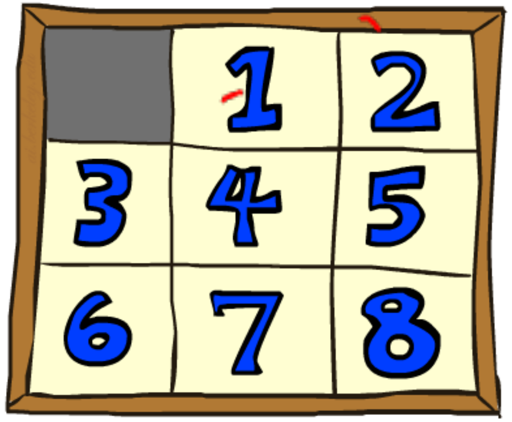
h=0

8 Puzzle I

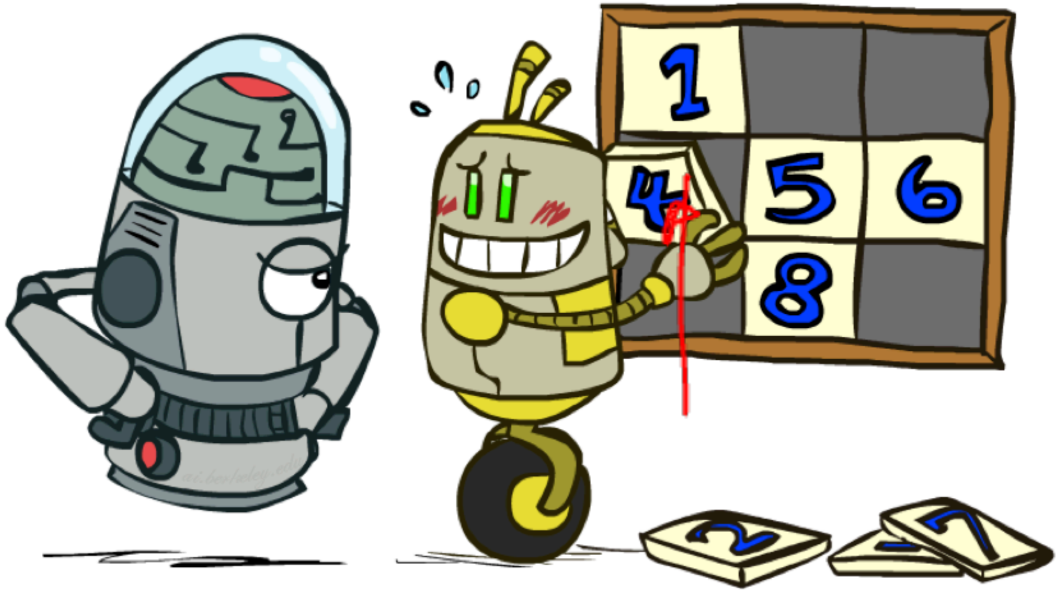
- Heuristic: Number of tiles misplaced
- Why is it admissible?
- h(start) = 8
- This is a *relaxed-problem* heuristic



Start State



Goal State



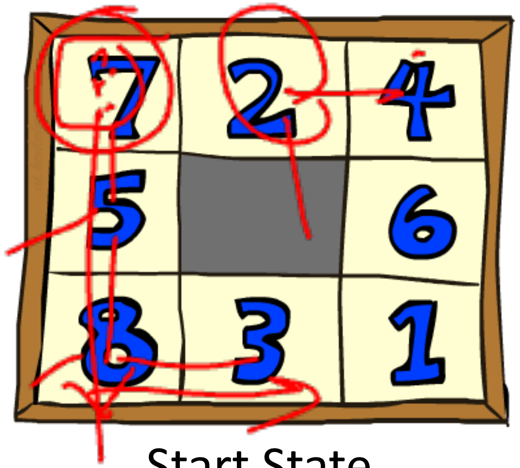
	Average nodes expanded when the optimal path has…		
	…4 steps	…8 steps	…12 steps
UCS	112	6,300	3.6×10^6
TILES	13	39	227

Statistics from Andrew Moore

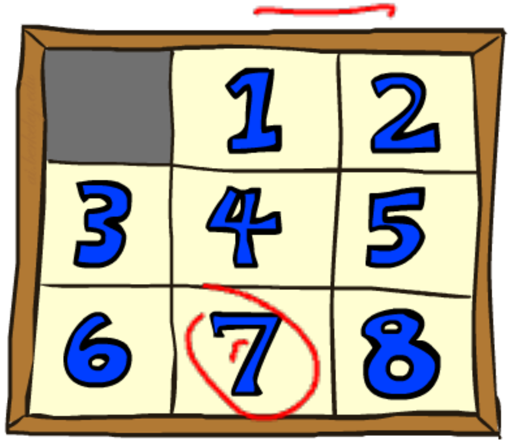
$0 < h_1 < h_2 < C^*$

8 Puzzle II

- What if we had an easier 8-puzzle where any tile could slide any direction at any time, ignoring other tiles?
- ~~Total *Manhattan* distance~~
- Why is it admissible?
- h(start) = 3 + 1 + 2 + … = 18



Start State

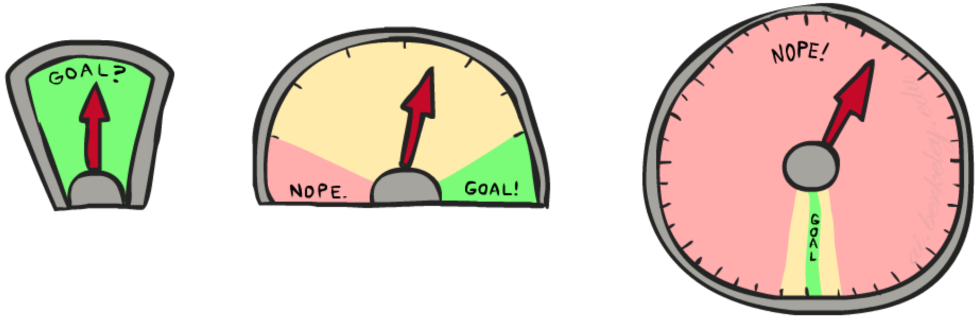


Goal State

	Average nodes expanded when the optimal path has…		
	…4 steps	…8 steps	…12 steps
TILES	13	39	227
MANHATTAN	12	25	73

8 Puzzle III

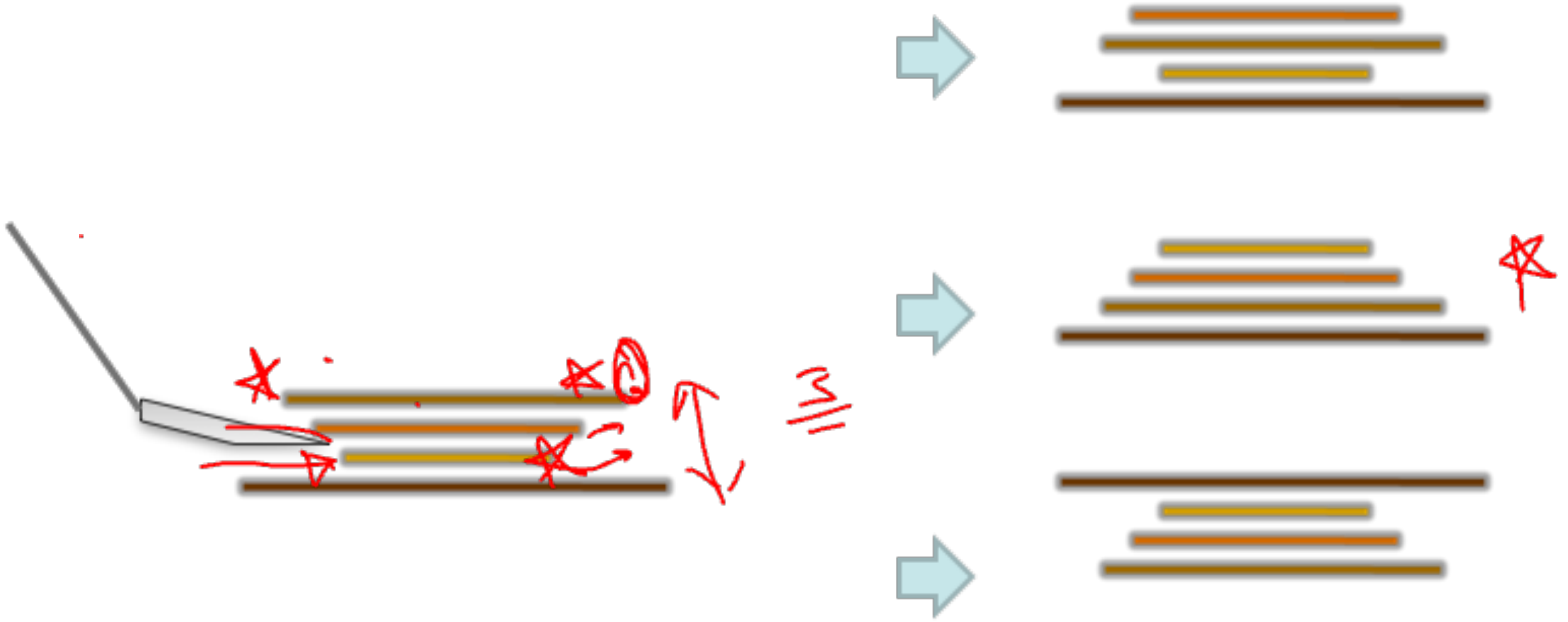
- How about using the *actual cost* as a heuristic?
  - Would it be admissible?
  - Would we save on nodes expanded?
  - What's wrong with it?



- With A*: a trade-off between quality of estimate and work per node
  - As heuristics get closer to the true cost, you will expand fewer nodes but usually do more work per node to compute the heuristic itself

Example: Pancake Problem

- Action: Flip over top n pancakes



- Cost: Number of pancakes

Semi-Lattice of Heuristics

Trivial Heuristics, Dominance

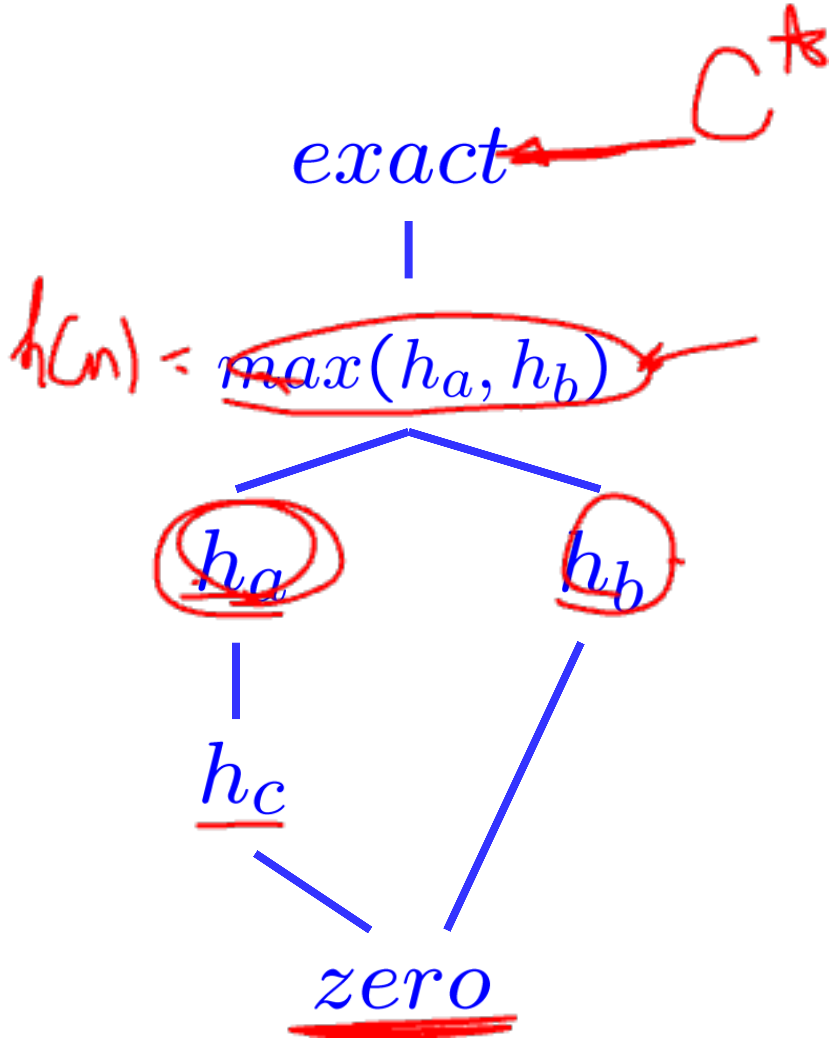
- Dominance: $h_a \geq h_c$ if

$$\forall n : h_a(n) \geq h_c(n)$$

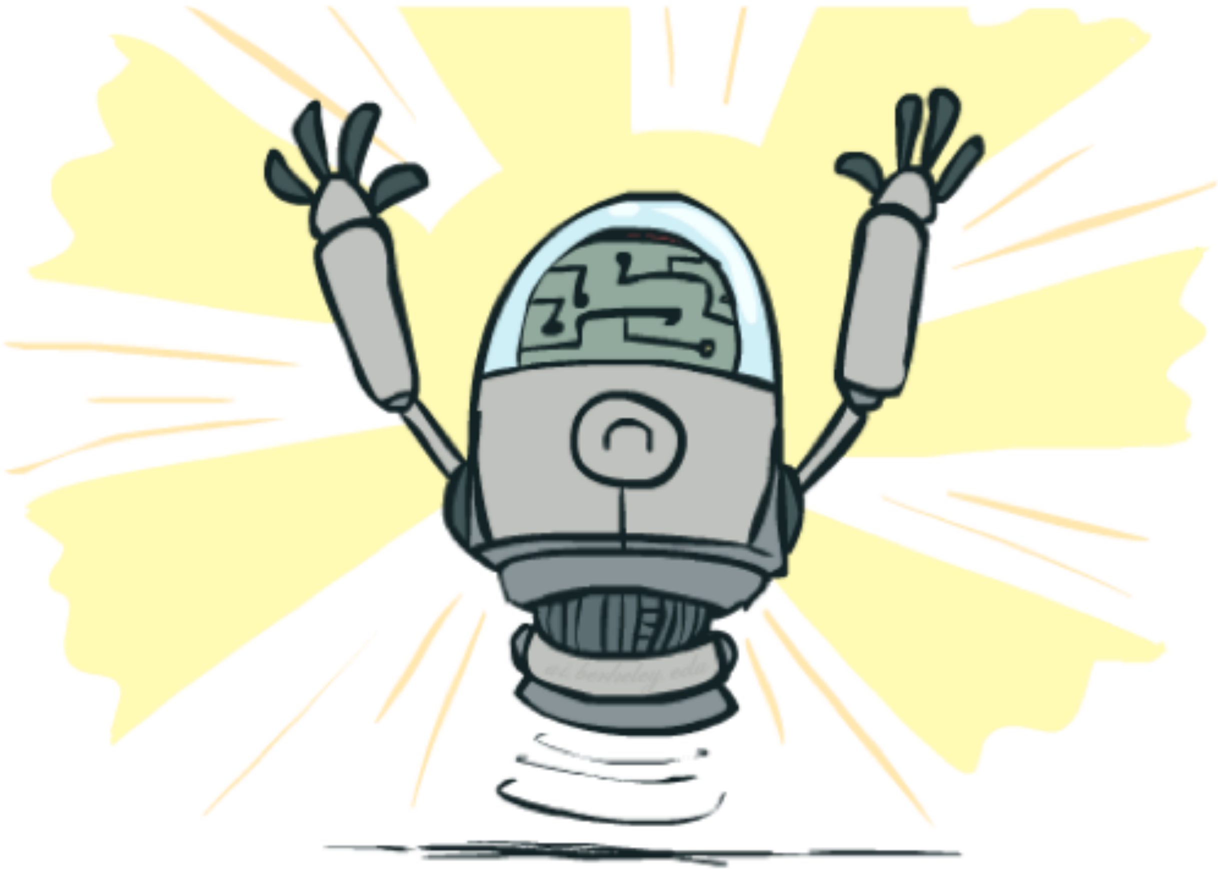
- Heuristics form a semi-lattice:
  - Max of admissible heuristics is admissible

$$h(n) = max(h_a(n), h_b(n))$$

- Trivial heuristics
  - Bottom of lattice is the zero heuristic (what does this give us?)
  - Top of lattice is the exact heuristic



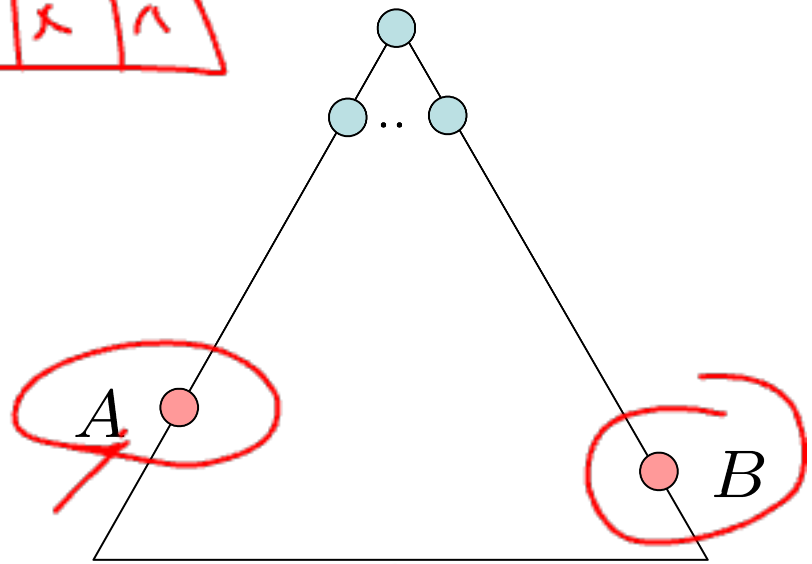
Optimality of A* Tree Search



Optimality of A* Tree Search

Assume:

- A is an optimal goal node
- B is a suboptimal goal node
- h is admissible

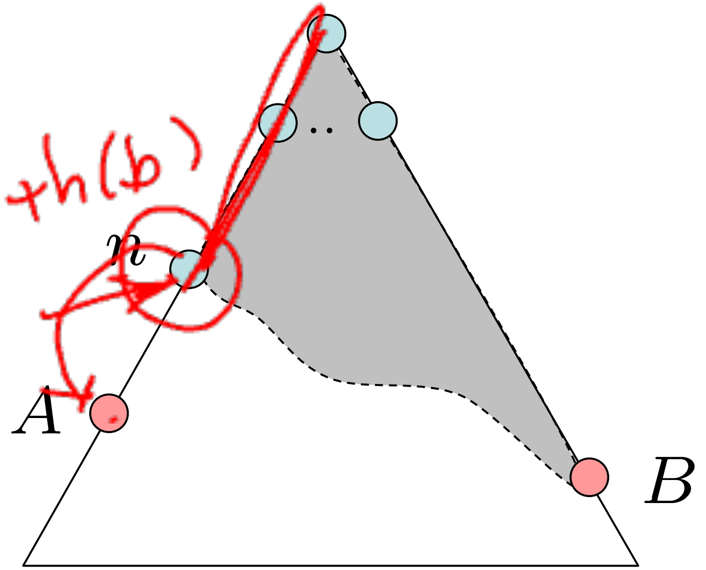


Proof Sketch:

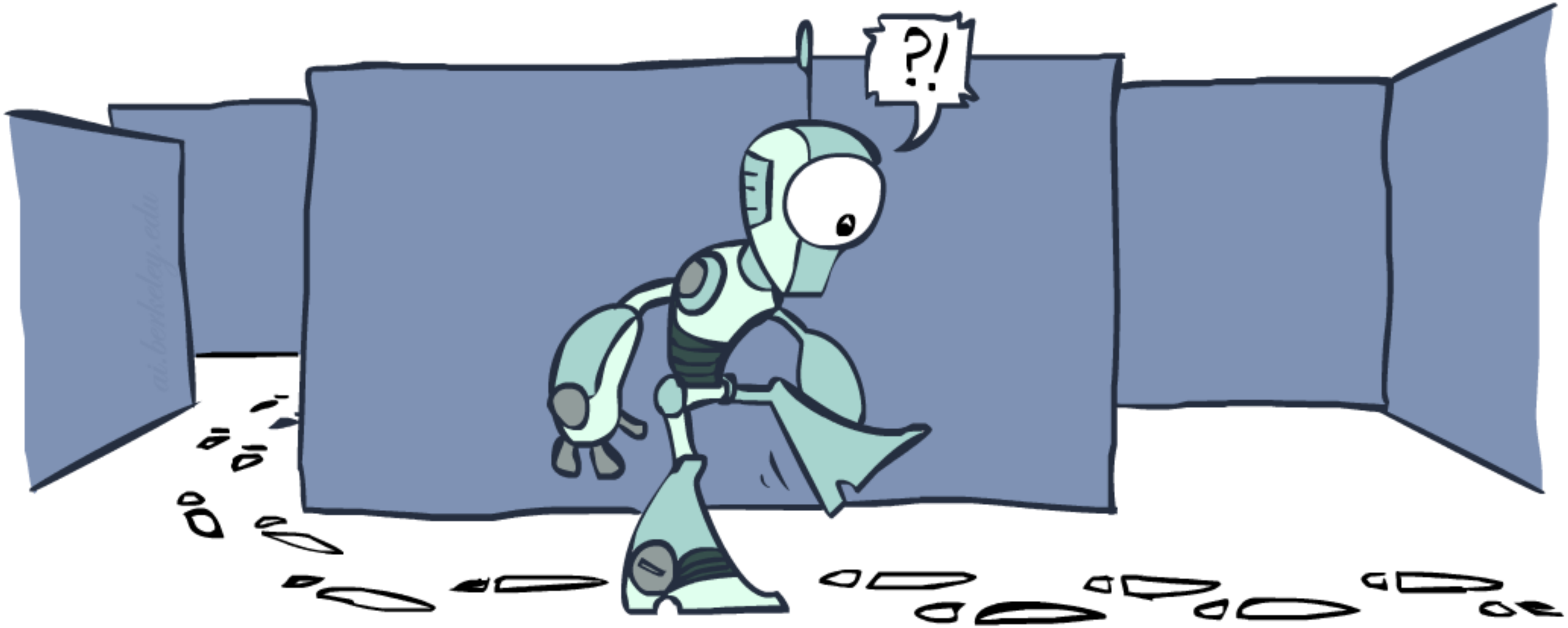
- All ancestors of A will exit the fringe before B
  - Because f(n) < f(B)
- A will exit the fringe before B

$g(n) + h(n) < g(b) + h(b)$

$< f(A)$

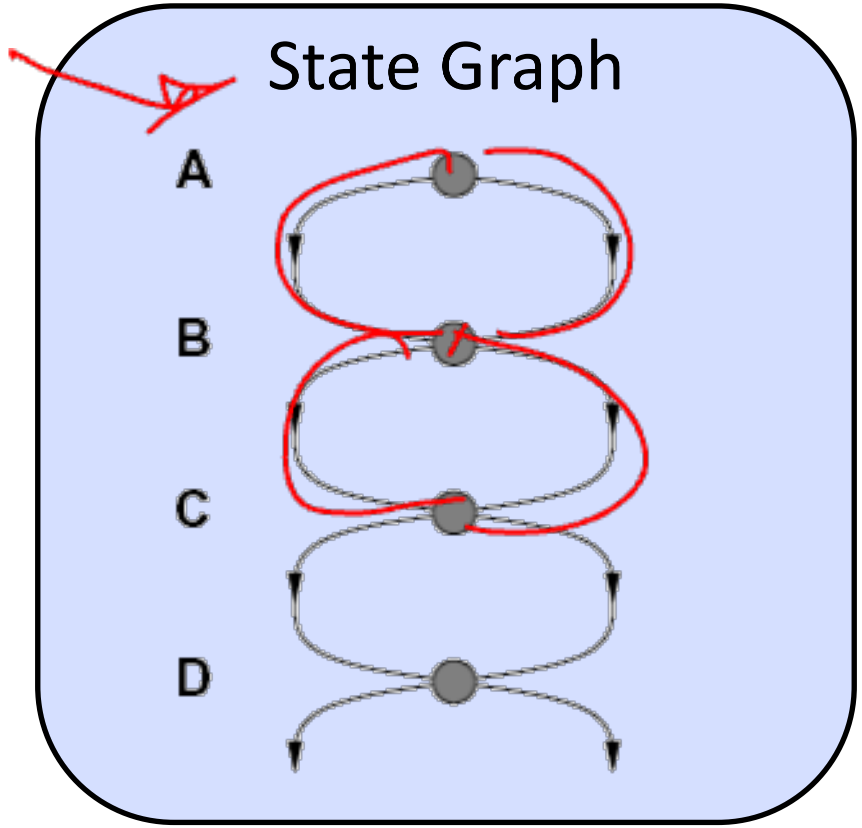


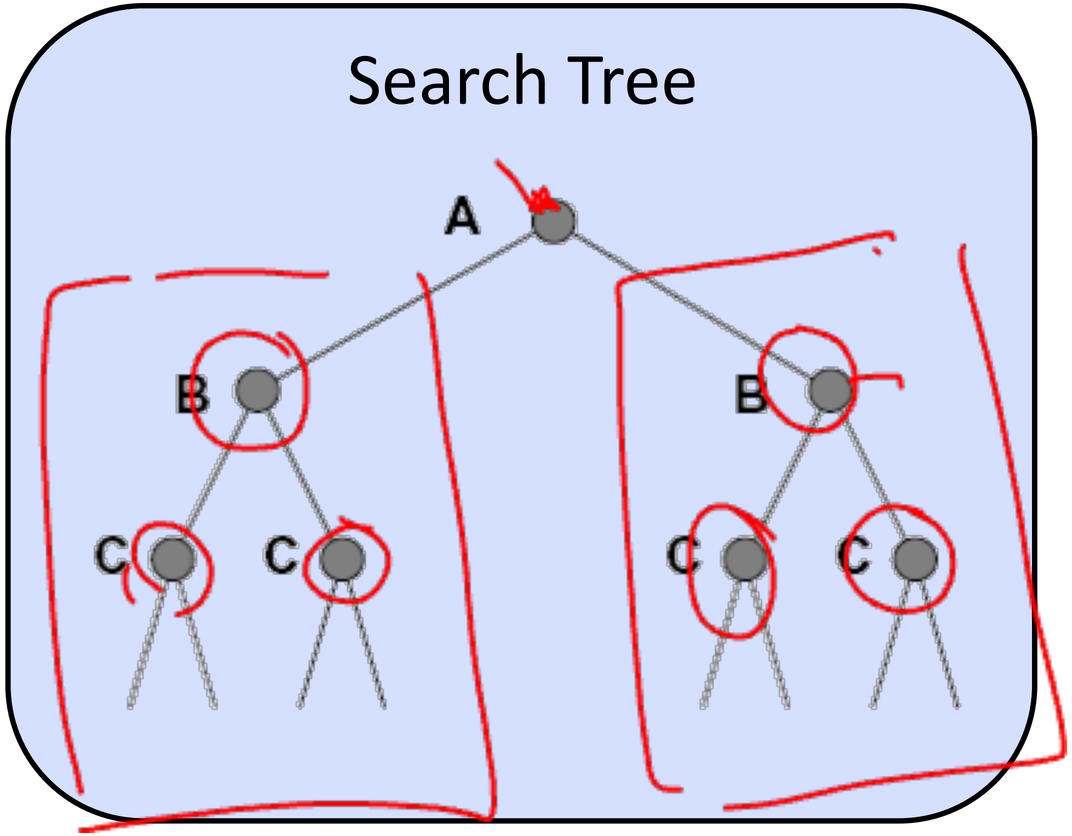
Graph Search



Tree Search: Extra Work!

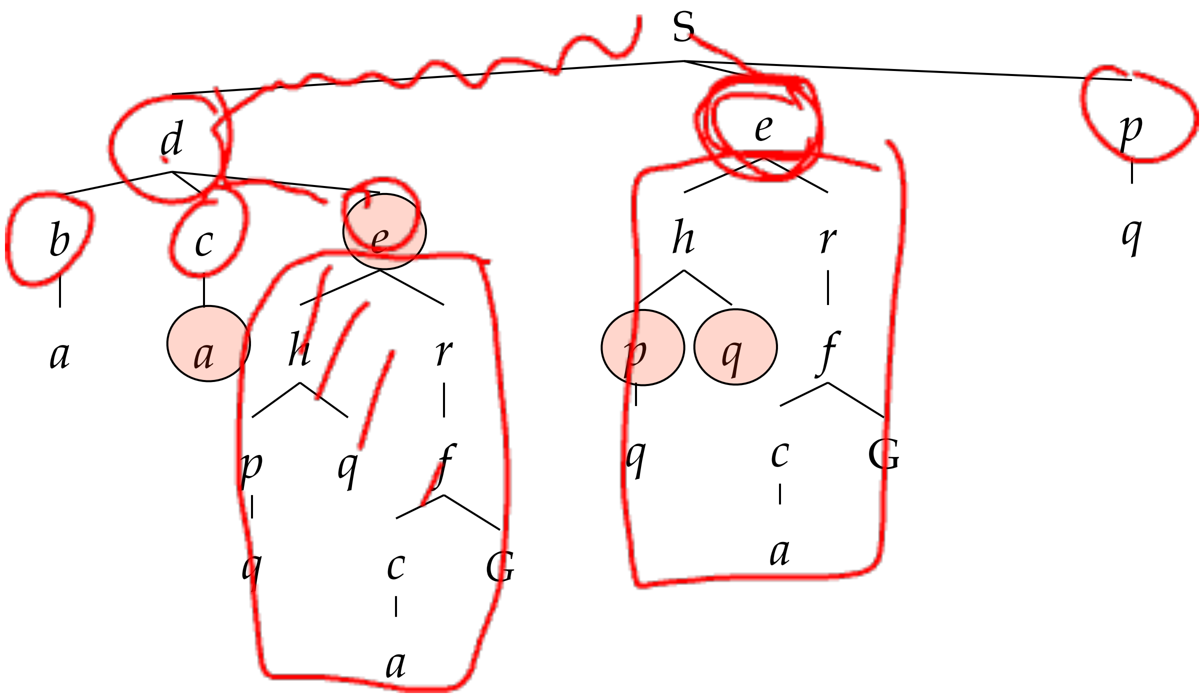
- Failure to detect repeated states can cause exponentially more work.





Graph Search

- In BFS, for example, we shouldn't bother expanding the circled nodes (why?)

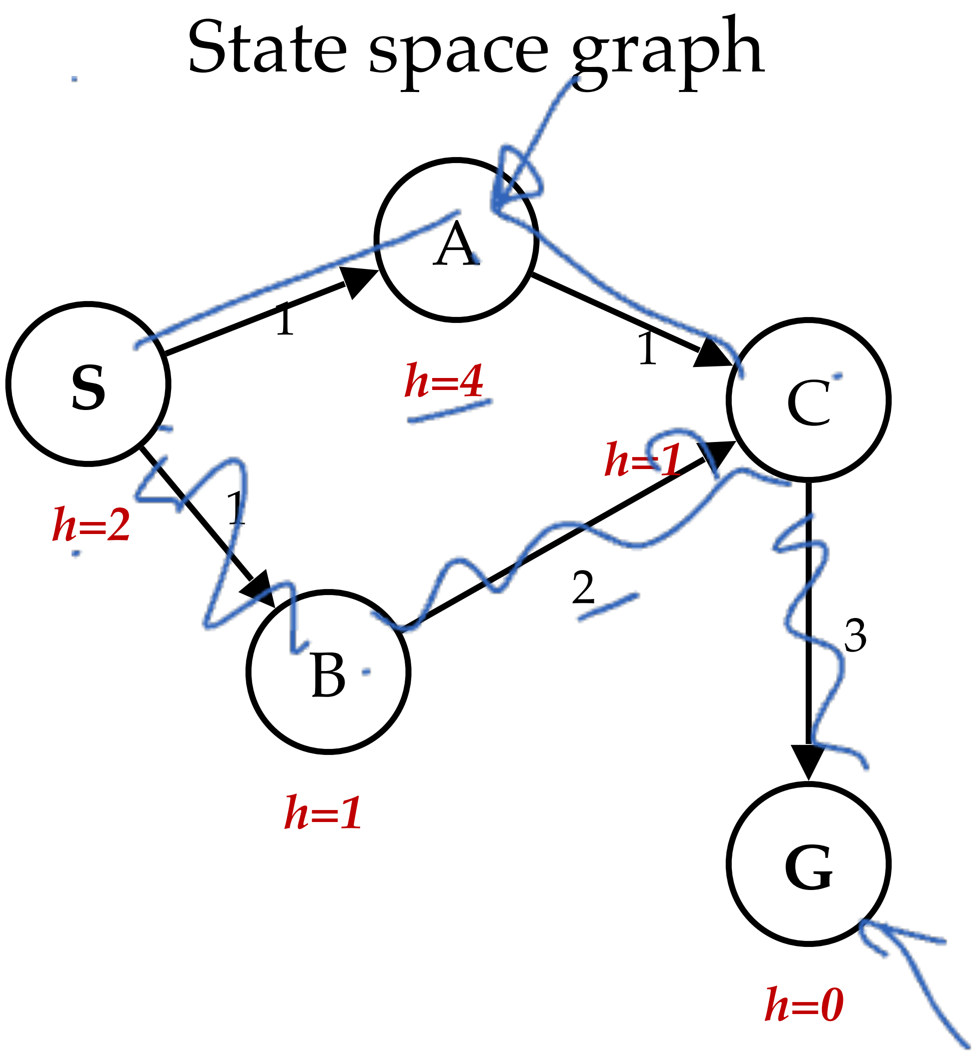


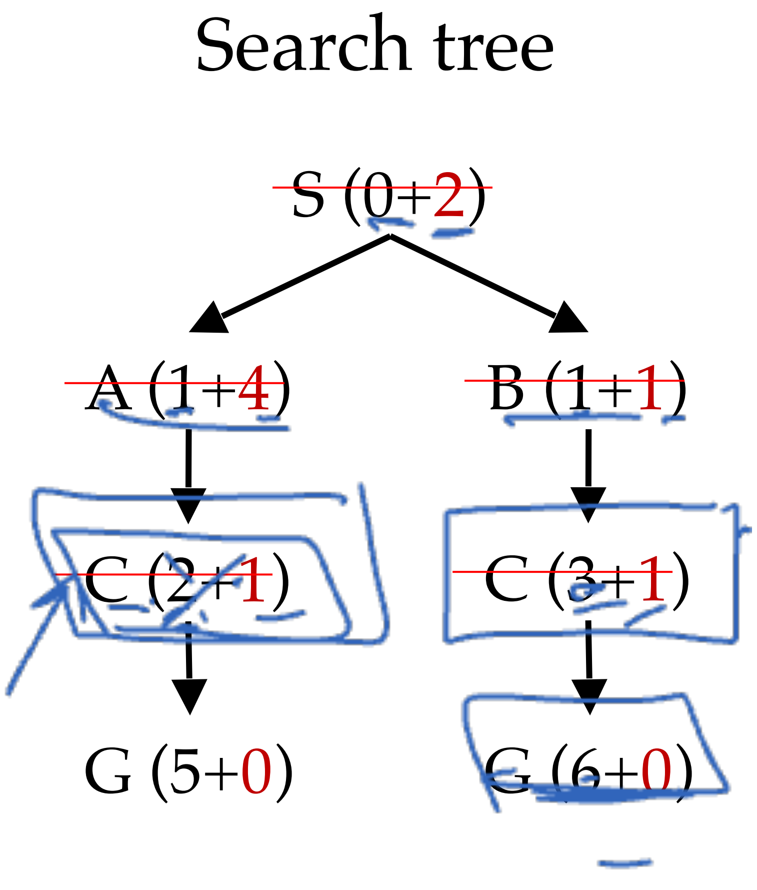
Graph Search

- Idea: never expand a state twice
- How to implement:
  - Tree search + set of expanded states (“closed set”)
  - Expand the search tree node-by-node, but…
  - Before expanding a node, check to make sure its state has never been expanded before
  - If not new, skip it, if new add to closed set

- Important: store the closed set as a set, not a list
- Can graph search wreck completeness? Why/why not?
- How about optimality?

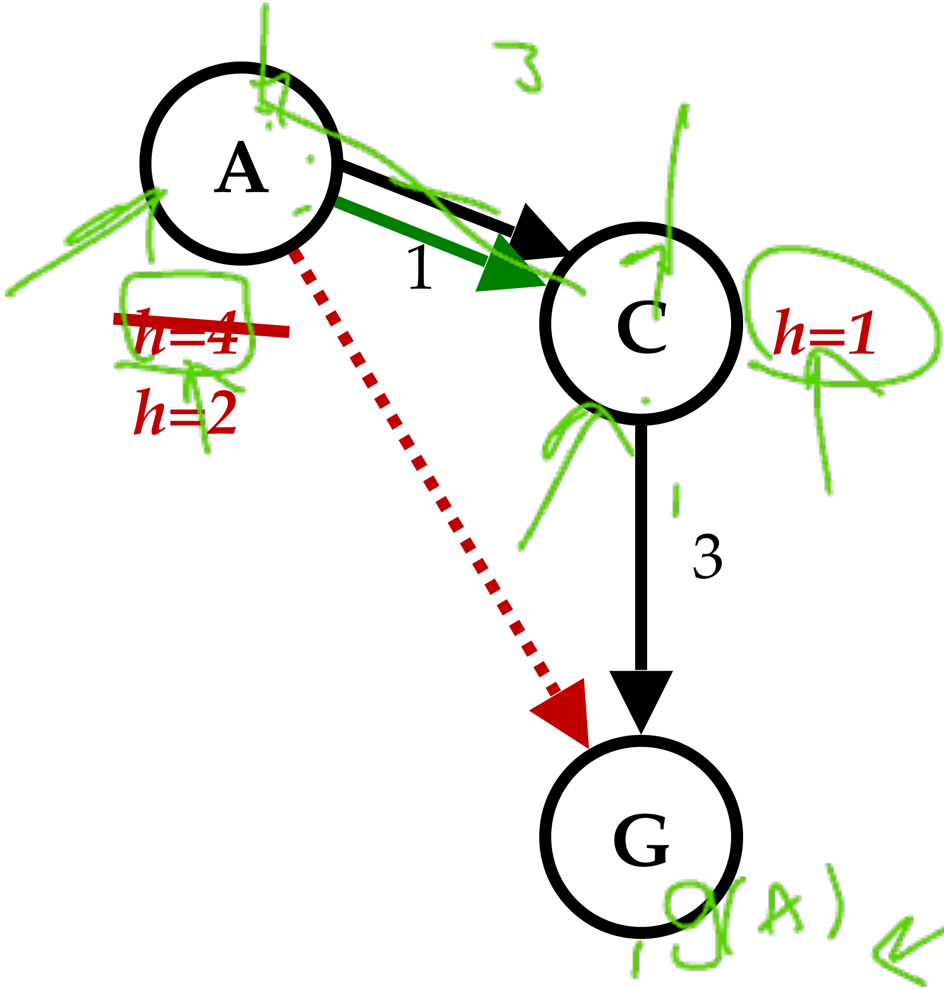
A* Graph Search Gone Wrong?





Closed Set: S B C A

Consistency of Heuristics



- Main idea: estimated heuristic costs ≤ actual costs
  - Admissibility: heuristic cost ≤ actual cost to goal

    h(A) ≤ actual cost from A to G
  - Consistency: heuristic "arc" cost ≤ actual cost for each arc

    h(A) – h(C) ≤ cost(A to C)
- Consequences of consistency:
  - The f value along a path never decreases

    h(A) ≤ cost(A to C) + h(C)
  - A* graph search is optimal

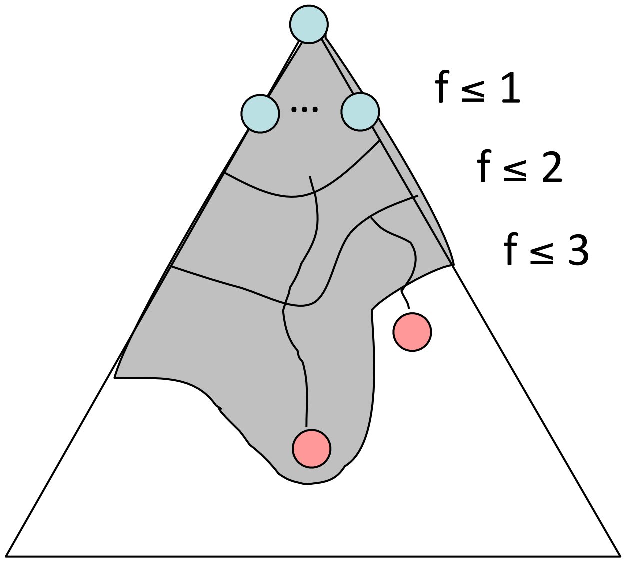
g(A)

A* Graph Search

- Sketch: consider what A* does with a consistent heuristic:

  graph

  - Fact 1: In tree search, A* expands nodes in increasing total f value (f-contours)
  - Fact 2: For every state s, nodes that reach s optimally are expanded before nodes that reach s suboptimally
  - Result: A* graph search is optimal



Optimality of A* Search

- With a admissible heuristic, Tree A* is optimal.
- With a consistent heuristic, Graph A* is optimal.
- With h=0, the same proof shows that UCS is optimal.

Pseudo-Code

```
function TREE-SEARCH(problem, fringe) return a solution, or failure
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, STATE[node]) then return node
        for child-node in EXPAND(STATE[node], problem) do
            fringe ← INSERT(child-node, fringe)
        end
    end
```

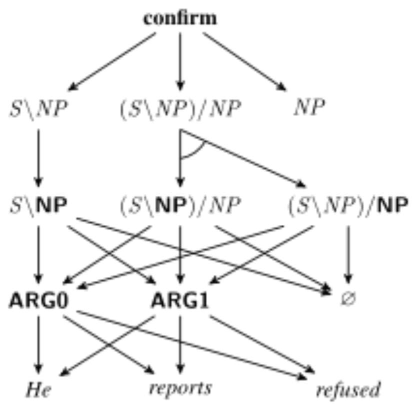
```
function GRAPH-SEARCH(problem, fringe) return a solution, or failure
    closed ← an empty set
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, STATE[node]) then return node
        if STATE[node] is not in closed then
            add STATE[node] to closed
            for child-node in EXPAND(STATE[node], problem) do
                fringe ← INSERT(child-node, fringe)
            end
    end
```

A* Applications

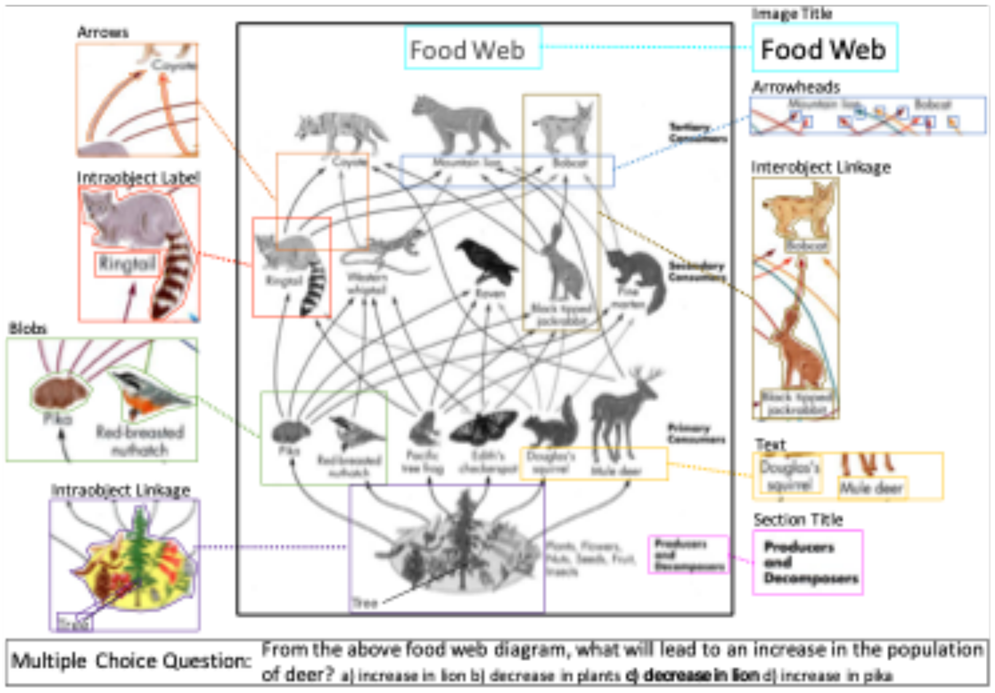
- Video games
- Pathing / routing problems
- Resource planning problems
- Robot motion planning
- Language analysis
- Machine translation
- Speech recognition
- …

A* in Recent Literature

- Joint A* CCG Parsing and Semantic Role Labeling (EMLN'15)

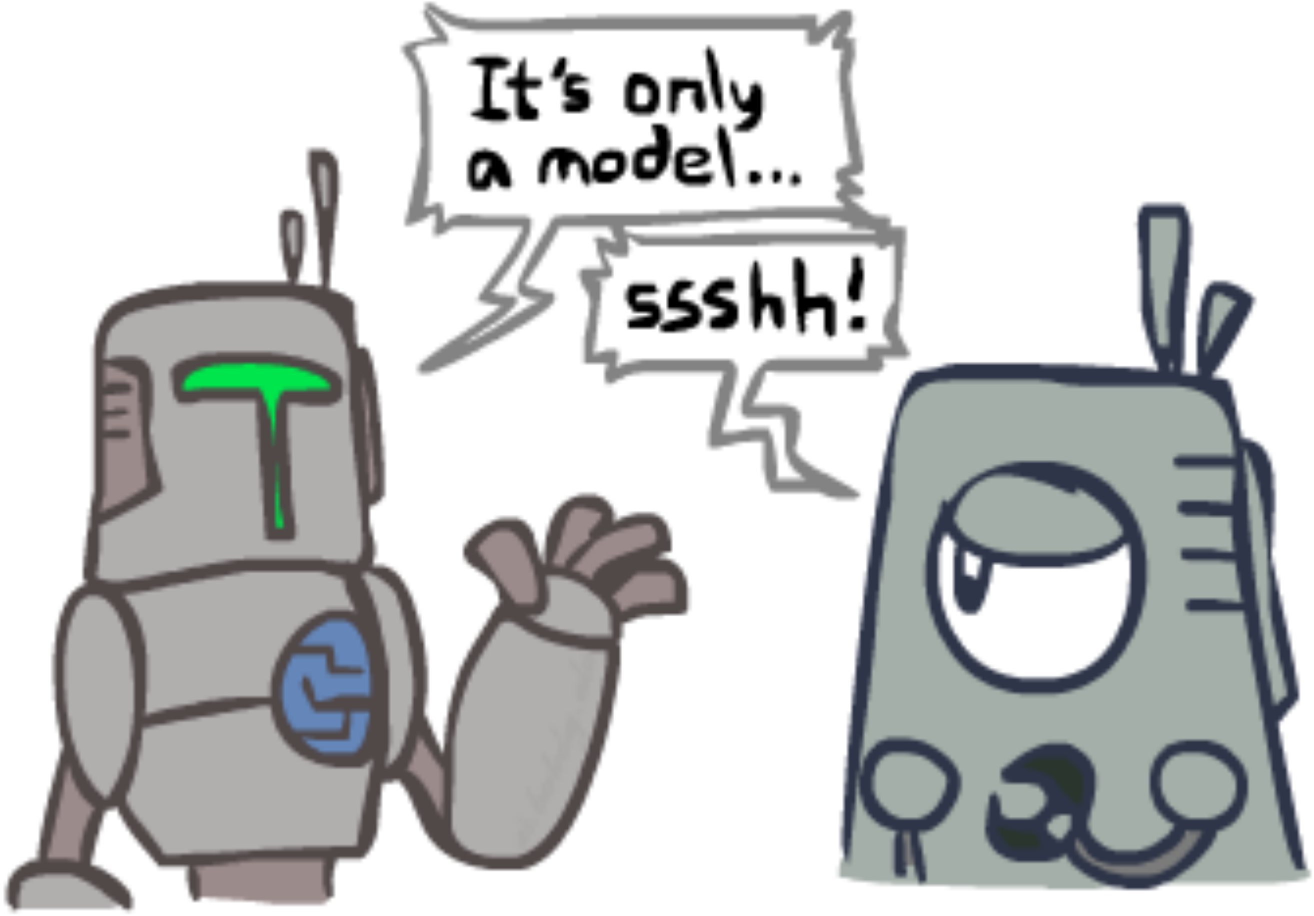


- Diagram Understanding (ECCV'17)



Search and Models

- Search operates over models of the world
  - The agent doesn't actually try all the plans out in the real world!
  - Planning is all "in simulation"
  - Your search is only as good as your models…



Search Gone Wrong?