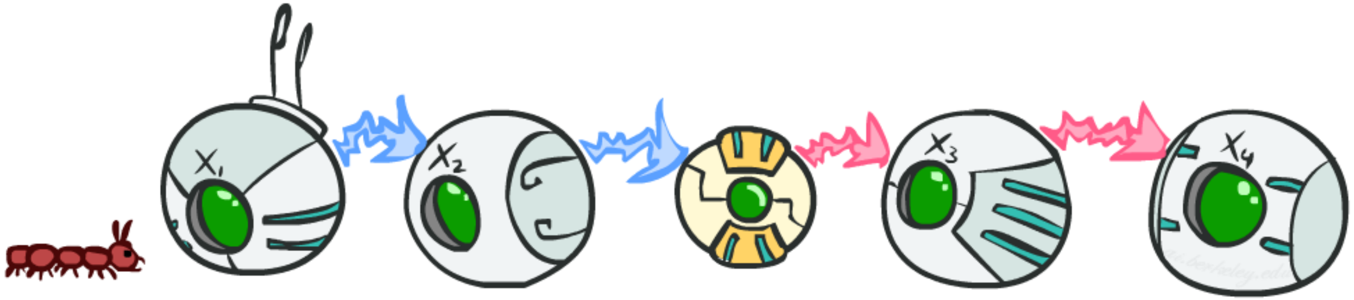
CSE 473: Artificial Intelligence

Hidden Markov Models



slides adapted from
Stuart Russel, Dan Klein, Pieter Abbeel from ai.berkeley.edu
And Hanna Hajishirzi, Jared Moore, Dan Weld

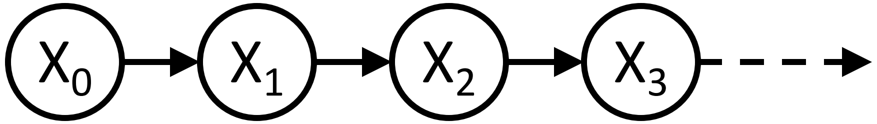
Uncertainty and Time

- Often, we want to reason about a ***sequence*** of observations
  - Speech recognition
  - Robot localization
  - User attention
  - Medical monitoring
- Generalize MDPs by adding sensing noise (and removing actions)

Video of Demo Pacman – Sonar

Markov Models (aka Markov chain/process)

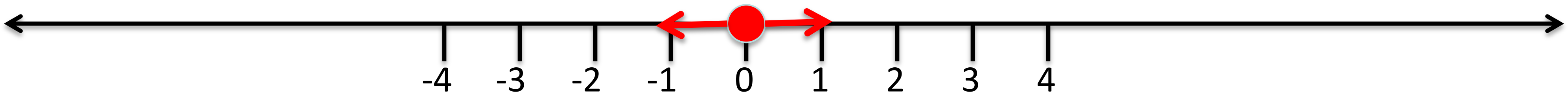
- Value of X at a given time is called the ***state*** (usually discrete, finite)



$\mathbf{P}(X_0)$ $\mathbf{P}(X_t \mid X_{t-1})$

- The ***transition model*** $\mathbf{P}(X_t \mid X_{t-1})$ specifies how the state evolves over time
- ***Stationarity*** assumption: transition probabilities are the same at all times
- ***Markov*** assumption: "future is independent of the past given the present"
  - X_{t+1} is independent of $X_0,\ldots, X_{t-1}$ given X_t
  - This is a ***first-order*** Markov model (a *k*th-order model allows dependencies on *k* earlier steps)
- Joint distribution $\mathbf{P}(X_0,\ldots, X_T) = \mathbf{P}(X_0) \prod_t \mathbf{P}(X_t \mid X_{t-1})$

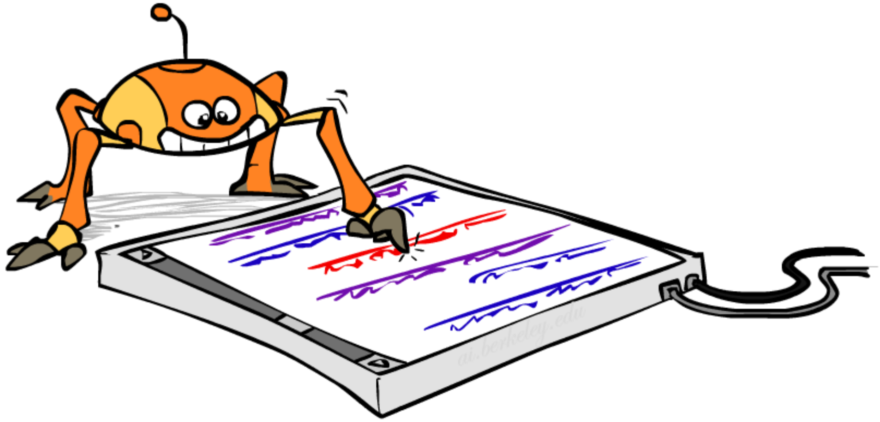
Example: Random walk in one dimension



- State: location on the unbounded integer line
- Initial probability: starts at 0
- Transition model: $P(X_t = k \mid X_{t-1} = k \pm 1) = 0.5$
- Applications: particle motion in crystals, stock prices, gambling, genetics, etc.
- Questions:
  - How far does it get as a function of t?
    - Expected distance is $O(\sqrt{t})$
  - Does it get back to 0 or can it go off for ever and not come back?
    - In 1D and 2D, returns w.p. 1; in 3D, returns w.p. 0.34053733

Example: Web browsing

- State: URL visited at step *t*
- Transition model:
  - With probability *p*, choose an outgoing link at random
  - With probability (1-*p*), choose an arbitrary new page
- Question: What is the ***stationary distribution*** over pages?
  - I.e., if the process runs forever, what fraction of time does it spend in any given page?
- Application: Google page rank



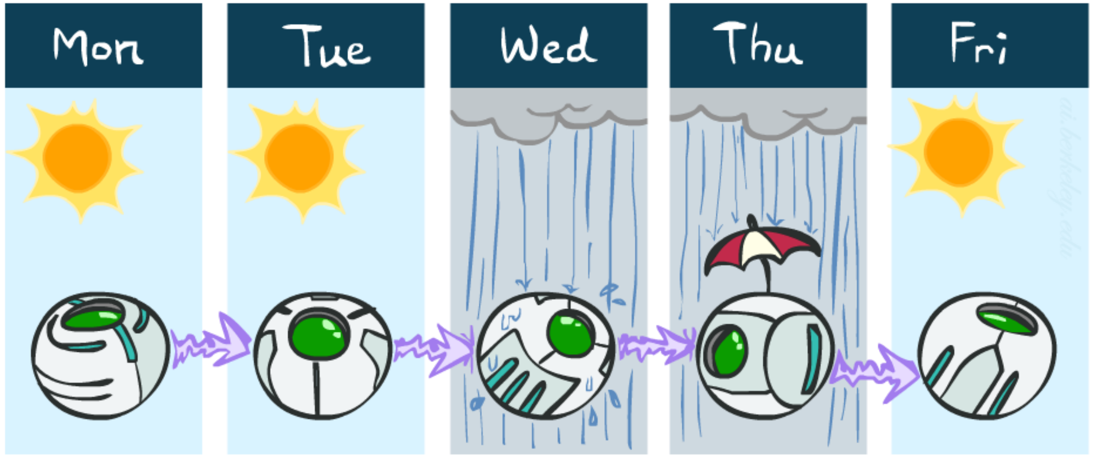
Example: Weather

- States {rain, sun}
- Initial distribution $P(X_0)$

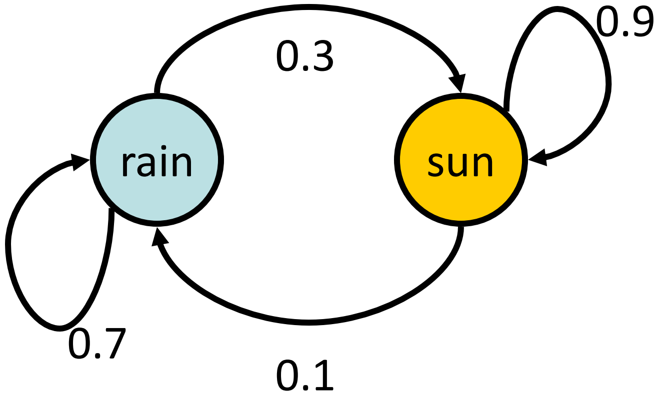
$P(X_0)$	
sun	rain
0.5	0.5

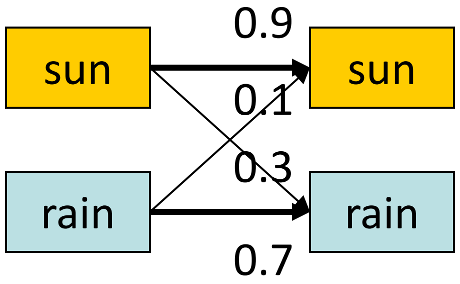
- Transition model $P(X_t \mid X_{t-1})$

X_{t-1}	$P(X_t \mid X_{t-1})$	
	sun	rain
sun	0.9	0.1
rain	0.3	0.7



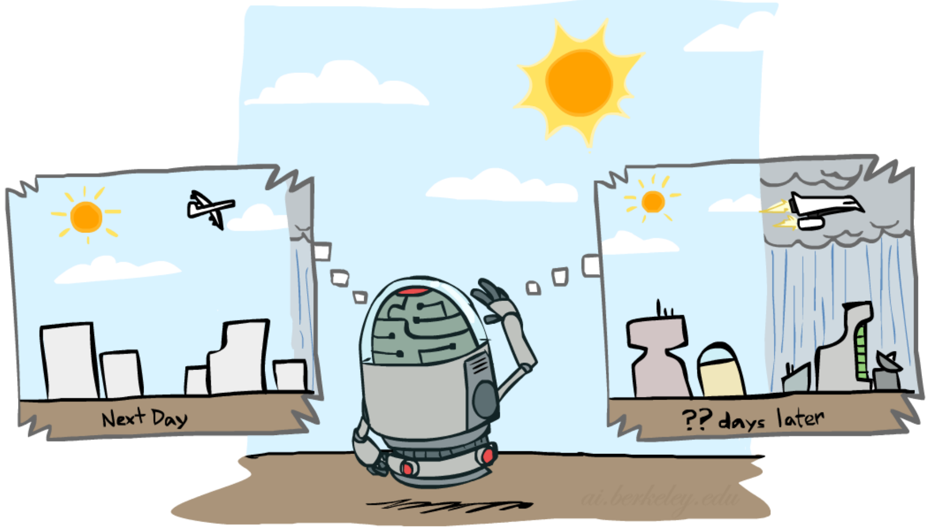
Two new ways of representing the same CPT





Weather prediction

X_{t-1}	$P(X_t \mid X_{t-1})$	
	sun	rain
sun	0.9	0.1
rain	0.3	0.7



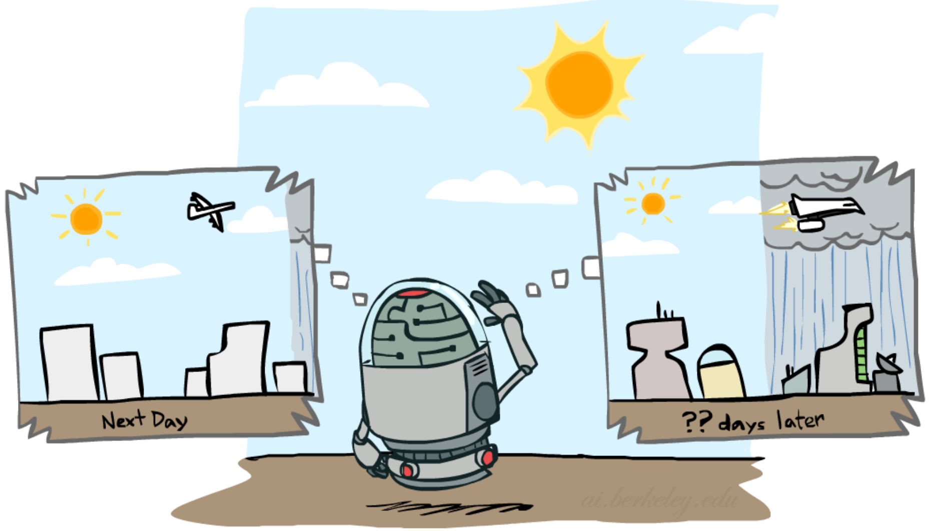
- Time 0: <0.5,0.5>

- What is the weather like at time 1?
  - $\mathbf{P}(X_1) = \sum_{x_0} \mathbf{P}(X_1, X_0 = x_0)$
    $= \sum_{x_0} P(X_0 = x_0)\, \mathbf{P}(X_1 \mid X_0 = x_0)$
    $= 0.5\langle 0.9, 0.1\rangle + 0.5\langle 0.3, 0.7\rangle = \langle 0.6, 0.4\rangle$

Weather prediction, contd.

- Time 1: <0.6,0.4>

X_{t-1}	$P(X_t \mid X_{t-1})$	
	sun	rain
sun	0.9	0.1
rain	0.3	0.7



- What is the weather like at time 2?
  - $\boldsymbol{P}(X_2) = \sum_{x_1} \boldsymbol{P}(X_2, X_1 = x_1)$

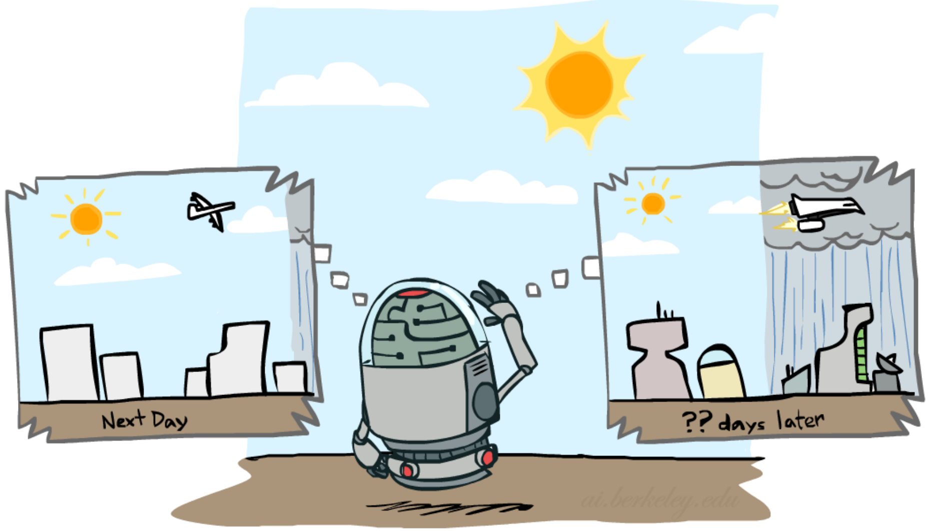
    $= \sum_{x_1} P(X_1 = x_1)\, \boldsymbol{P}(X_2 \mid X_1 = x_1)$

    $= 0.6\langle 0.9, 0.1\rangle + 0.4\langle 0.3, 0.7\rangle = \langle 0.66, 0.34\rangle$

Weather prediction, contd.

- Time 2: <0.66,0.34>

X_{t-1}	$P(X_t \mid X_{t-1})$	
	sun	rain
sun	0.9	0.1
rain	0.3	0.7



- What is the weather like at time 3?
  - $\mathbf{P}(X_3) = \sum_{x_2} \mathbf{P}(X_3, X_2 = x_2)$

    $= \sum_{x_2} P(X_2 = x_2)\, \mathbf{P}(X_3 \mid X_2 = x_2)$

    $= 0.66\langle 0.9, 0.1\rangle + 0.34\langle 0.3, 0.7\rangle = \langle 0.696, 0.304\rangle$

Forward algorithm (simple form)

- What is the state at time t?
  - $\mathbf{P}(X_t) = \sum_{x_{t-1}} \mathbf{P}(X_t, X_{t-1}=x_{t-1})$
    $= \sum_{x_{t-1}} P(X_{t-1}=x_{t-1})\, \mathbf{P}(X_t \mid X_{t-1}=x_{t-1})$
- Iterate this update starting at t=0

Probability from previous iteration

Transition model

And the same thing in linear algebra

- What is the weather like at time 2?

  $\mathbf{P}(X_2) = 0.6\langle 0.9, 0.1\rangle + 0.4\langle 0.3, 0.7\rangle = \langle 0.66, 0.34\rangle$

- In matrix-vector form:

$$\mathbf{P}(X_2) = \begin{pmatrix} \boxed{\begin{matrix} 0.9 \\ 0.1 \end{matrix}} & \begin{matrix} 0.3 \\ 0.7 \end{matrix} \end{pmatrix} \begin{pmatrix} 0.6 \\ 0.4 \end{pmatrix} = \begin{pmatrix} 0.66 \\ 0.34 \end{pmatrix}$$

X_{t-1}	$P(X_t \mid X_{t-1})$	
	sun	rain
sun	0.9	0.1
rain	0.3	0.7

- I.e., multiply by T^{T}, transpose of transition matrix

Stationary Distributions

- The limiting distribution is called the ***stationary distribution*** P_∞ of the chain
- It satisfies $P_\infty = P_{\infty+1} = T^T P_\infty$
- Solving for P_∞ in the example:

$$\begin{pmatrix} 0.9 & 0.3 \\ 0.1 & 0.7 \end{pmatrix} \begin{pmatrix} p \\ 1-p \end{pmatrix} = \begin{pmatrix} p \\ 1-p \end{pmatrix}$$

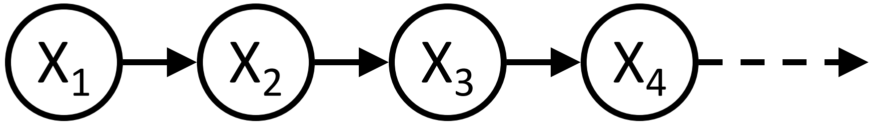
$0.9p + 0.3(1-p) = p$

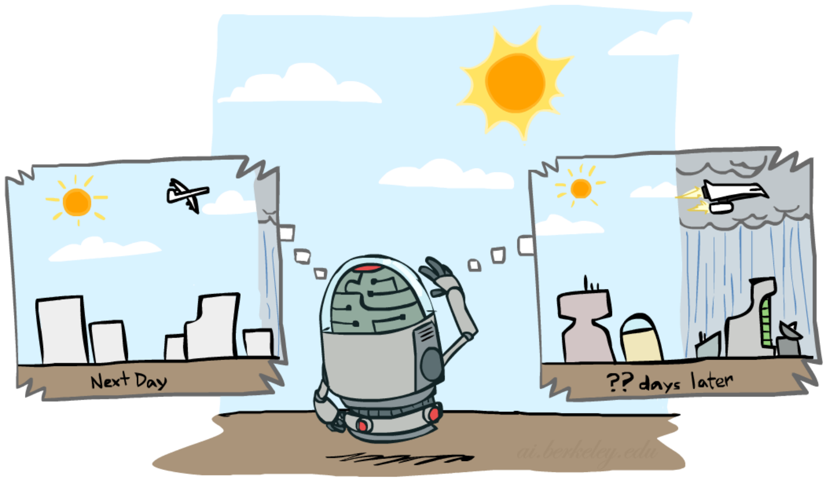
$p = 0.75$

Stationary distribution is <0.75,0.25> ***regardless of starting distribution***

Stationary Distributions

- Question: What's **P**(X) at time t = infinity?





$$P_\infty(sun) = P(sun|sun)P_\infty(sun) + P(sun|rain)P_\infty(rain)$$
$$P_\infty(rain) = P(rain|sun)P_\infty(sun) + P(rain|rain)P_\infty(rain)$$

$$P_\infty(sun) = 0.9P_\infty(sun) + 0.3P_\infty(rain)$$
$$P_\infty(rain) = 0.1P_\infty(sun) + 0.7P_\infty(rain)$$

$$P_\infty(sun) = 3P_\infty(rain)$$
$$P_\infty(rain) = 1/3P_\infty(sun)$$

Also: $P_\infty(sun) + P_\infty(rain) = 1$

$$\Rightarrow$$

$$P_\infty(sun) = 3/4$$
$$P_\infty(rain) = 1/4$$

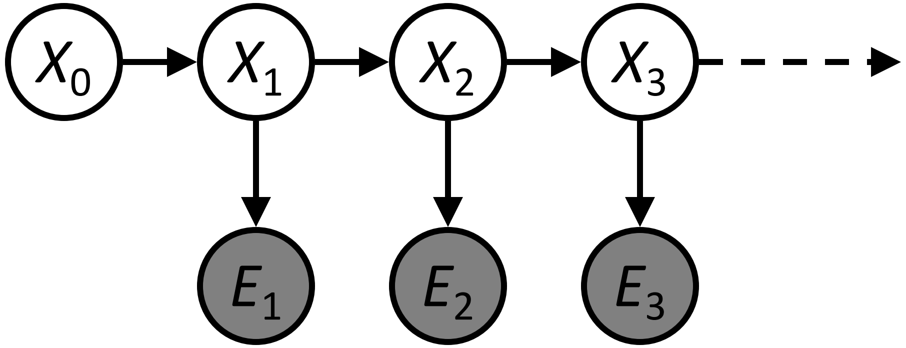
X_{t-1}	X_t	$P(X_t \mid X_{t-1})$
sun	sun	0.9
sun	rain	0.1
rain	sun	0.3
rain	rain	0.7

Hidden Markov Models

Hidden Markov Models

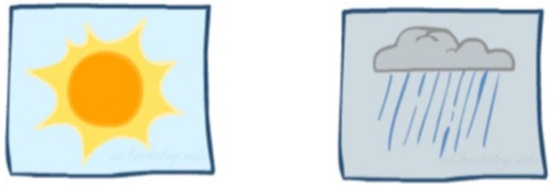
- Usually the true state is not observed directly

- Hidden Markov models (HMMs)
  - Underlying Markov chain over states X
  - You observe evidence E at each time step
  - X_t is a single discrete variable; E_t may be continuous and may consist of several variables

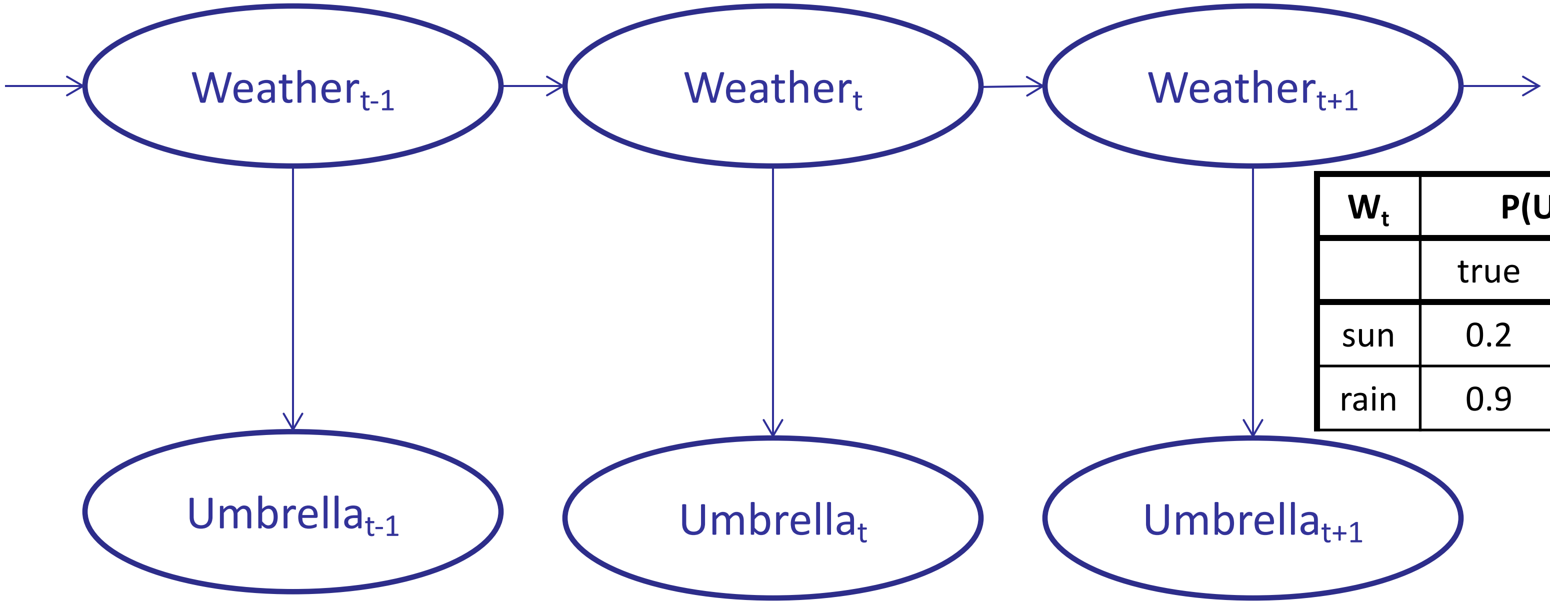


Example: Weather HMM

- An HMM is defined by:
  - Initial distribution: $\boldsymbol{P}(X_0)$
  - Transition model: $\boldsymbol{P}(X_t | X_{t-1})$
  - Sensor model: $\boldsymbol{P}(E_t | X_t)$



W_{t-1}	**$P(W_t \mid W_{t-1})$**	
	sun	rain
sun	0.9	0.1
rain	0.3	0.7



W_t	**$P(U_t \mid W_t)$**	
	true	false
sun	0.2	0.8
rain	0.9	0.1

HMM as probability model

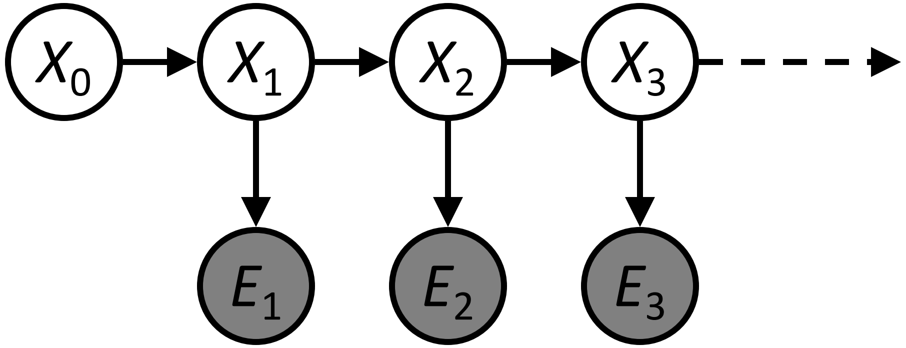
- Joint distribution for Markov model:

$$\mathbf{P}(X_0,\ldots,X_T) = \mathbf{P}(X_0) \prod_{t=1:T} \mathbf{P}(X_t \mid X_{t-1})$$

- Joint distribution for hidden Markov model:

$$\mathbf{P}(X_0,X_1,E_1,\ldots,X_T,E_T) = \mathbf{P}(X_0) \prod_{t=1:T} \mathbf{P}(X_t \mid X_{t-1})\, \boxed{\mathbf{P}(E_t \mid X_t)}$$

- Future states are independent of the past given the present
- Current evidence is independent of everything else given the current state
- **Question: Are evidence variables independent of each other?**



Useful notation:

$$X_{a:b} = X_a, X_{a+1}, \ldots, X_b$$

Real HMM Examples

- Speech recognition HMMs:
  - Observations are acoustic signals (continuous valued)
  - States are specific positions in specific words (so, tens of thousands)

- Machine translation HMMs:
  - Observations are words (tens of thousands)
  - States are translation options

- Robot tracking:
  - Observations are range readings (continuous)
  - States are positions on a map (continuous)

- Molecular biology:
  - Observations are nucleotides ACGT
  - States are coding/non-coding/start/stop/splice-site etc.

Inference tasks

- ***Filtering***: $\mathbf{P}(X_t | e_{1:t})$
  - ***belief state***—input to the decision process of a rational agent
- ***Prediction***: $\mathbf{P}(X_{t+k} | e_{1:t})$ for $k > 0$
  - evaluation of possible action sequences; like filtering without the evidence
- ***Smoothing***: $\mathbf{P}(X_k | e_{1:t})$ for $0 \leq k < t$
  - better estimate of past states, essential for learning
- ***Most likely explanation***: $\arg\max_{x_{1:t}} P(x_{1:t} \mid e_{1:t})$
  - speech recognition, decoding with a noisy channel

Filtering / Monitoring

- Filtering, or monitoring, or state estimation, is the task of maintaining the distribution $\boldsymbol{f}_{1:t} = \boldsymbol{P}(X_t | e_{1:t})$ over time

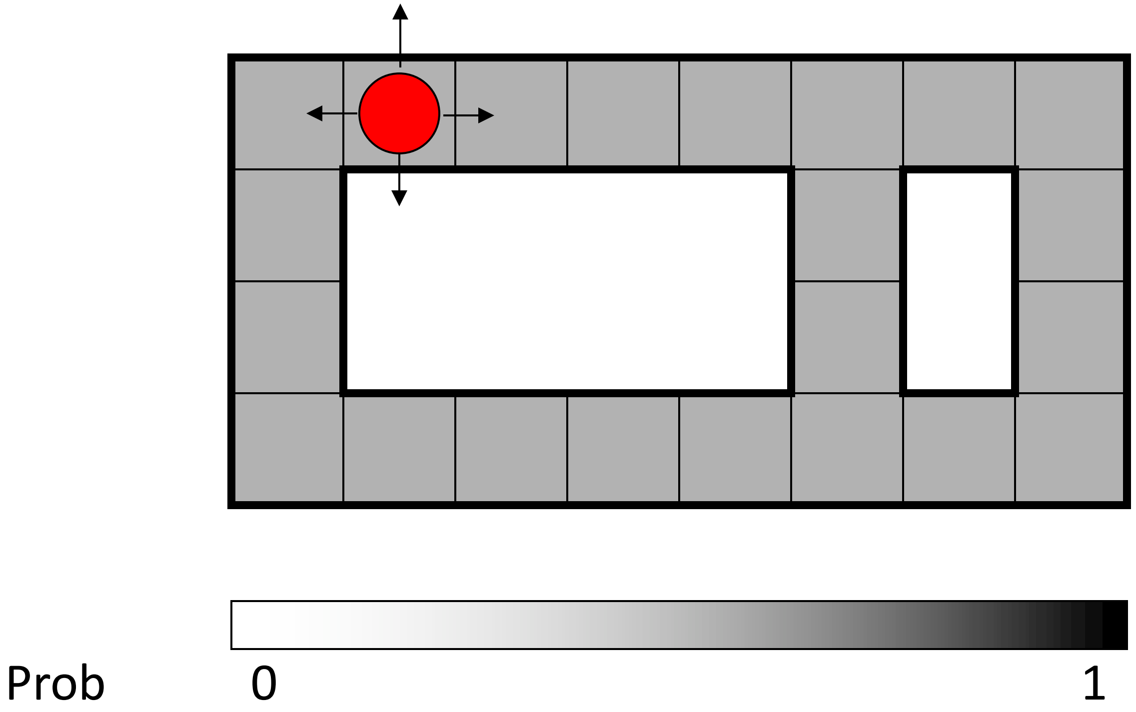
- We start with $\boldsymbol{f}_0$ in an initial setting, usually uniform

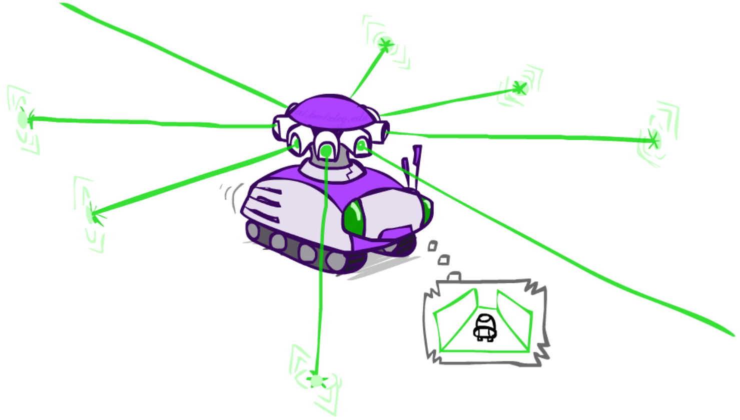
- Filtering is a fundamental task in engineering and science

- The Kalman filter (continuous variables, linear dynamics, Gaussian noise) was invented in 1960 and used for trajectory estimation in the Apollo program; core ideas used by Gauss for planetary observations

Example: Robot Localization

Example from
Michael Pfeiffer



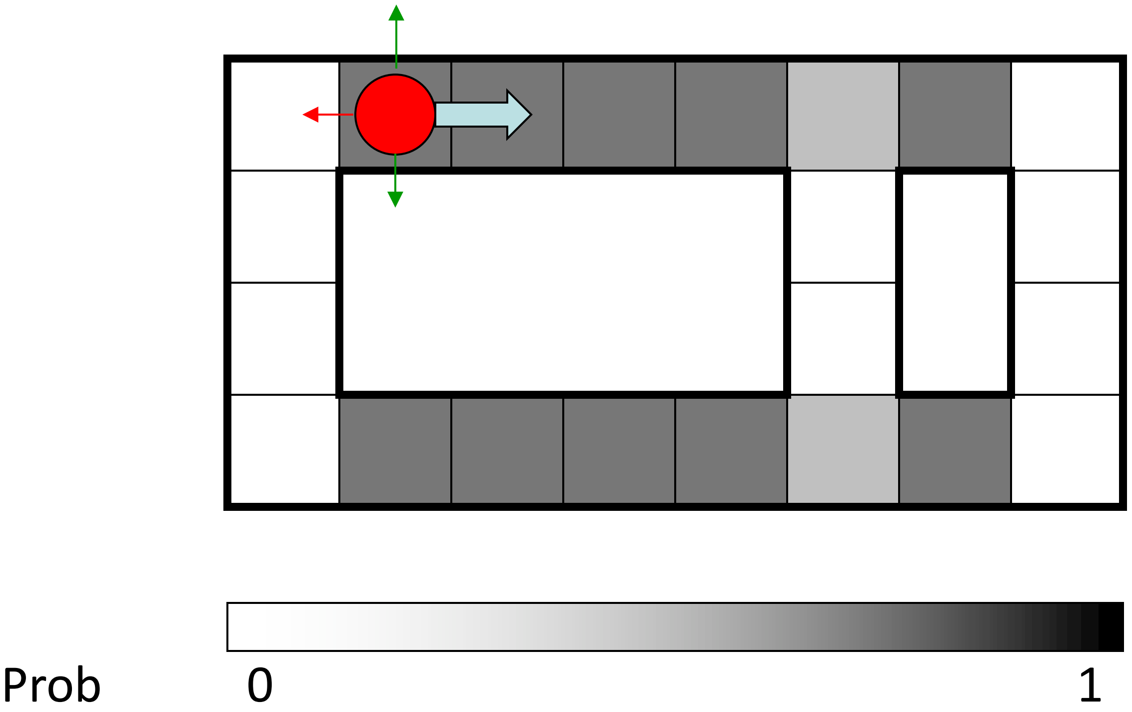


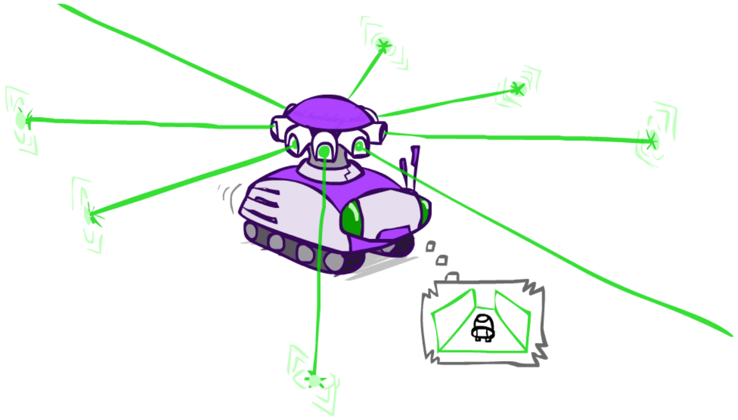
t=0

Sensor model: four bits for wall/no-wall in each direction, never more than 1 mistake

Transition model: action may fail with small prob.

Example: Robot Localization



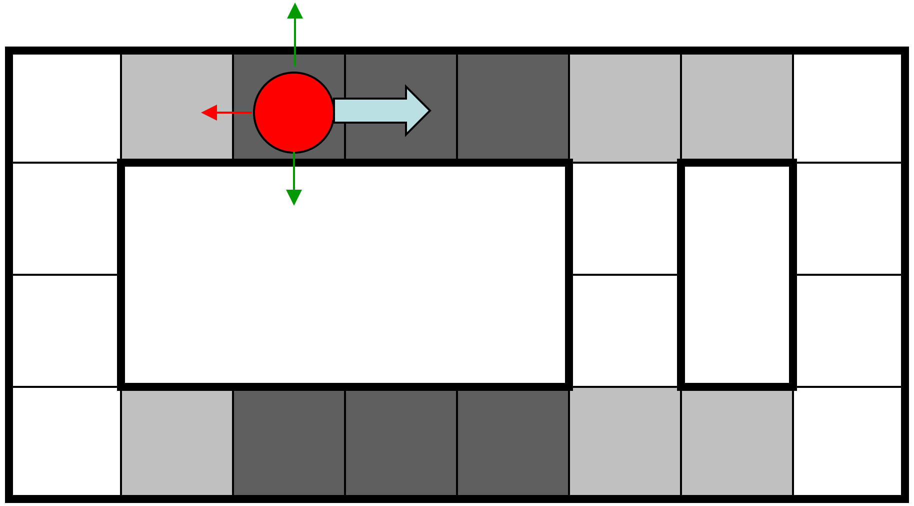


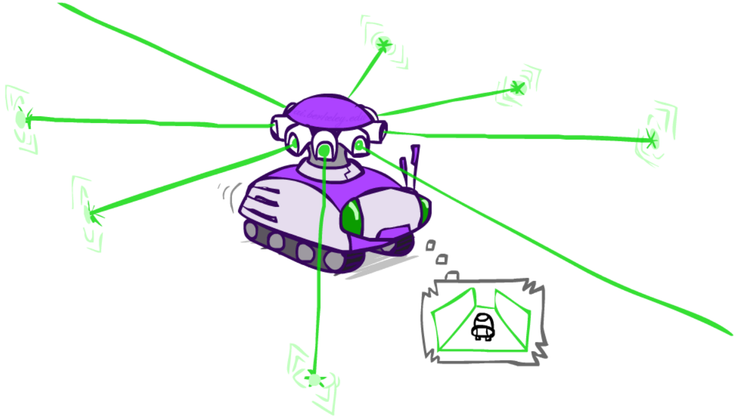
t=1

Lighter grey: was ***possible*** to get the reading,

but ***less likely*** (required 1 mistake)

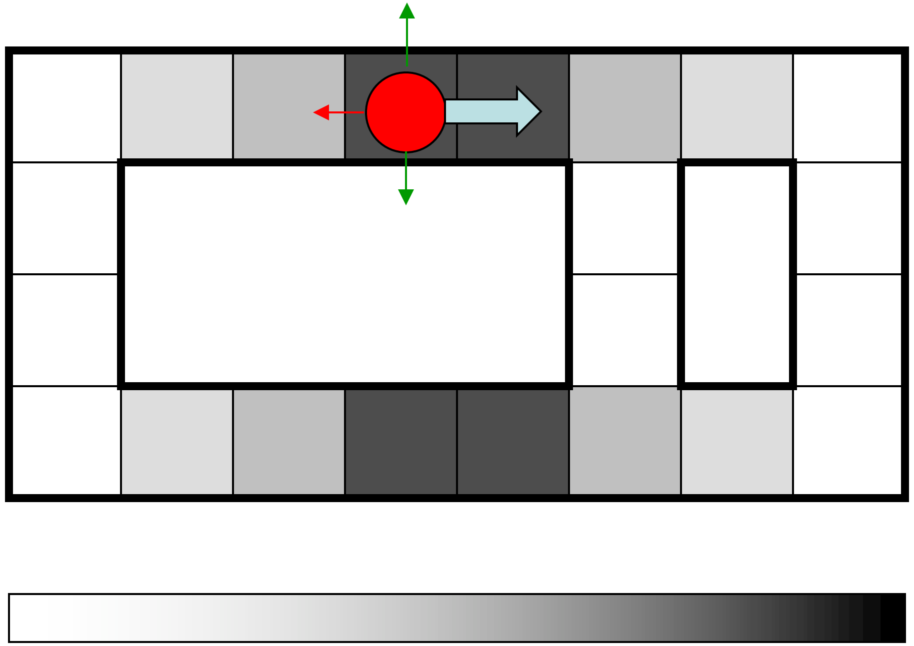
Example: Robot Localization

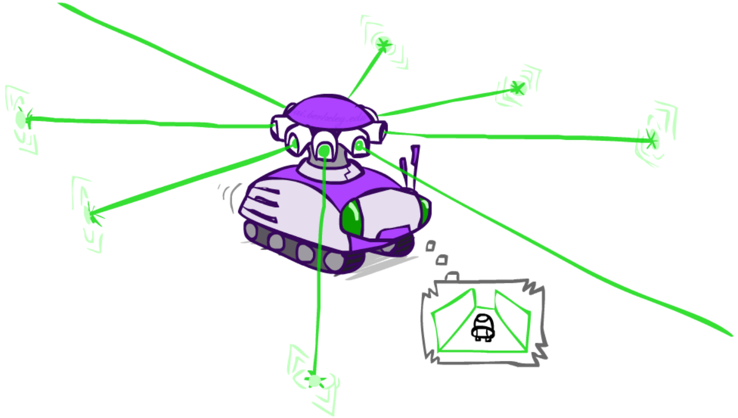




t=2

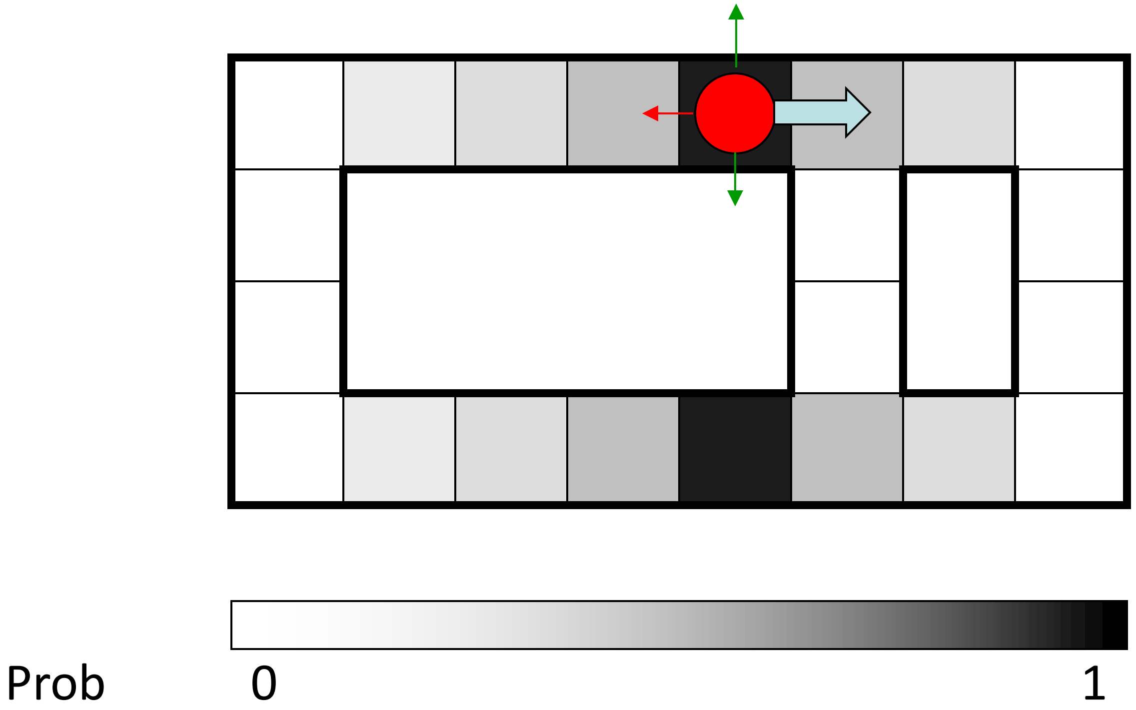
Example: Robot Localization

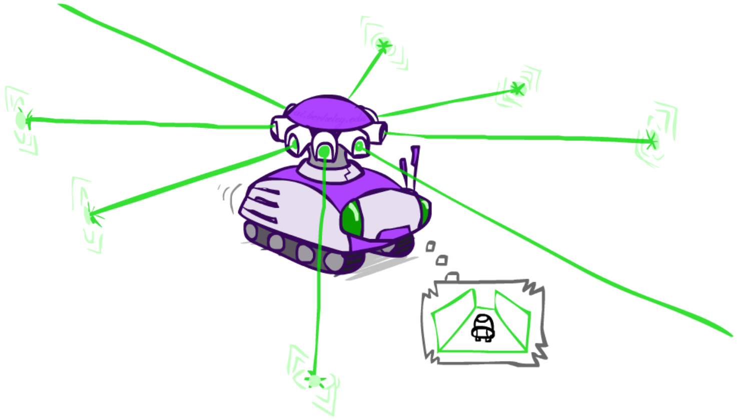




t=3

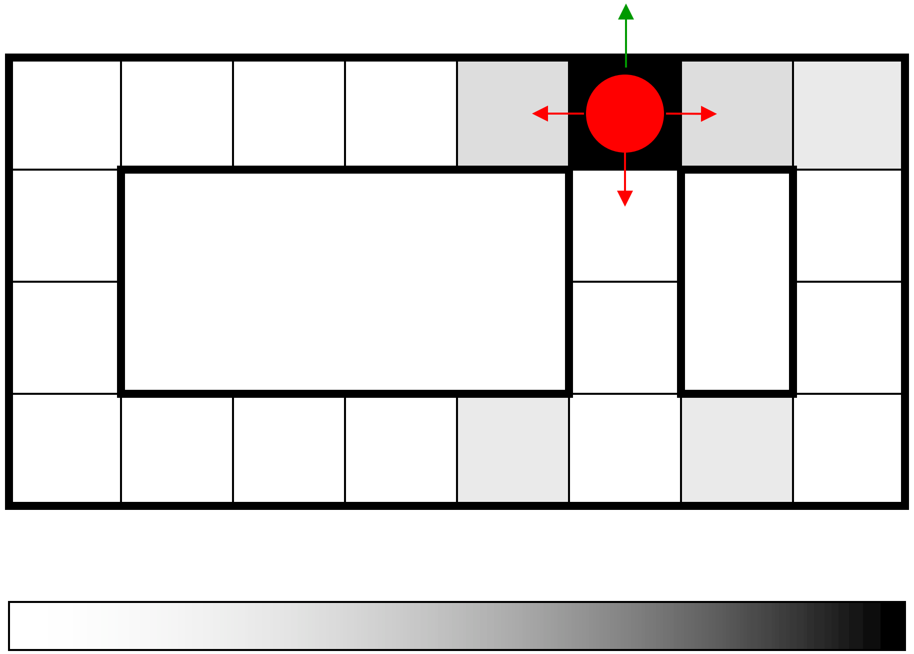
Example: Robot Localization

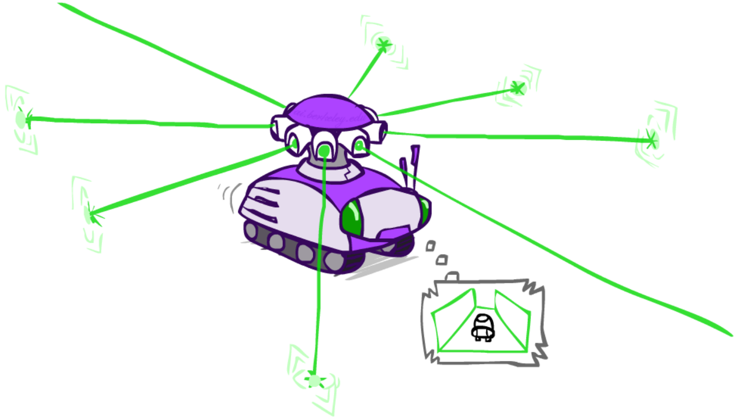




t=4

Example: Robot Localization





t=5

Filtering algorithm

- Aim: devise a ***recursive filtering*** algorithm of the form
  - $\mathbf{P}(X_{t+1} | e_{1:t+1}) = g(e_{t+1}, \mathbf{P}(X_t | e_{1:t}))$
- $$\begin{aligned} \mathbf{P}(X_{t+1} | e_{1:t+1}) &= \mathbf{P}(X_{t+1} | e_{1:t}, e_{t+1}) \\ &= \textstyle\sum_{x_t} \mathbf{P}(x_t, X_{t+1} | e_{1:t}, e_{t+1}) \\ &= \textstyle\sum_{x_t} \alpha\, \mathbf{P}(x_t, X_{t+1}, e_{t+1} | e_{1:t}) \\ &= \textstyle\sum_{x_t} \alpha\, \mathbf{P}(e_{t+1} | X_{t+1})\, \mathbf{P}(x_t | e_{1:t})\, \mathbf{P}(X_{t+1} | x_t, e_{1:t}) \\ &= \alpha\, \mathbf{P}(e_{t+1} | X_{t+1}) \textstyle\sum_{x_t} P(x_t | e_{1:t})\, \mathbf{P}(X_{t+1} | x_t) \end{aligned}$$

Marginal Probability

Normalization Trick / Bayes Rule

Definition of HMM

Simple factoring of a constant

Filtering algorithm

- $\mathbf{P}(X_{t+1} \mid e_{1:t+1}) = \underbrace{\alpha}_{\text{Normalize}} \underbrace{\mathbf{P}(e_{t+1} \mid X_{t+1})}_{\text{Update}} \underbrace{\sum_{x_t} P(x_t \mid e_{1:t})\, \mathbf{P}(X_{t+1} \mid x_t)}_{\text{Predict}}$

- $\boldsymbol{f}_{1:t+1} = \textsc{Forward}(\boldsymbol{f}_{1:t}, e_{t+1})$
- Cost per time step: $O(|X|^2)$ where $|X|$ is the number of states
- Time and space costs are ***constant***, independent of t
- $O(|X|^2)$ is infeasible for models with many state variables
  - Will introduce approximate filtering algorithms soon

Summary: Filtering

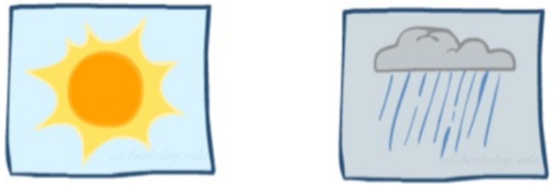
- Filtering is the inference process of finding a distribution over X_T given e_1 through e_T : $P(X_T \mid e_{1:t})$
- We first compute $P(X_1 \mid e_1)$: $P(x_1|e_1) \propto P(x_1) \cdot P(e_1|x_1)$
- For each t from 2 to T, we have $P(X_{t-1} \mid e_{1:t-1})$
- Elapse time: compute $P(X_t \mid e_{1:t-1})$

$$P(x_t|e_{1:t-1}) = \sum_{x_{t-1}} P(x_{t-1}|e_{1:t-1}) \cdot P(x_t|x_{t-1})$$

- Observe: compute $P(X_t \mid e_{1:t-1} , e_t) = P(X_t \mid e_{1:t})$

$$P(x_t|e_{1:t}) \propto P(x_t|e_{1:t-1}) \cdot P(e_t|x_t)$$

Example: Weather HMM



$f(sun) = 0.5$
$f(rain) = 0.5$

predict

$P(s_1 \mid w_0) = .6$ = .5*.9+.5*.3
$P(r_1 \mid w_0) = .4$ = 5*.1+.5*.7

update

$f(sun) = 0.25$ = .2*.6/(.2*.6 + .9*.4)
$f(rain) = 0.75$ = .9*.4/(.2*.6 + .9*.4)

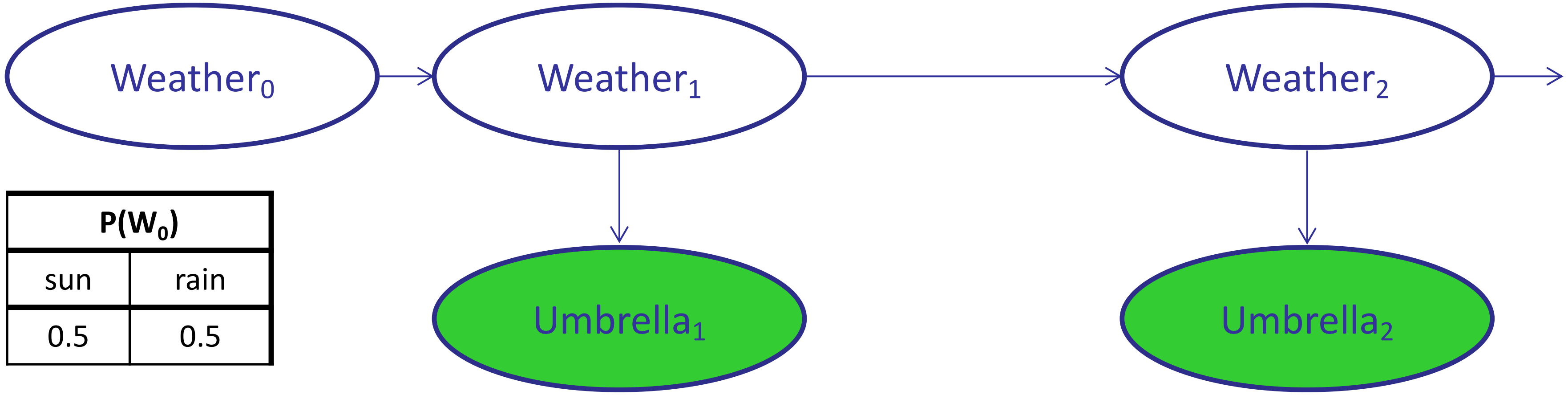
predict

$P(s_2 \mid w_1) = 0.45$ = .25*.9 + .75 *.3
$P(r_2 \mid w_1) = 0.55$ = .25*.1 + . 75*.7

update

$f(sun) = 0.154$ = .2*.45/(.2*.45 + .9*.55)
$f(rain) = 0.846$ = .9*.55/(.2*.45+ .9*.55)

W_{t-1}	**$P(W_t \mid W_{t-1})$**	
	sun	rain
sun	0.9	0.1
rain	0.3	0.7



$P(W_0)$	
sun	rain
0.5	0.5

W_t	**$P(U_t \mid W_t)$**	
	true	false
sun	0.2	0.8
rain	0.9	0.1

$$\mathbf{P}(X_{t+1} \mid e_{1:t+1}) = \alpha\, \mathbf{P}(e_{t+1} \mid X_{t+1}) \sum_{x_t} P(x_t \mid e_{1:t})\, \mathbf{P}(X_{t+1} \mid x_t)$$

Video of Demo Pacman – Sonar

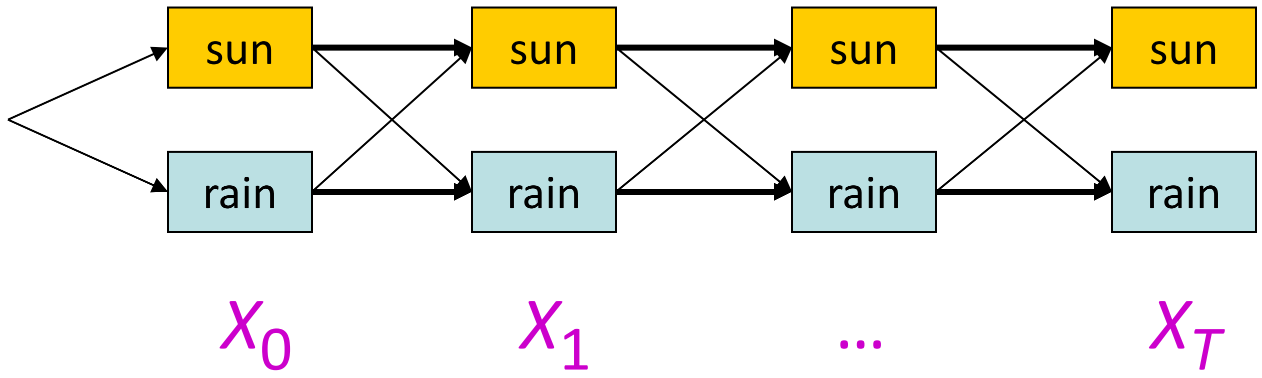
Most Likely Explanation

Inference tasks

- ***Filtering***: $\mathbf{P}(X_t | e_{1:t})$
  - ***belief state***—input to the decision process of a rational agent
- ***Prediction***: $\mathbf{P}(X_{t+k} | e_{1:t})$ for $k > 0$
  - evaluation of possible action sequences; like filtering without the evidence
- ***Smoothing***: $\mathbf{P}(X_k | e_{1:t})$ for $0 \leq k < t$
  - better estimate of past states, essential for learning
- ***Most likely explanation***: $\arg\max_{x_{1:t}} P(x_{1:t} \mid e_{1:t})$
  - speech recognition, decoding with a noisy channel

Most likely explanation = most probable path*

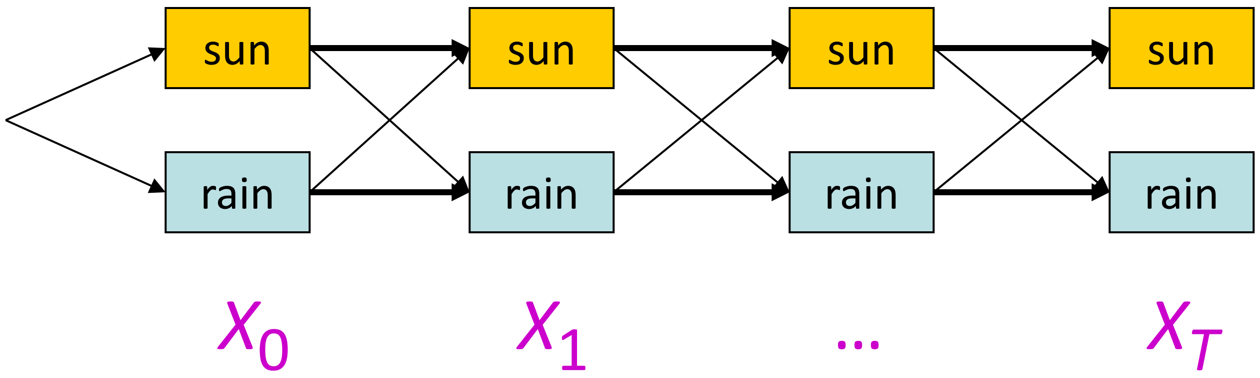
- ***State trellis***: graph of states and transitions over time



$$\arg\max_{x_{1:t}} P(x_{1:t} \mid e_{1:t})$$
$$= \arg\max_{x_{1:t}} \alpha\, P(x_{1:t}, e_{1:t})$$
$$= \arg\max_{x_{1:t}} P(x_{1:t}, e_{1:t})$$
$$= \arg\max_{x_{1:t}} P(x_0) \prod_t P(x_t \mid x_{t-1})\, P(e_t \mid x_t)$$

- Each arc represents some transition $x_{t-1} \rightarrow x_t$
- Each arc has weight $P(x_t \mid x_{t-1})\, P(e_t \mid x_t)$ (arcs to initial states have weight $P(x_0)$)
- The ***product*** of weights on a path is proportional to that state sequence's probability
- Forward algorithm computes sums of paths, ***Viterbi algorithm*** computes best paths

Forward / Viterbi algorithms*



Forward Algorithm (sum)

For each state at time *t*, keep track of the ***total probability of all paths*** to it

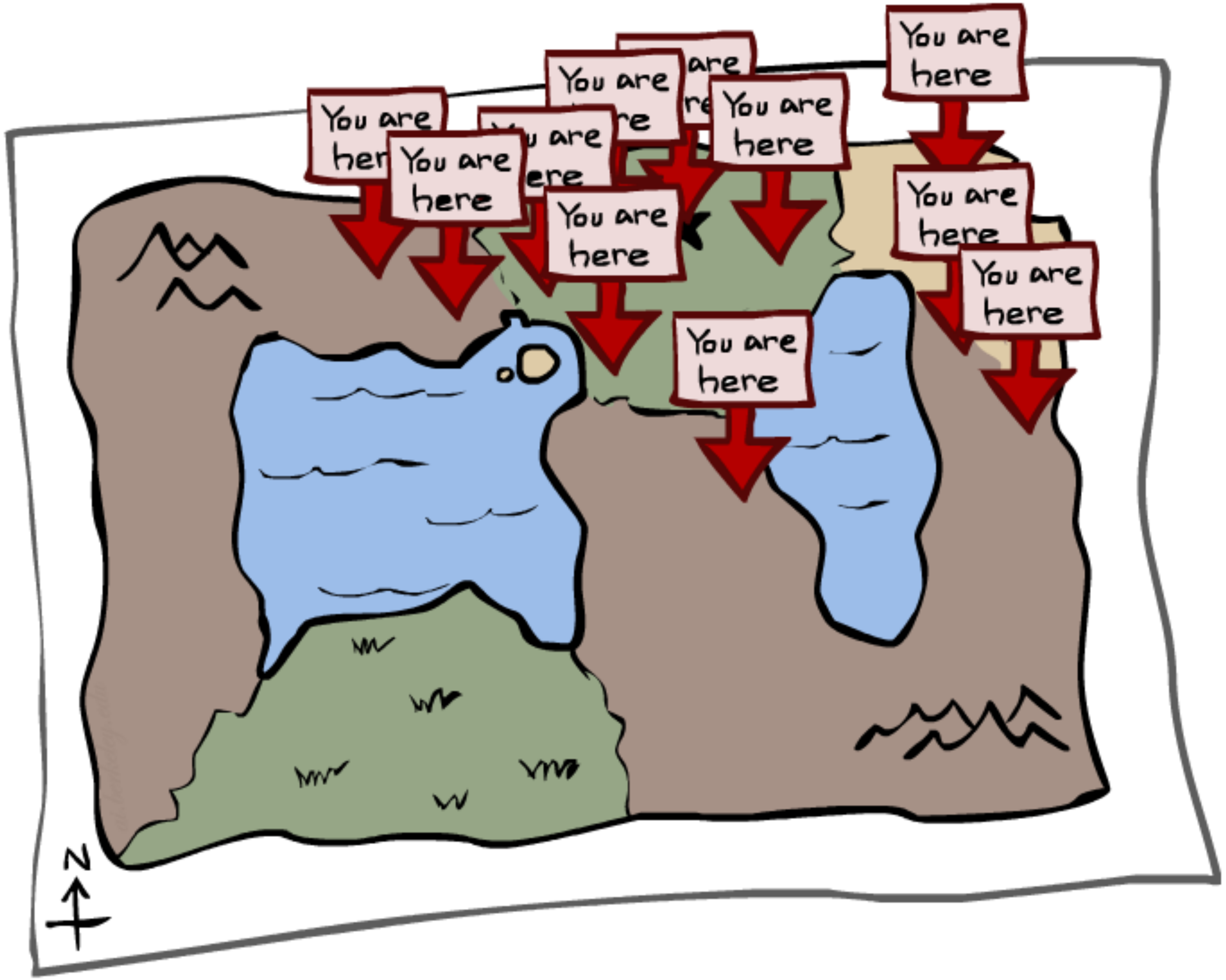
$$f_{1:t+1} = \textsc{Forward}(f_{1:t}, e_{t+1})$$
$$= \alpha\, P(e_{t+1} | X_{t+1}) \sum_{x_t} \mathbf{P}(X_{t+1} | x_t)\, f_{1:t}$$

Viterbi Algorithm (max)

For each state at time *t*, keep track of the ***maximum probability of any path*** to it

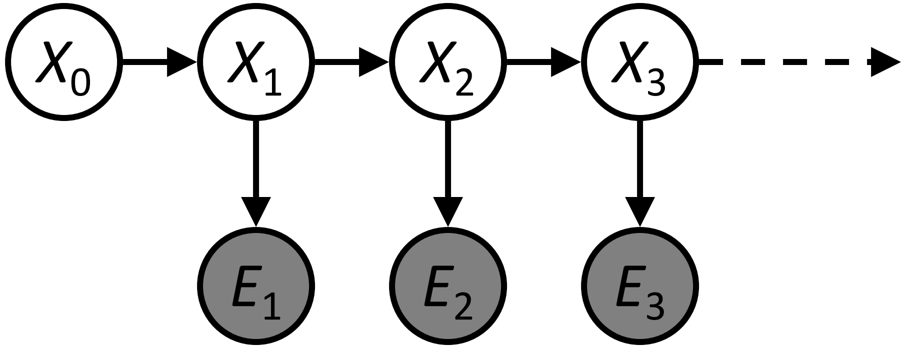
$$m_{1:t+1} = \textsc{Viterbi}(m_{1:t}, e_{t+1})$$
$$= \mathbf{P}(e_{t+1} | X_{t+1}) \max_{x_t} \mathbf{P}(X_{t+1} | x_t)\, m_{1:t}$$

Particle Filtering



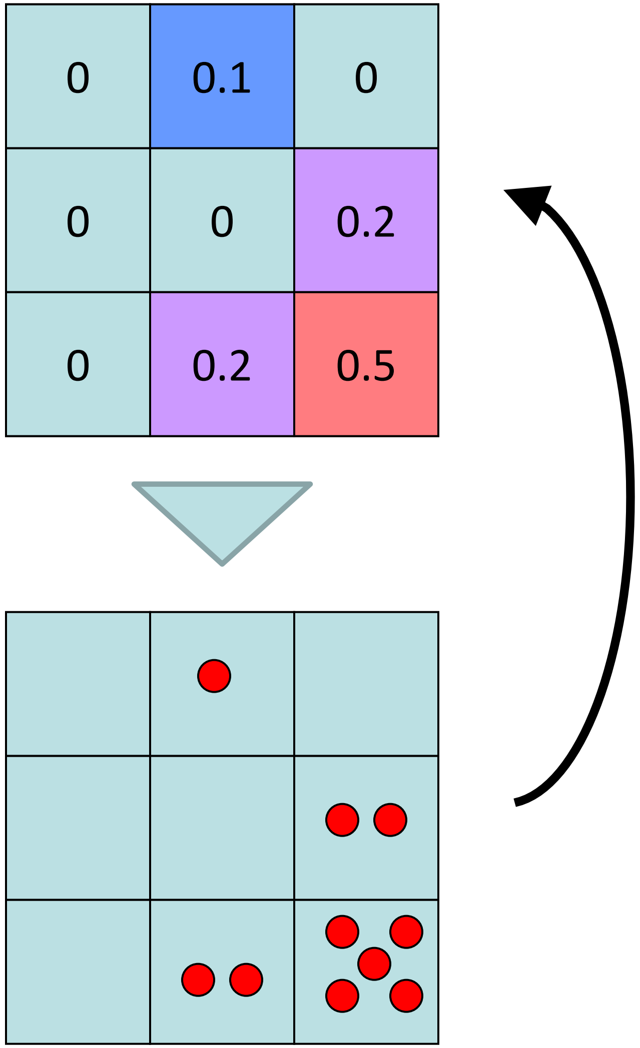
We need a new algorithm!

- When $|X|$ is grows, exact inference becomes infeasible
  - $O(|X|^2)$ cost per time step
  - (e.g., 3 ghosts in a 10x20 world, continuous domains)



Particle Filtering

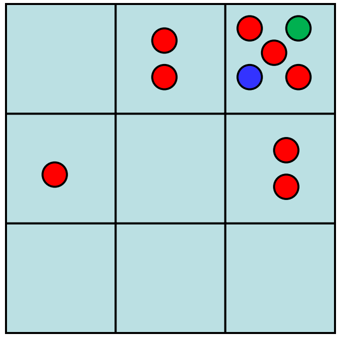
- Represent belief state by a set of samples
  - Samples are called ***particles***
  - Time per step is linear in the number of samples
  - But: number needed may be large
- This is how robot localization works in practice



0	0.1	0
0	0	0.2
0	0.2	0.5

Representation: Particles

- Our representation of $P(X)$ is now a list of $N << |X|$ particles
- $P(x)$ approximated by number of particles with value x
  - So, many x may have $P(x) = 0$!
  - More particles => more accuracy (cf. frequency histograms)
  - Usually we want a ***low-dimensional*** marginal
    - E.g., “Where is ghost 1?” rather than “Are ghosts 1,2,3 in [2,6], [5,6], and [8,11]?”



Particles:
(1,2)
(2,3)
(2,3)
(3,2)
(3,2)
(3,3)
(3,3)
(3,3)
(3,3)
(3,3)

Particle Filtering: Prediction step

- Particle *j* in state $x_t^{(j)}$ samples a new state directly from the transition model:
  - $x_{t+1}^{(j)} \sim P(X_{t+1} \mid x_t^{(j)})$
  - Here, most samples move clockwise, but some move in another direction or stay in place

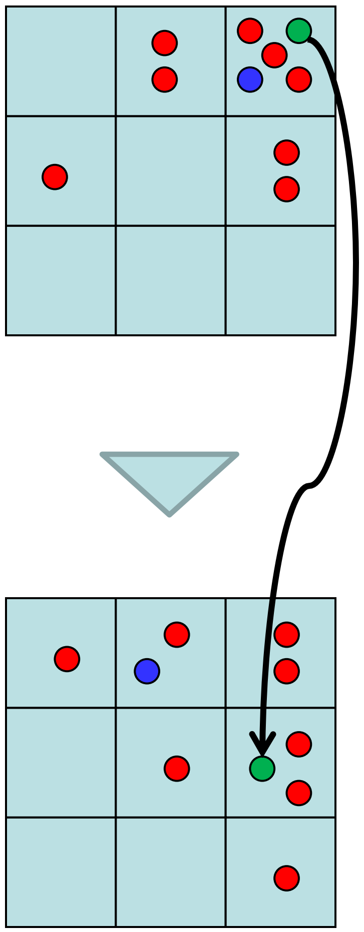
- For example:

$$x_{t+1}^{(j)} \sim P(X_{t+1} \mid x_t^{(green)}) = \langle P((3,3) \mid (3,3)), P((2,3) \mid (3,3)), P((3,2) \mid (3,3)) \rangle$$
$$= \langle 1/3, 1/3, 1/3 \rangle$$

(What if the transition model is almost deterministic?)

Particles:
(1,2)
(2,3)
(2,3)
(3,2)
(3,2)
(3,3)
(3,3)
(3,3)
(3,3)
(3,3)

Particles:
(1,3)
(2,2)
(2,3)
(2,3)
(3,1)
(3,2)
(3,2)
(3,2)
(3,3)
(3,3)

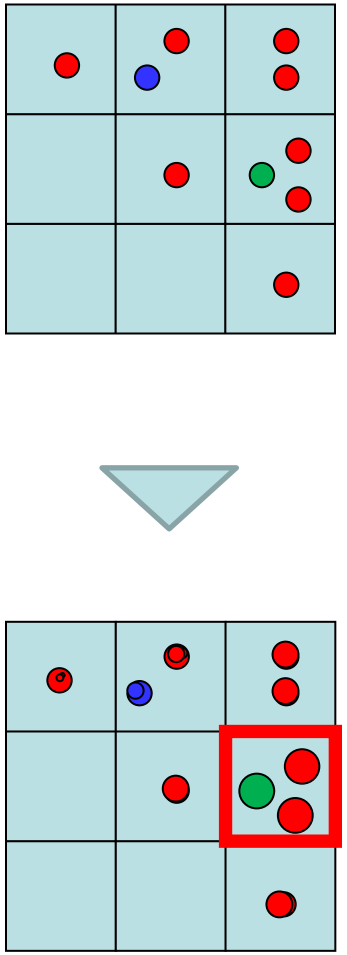


Particle Filtering: Update step

- After observing e_{t+1} :
  - As in likelihood weighting, weight each sample based on the evidence
    - $w^{(j)} = P(e_{t+1} \mid x_{t+1}^{(j)})$
  - Normalize the weights: particles that fit the data better get higher weights, others get lower weights
- For example, say $e_{t+1} = (3,2)$
  - $w^{(green)} = P((3,2) \mid (3,2)) = .9$
  - $w^{(blue)} = P((3,2) \mid (2,3)) = .2$

Particles:

(1,2)
(2,3)
(2,3)
(3,2)
(3,2)
(3,3)
(3,3)
(3,3)
(3,3)
(3,3)

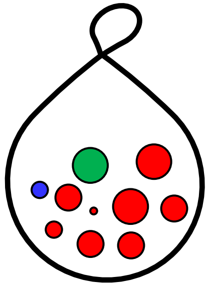


Particles:

(1,3) w=.1
(2,2) w=.4
(2,3) w=.2
(2,3) w=.2
(3,1) w=.4
(3,2) w=.9
(3,2) w=.9
(3,2) w=.9
(3,3) w=.4
(3,3) w=.4

Particle Filtering: Resample

- Rather than tracking weighted samples, we ***resample***
- *N* times, we choose from our weighted sample distribution
  - $x_{t+1}^{(j)} \sim N(X_{t+1} | e_{1:t}) / N = \alpha\, W(X_{t+1} | e_{1:t})$
  - (i.e., draw with replacement)
- Now the update is complete for this time step, continue with the next one (with weights reset to 1)



.02	.08	.17
0	.08	.56
0	0	.08

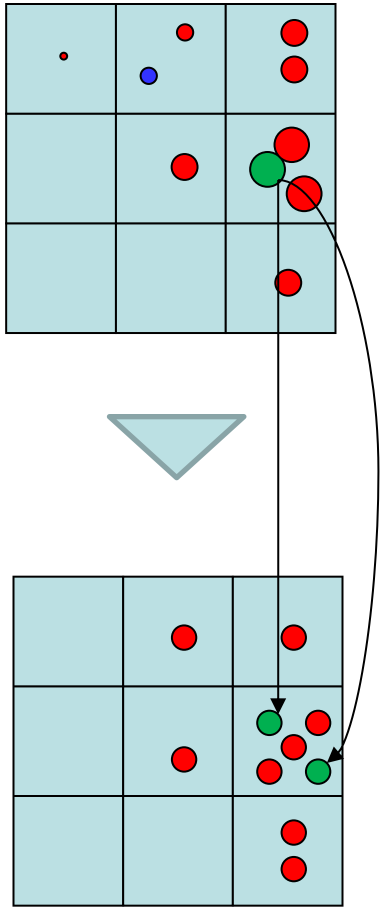
```
**routine** weighted-sample:
  return random() in $\alpha\, W(X_{t+1} | e_{1:t})$
```

Particles:

(1,3) w=.1
(2,2) w=.4
(2,3) w=.2
(2,3) w=.2
(3,1) w=.4
(3,2) w=.9
(3,2) w=.9
(3,2) w=.9
(3,3) w=.4
(3,3) w=.4

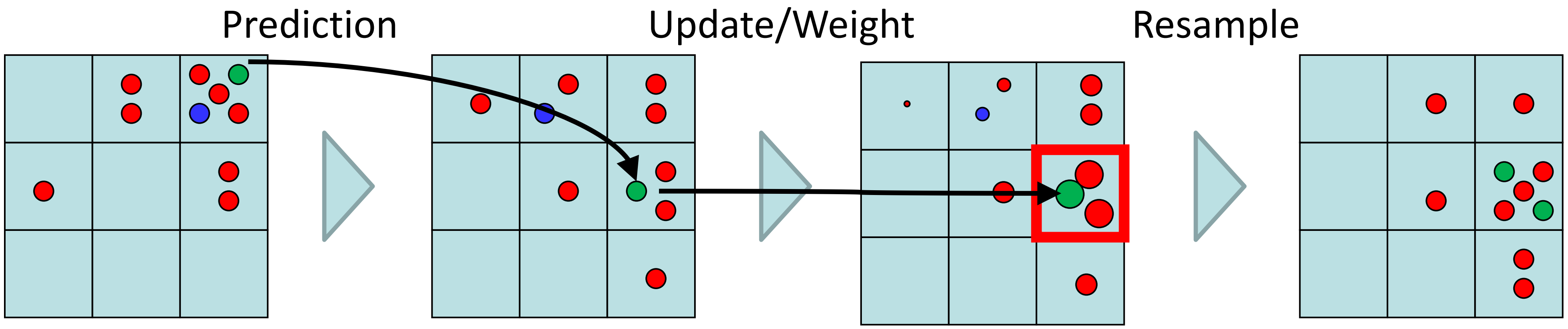
(New) Particles:

(2,2)
(2,3)
(3,1)
(3,1)
(3,2)
(3,2)
(3,2)
(3,2)
(3,2)
(3,3)



Summary: Particle Filtering

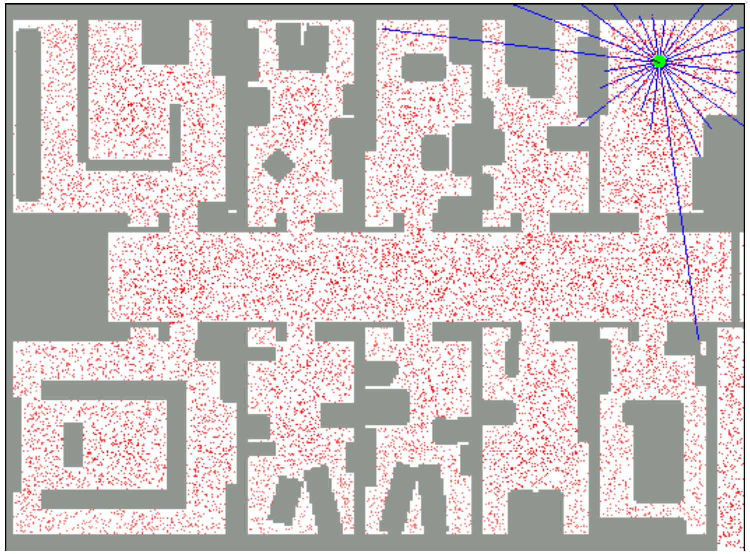
- Particles: track samples of states rather than an explicit distribution

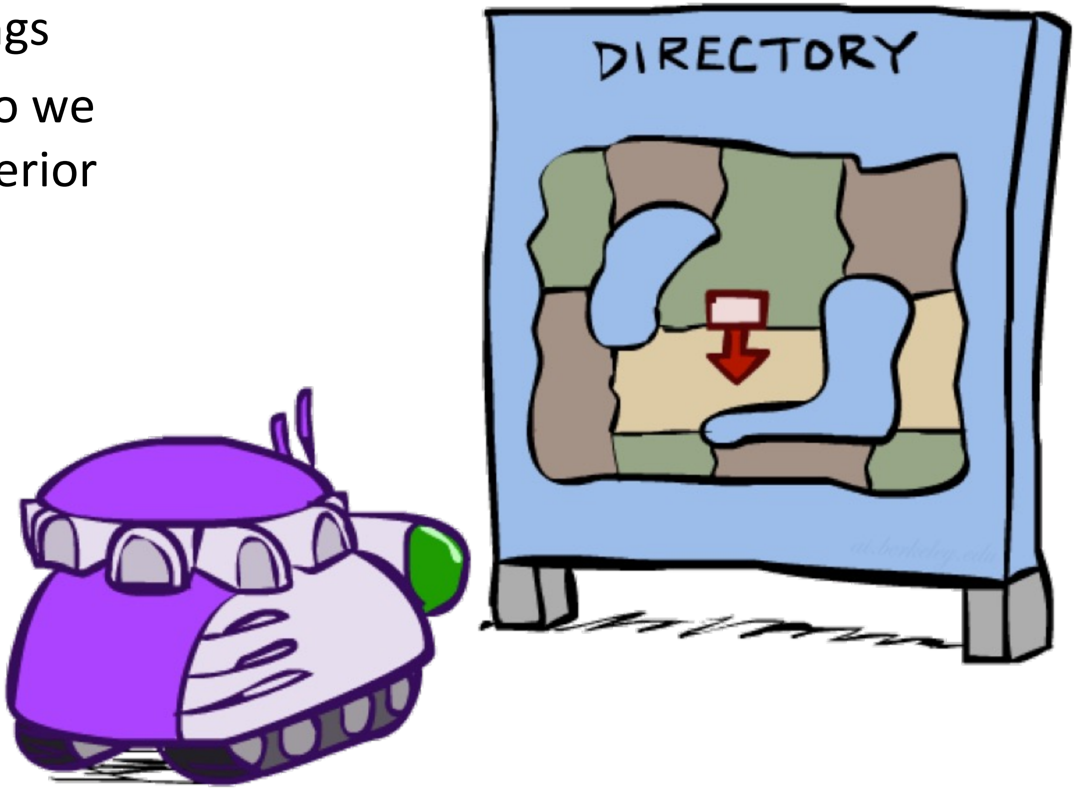


Particles:	Particles:	Particles:	(New) Particles:
(1,2)	(1,3)	(1,3) w=.1	(2,2)
(2,3)	(2,2)	(2,2) w=.4	(2,3)
(2,3)	(2,3)	(2,3) w=.2	(3,1)
(3,2)	(2,3)	(2,3) w=.2	(3,1)
(3,2)	(3,1)	(3,1) w=.4	(3,2)
(3,3)	(3,2)	(3,2) w=.9	(3,2)
(3,3)	(3,2)	(3,2) w=.9	(3,2)
(3,3)	(3,2)	(3,2) w=.9	(3,2)
(3,3)	(3,3)	(3,3) w=.4	(3,2)
(3,3)	(3,3)	(3,3) w=.4	(3,3)

Robot Localization

- In robot localization:
  - We know the map, but not the robot's position
  - Observations may be vectors of range finder readings
  - State space and readings are typically continuous so we cannot usually represent or compute an exact posterior
  - Particle filtering is a main technique



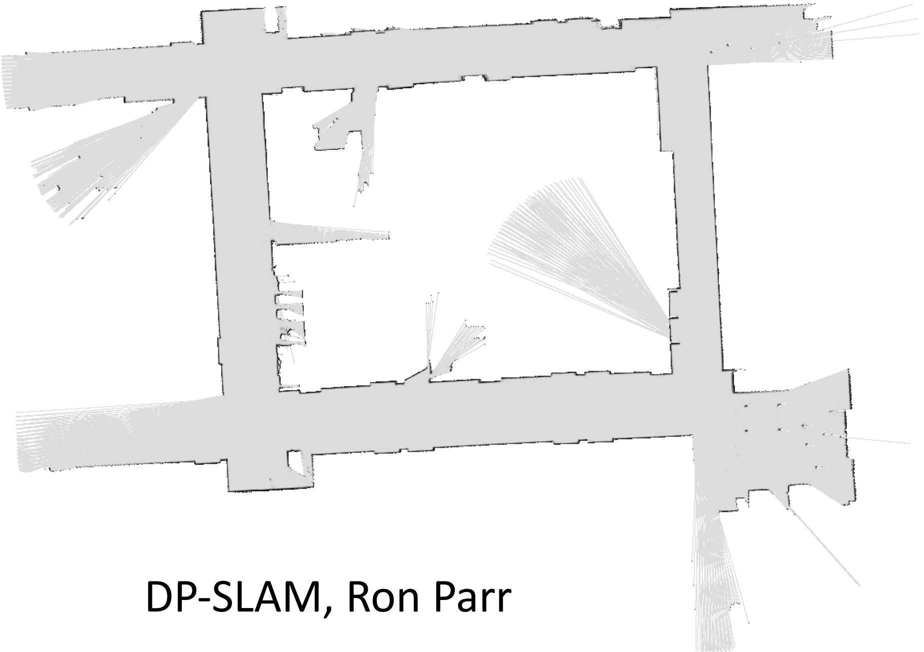


Particle Filter Localization (Sonar)

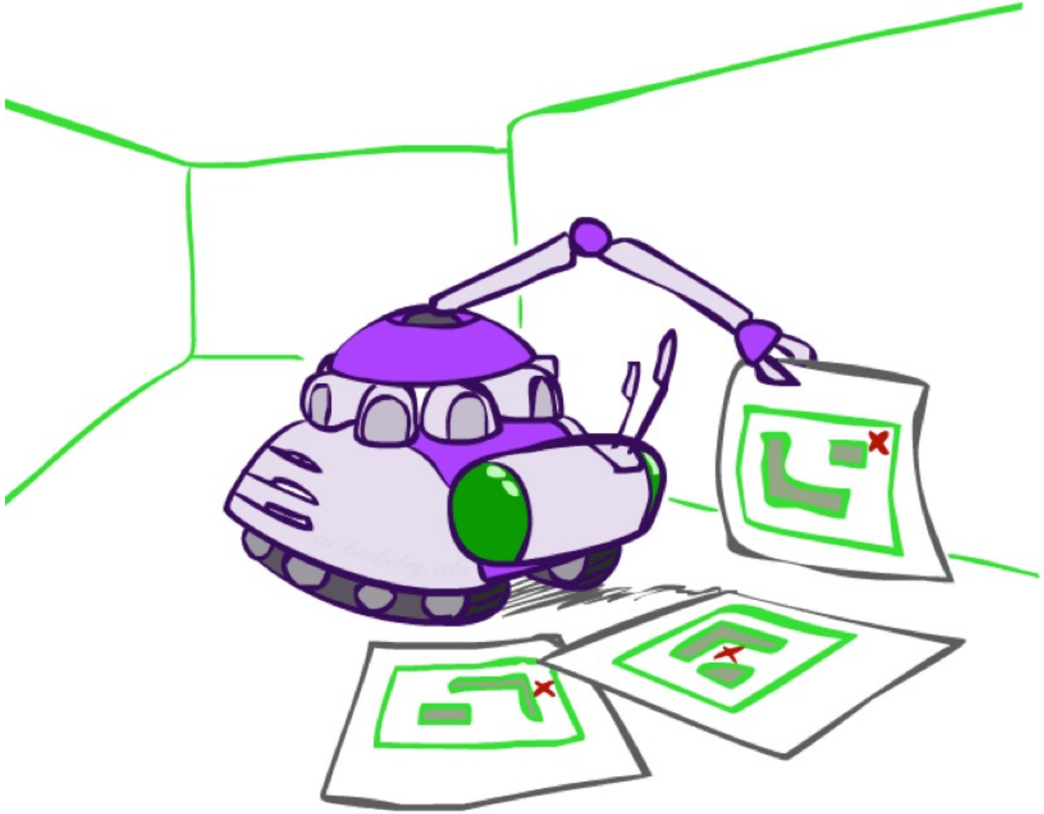
[Video: global-sonar-uw-annotated.avi]

Robot Mapping

- SLAM: Simultaneous Localization And Mapping
  - Robot does not know map or location
  - State $x_t^{(j)}$ consists of position+orientation, map!
  - (Each map usually inferred exactly given sampled position+orientation sequence)

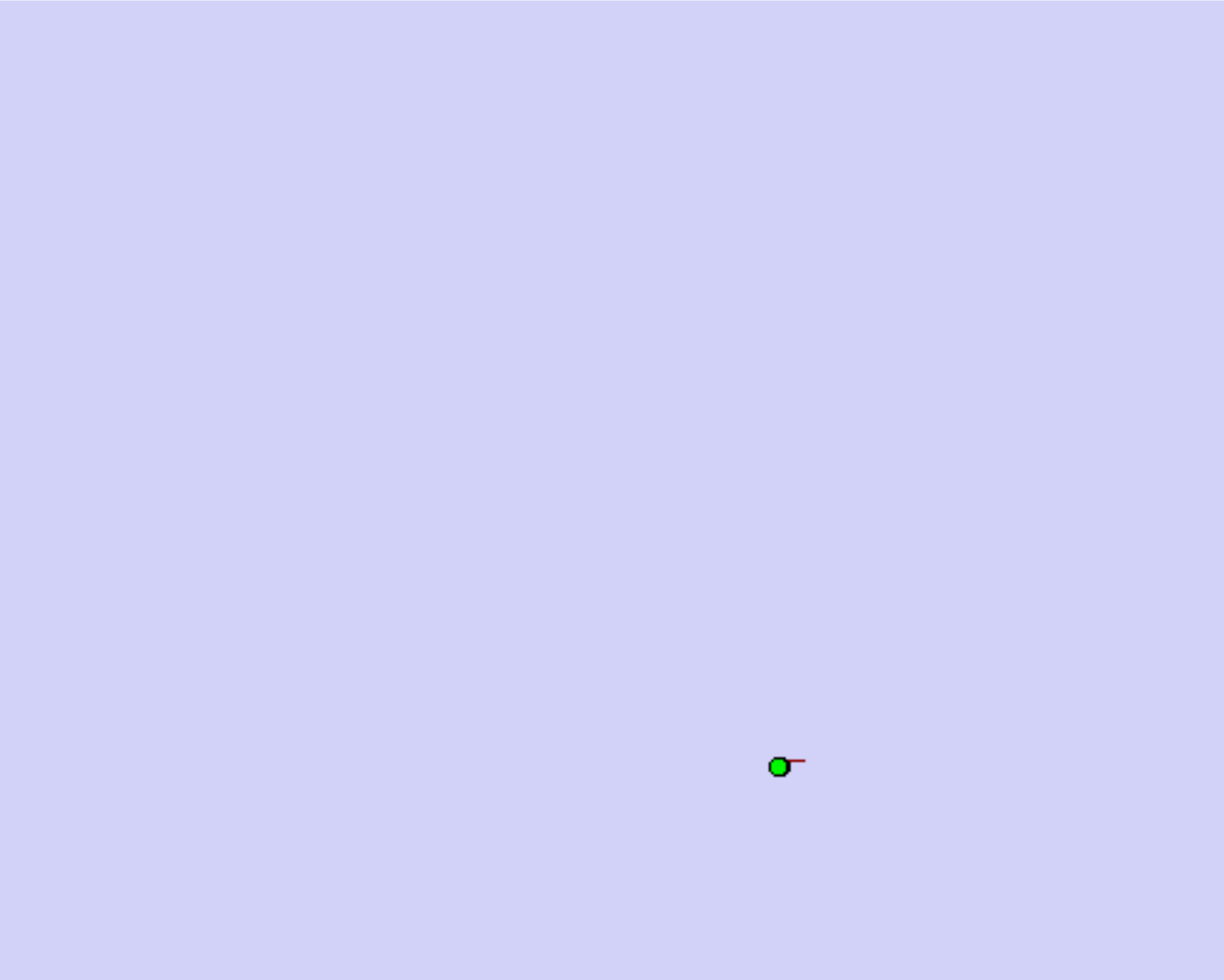


DP-SLAM, Ron Parr



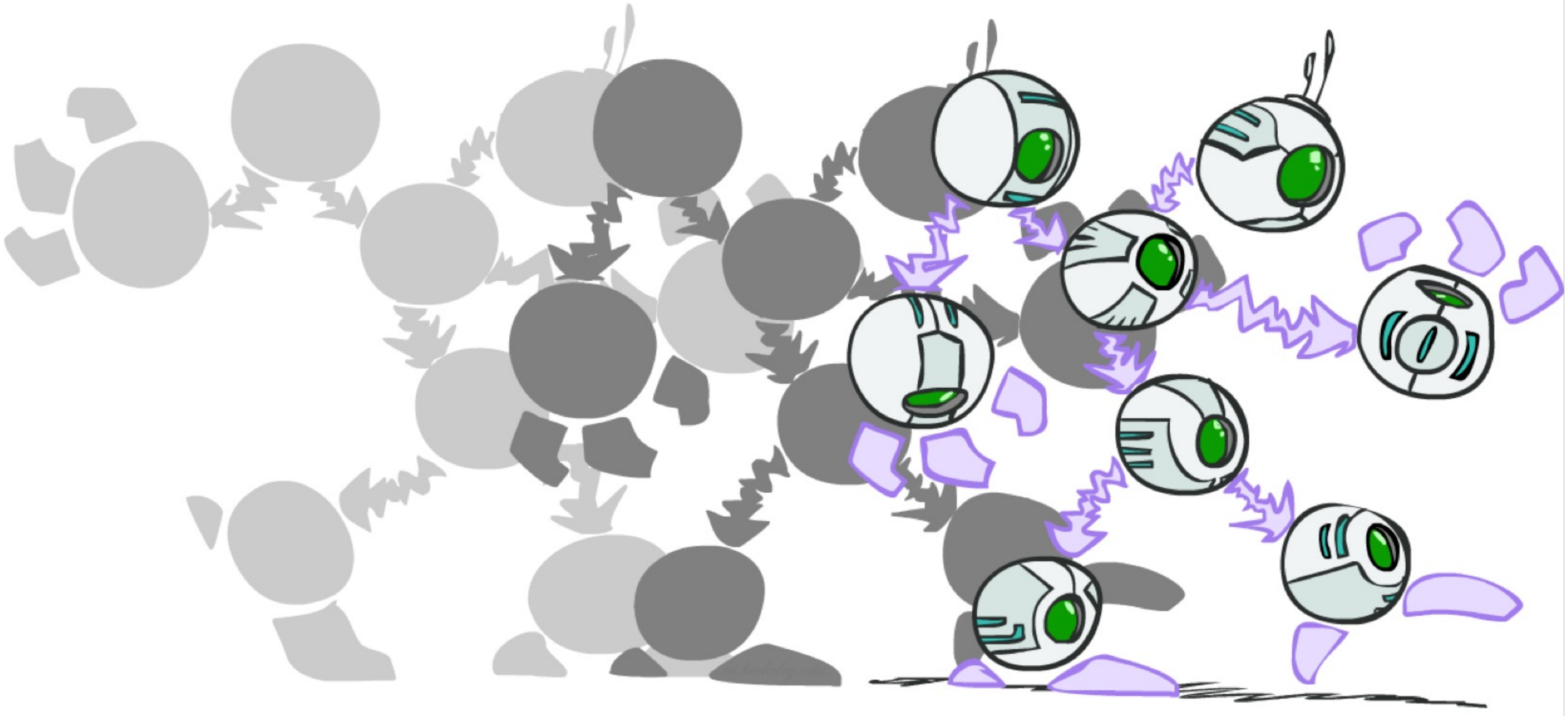
[Demo: PARTICLES-SLAM-mapping1-new.avi]

Particle Filter SLAM – Video



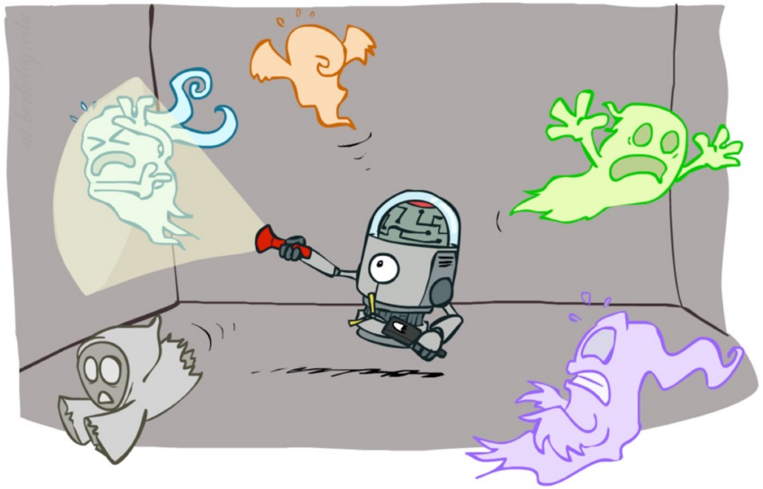
[Demo: PARTICLES-SLAM-fastslam.avi]

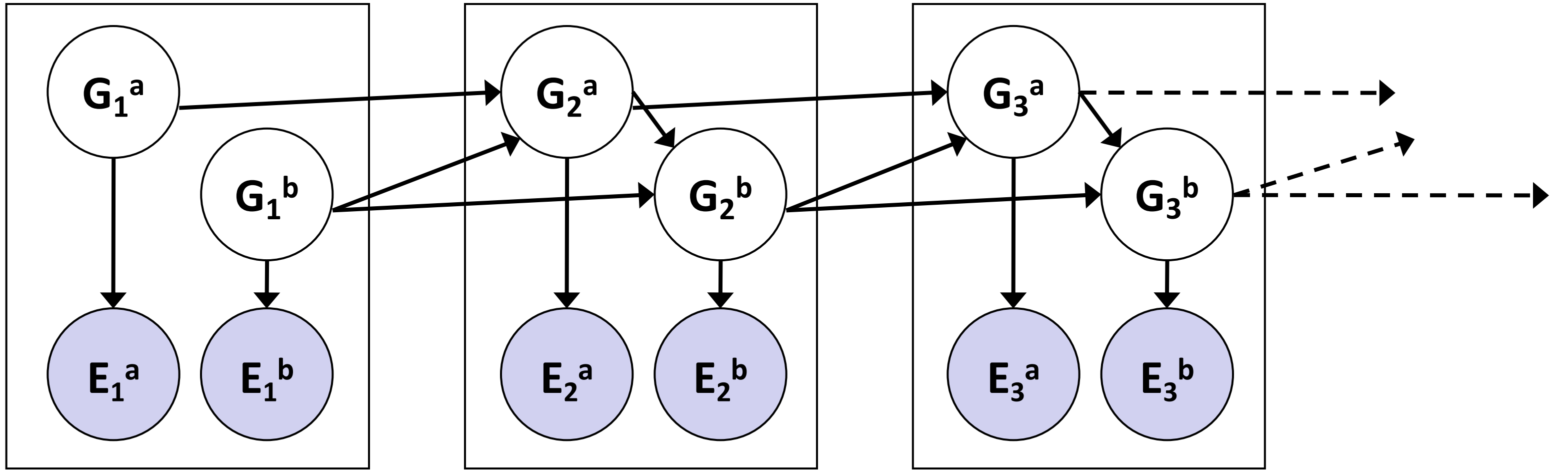
Dynamic Bayes' Nets

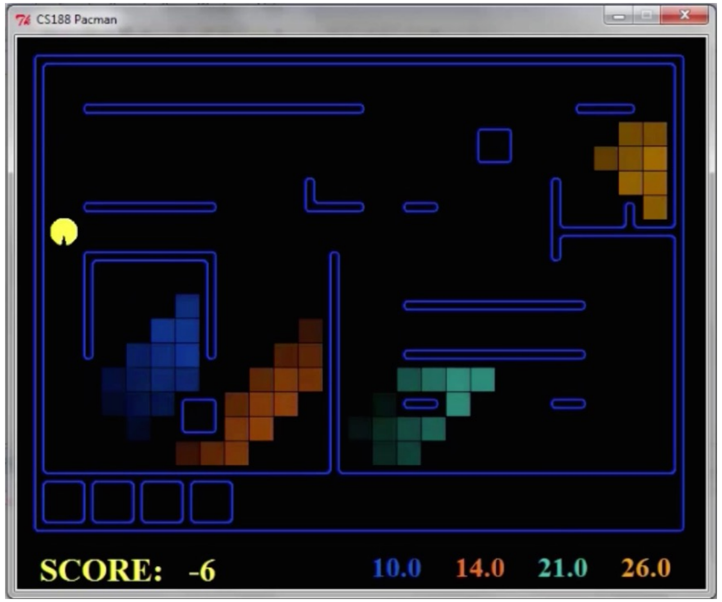


Dynamic Bayes Nets (DBNs)

- We want to track multiple variables over time, using multiple sources of evidence
- Idea: Repeat a fixed Bayes net structure at each time
- Variables at time *t* can have parents at time *t-1*

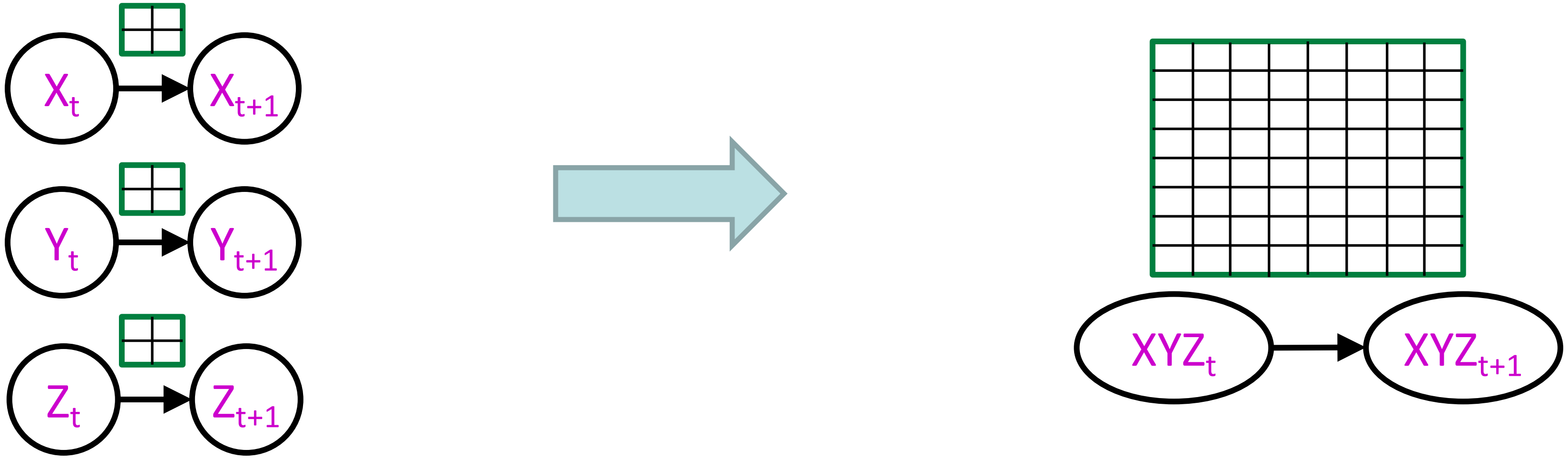






DBNs and HMMs

- Every HMM is a single-variable DBN
- Every discrete DBN is an HMM
  - HMM state is Cartesian product of DBN state variables

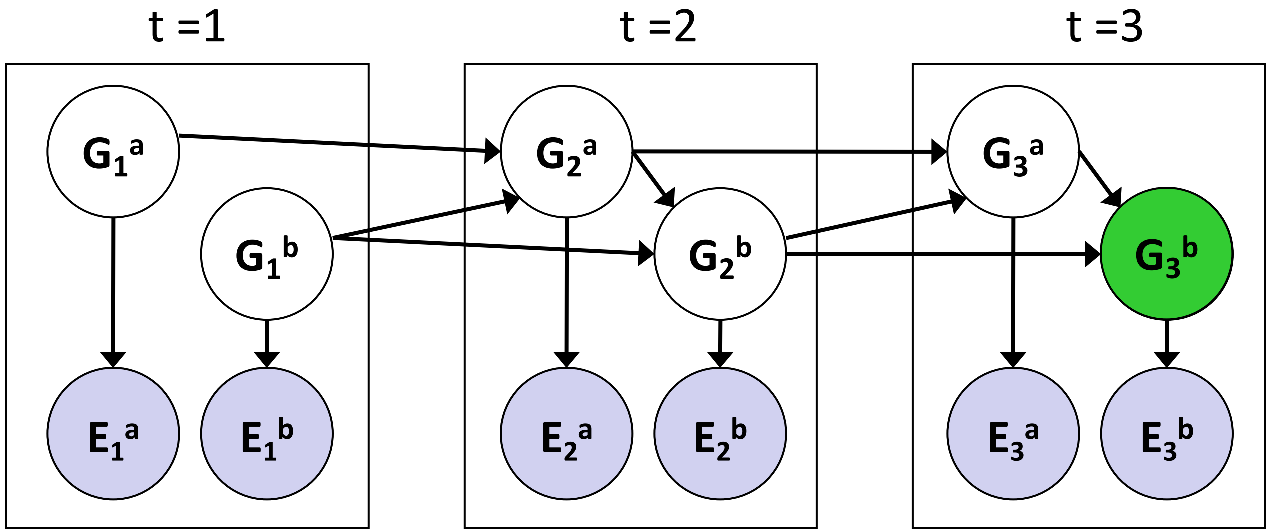


- Sparse dependencies => exponentially fewer parameters in DBN
  - E.g., 20 state variables, 3 parents each;

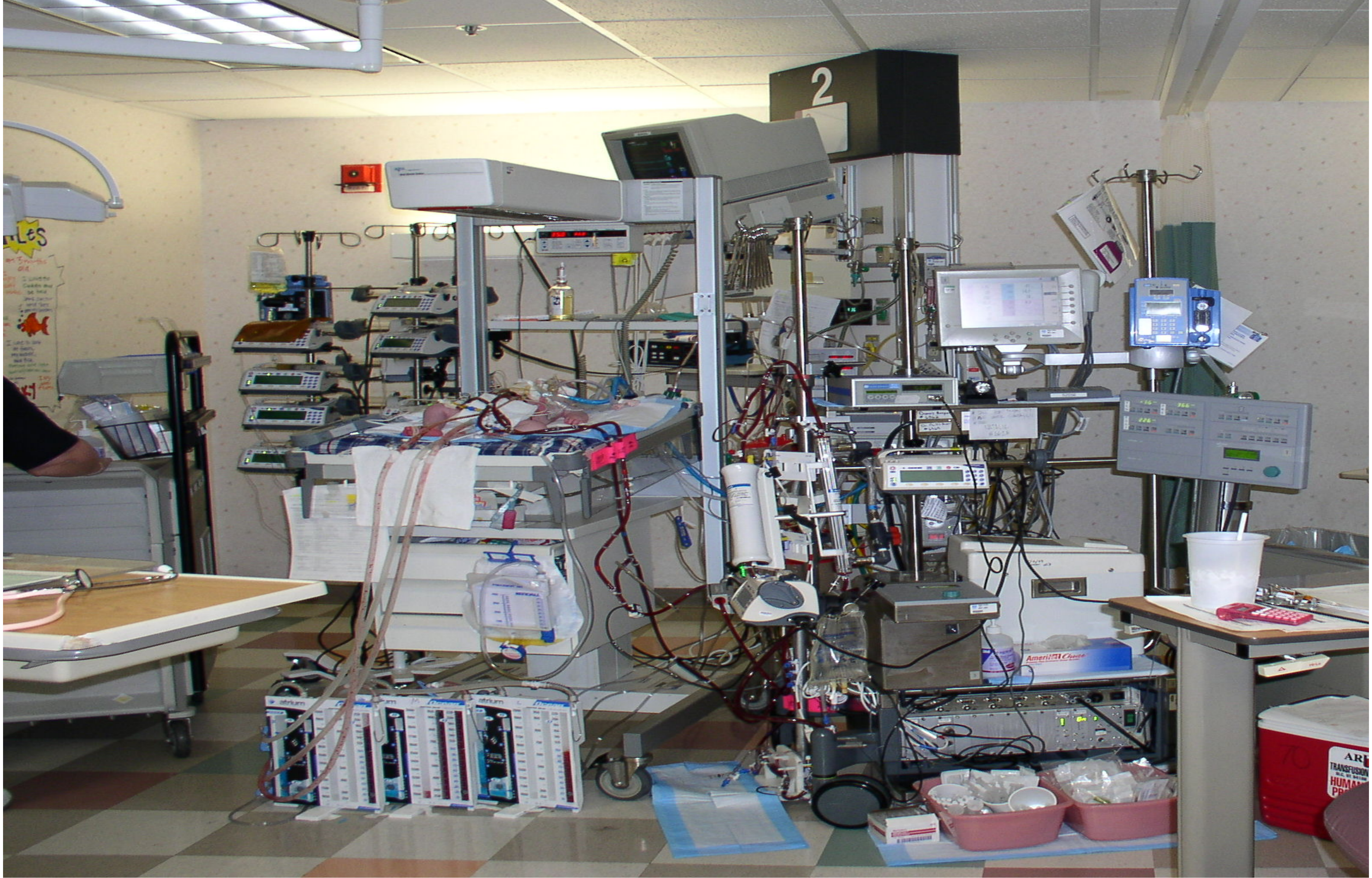
    DBN has 20 x 2^3 = 160 parameters, HMM has 2^{20} x 2^{20} =~ 10^{12} parameters

Exact Inference in DBNs

- Variable elimination applies to dynamic Bayes nets
- Offline: "unroll" the network for T time steps, then eliminate variables to find $P(X_T|e_{1:T})$



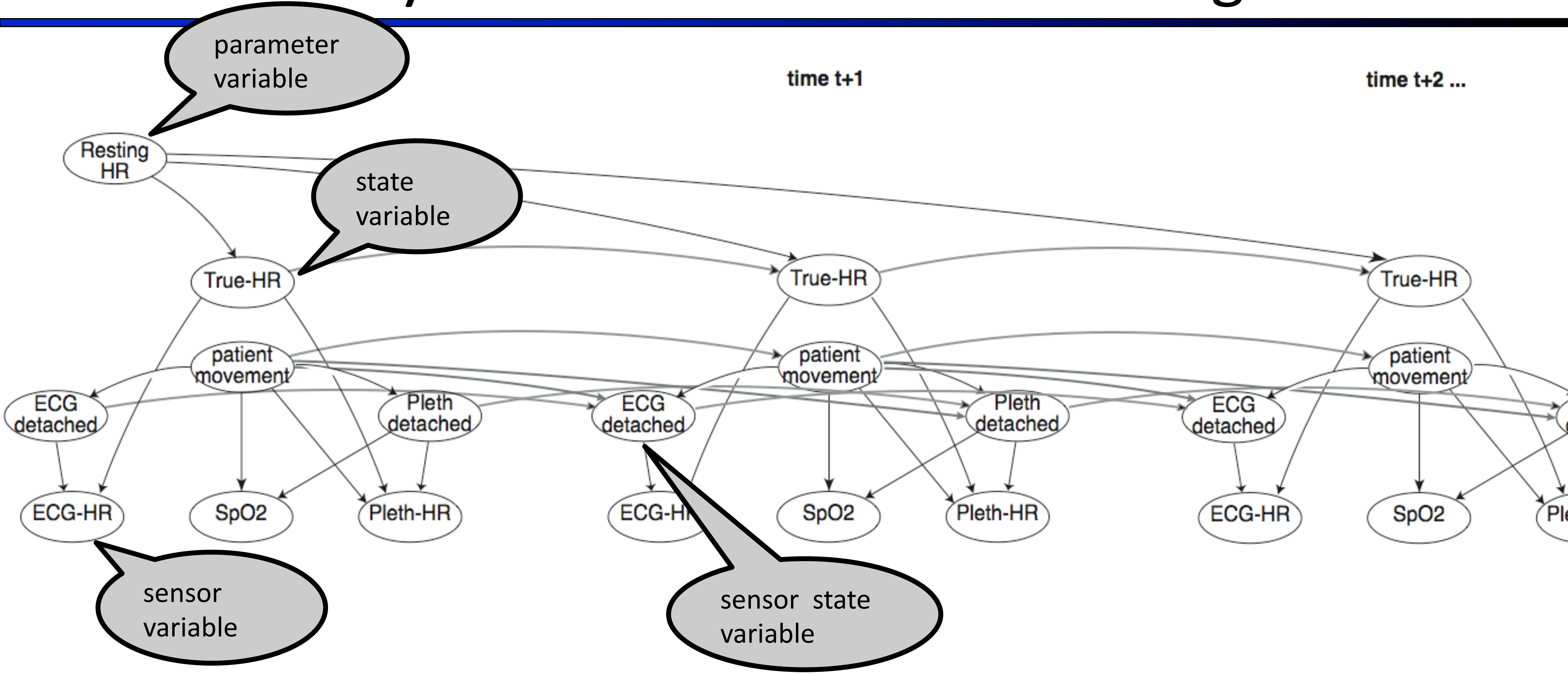
- Online: eliminate all variables from the previous time step; store factors for current time only
- Problem: largest factor contains all variables for current time (plus a few more)


2
atrium
atrium
atrium

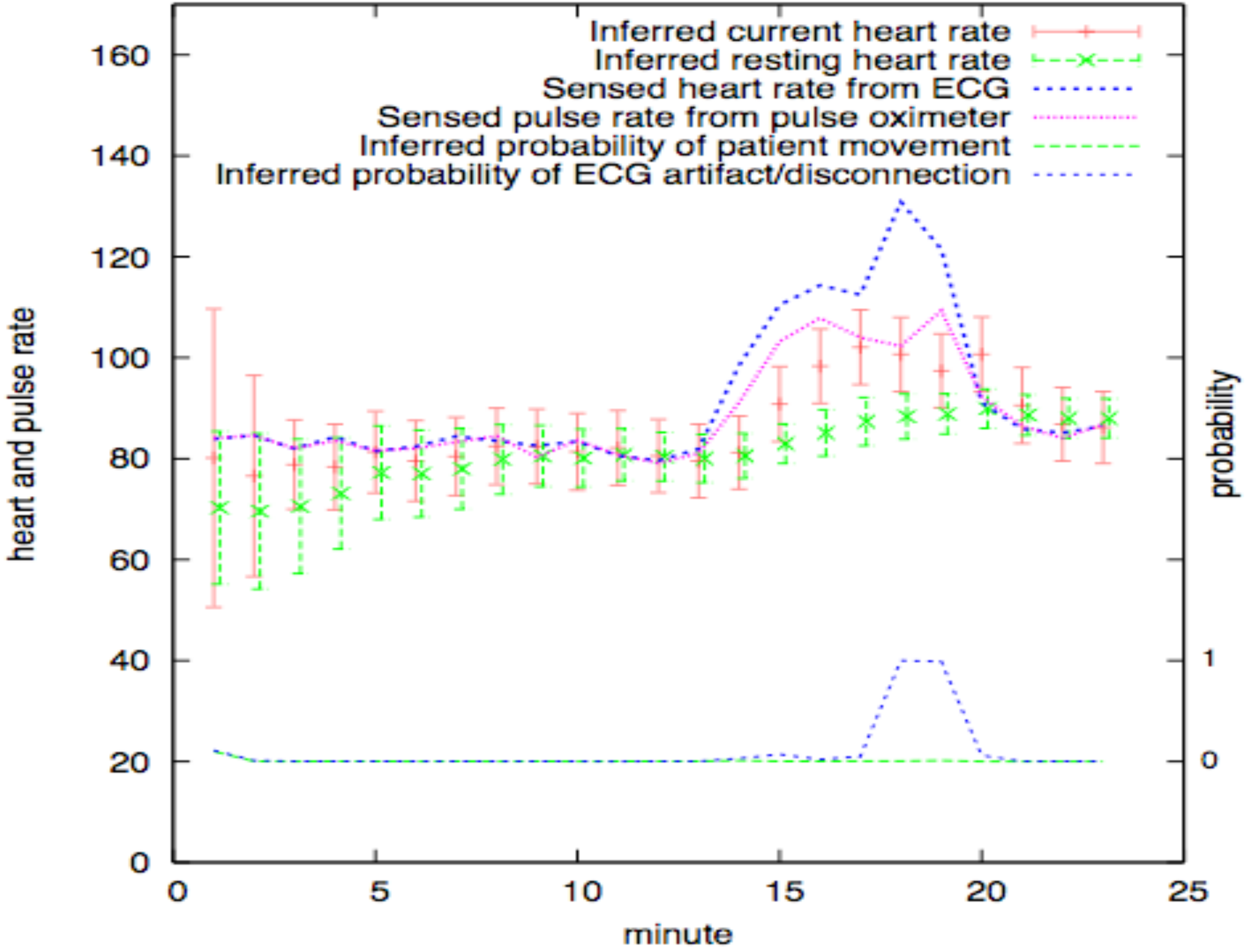
Application: ICU monitoring

- ***State***: variables describing physiological state of patient
- ***Evidence***: values obtained from monitoring devices
- ***Transition model***: physiological dynamics, sensor dynamics
- ***Query variables***: pathophysiological conditions (a.k.a. bad things)

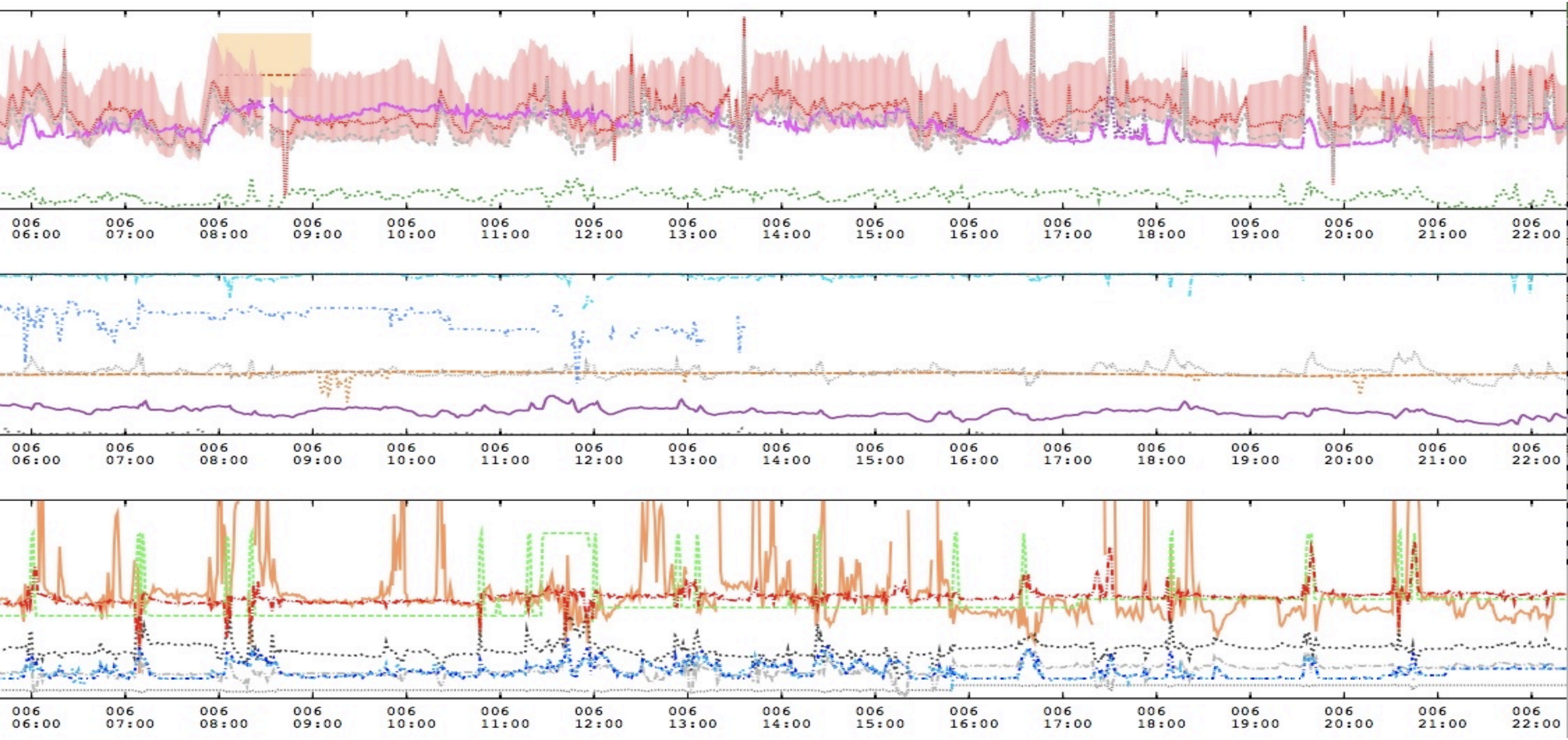
Toy DBN: heart rate monitoring

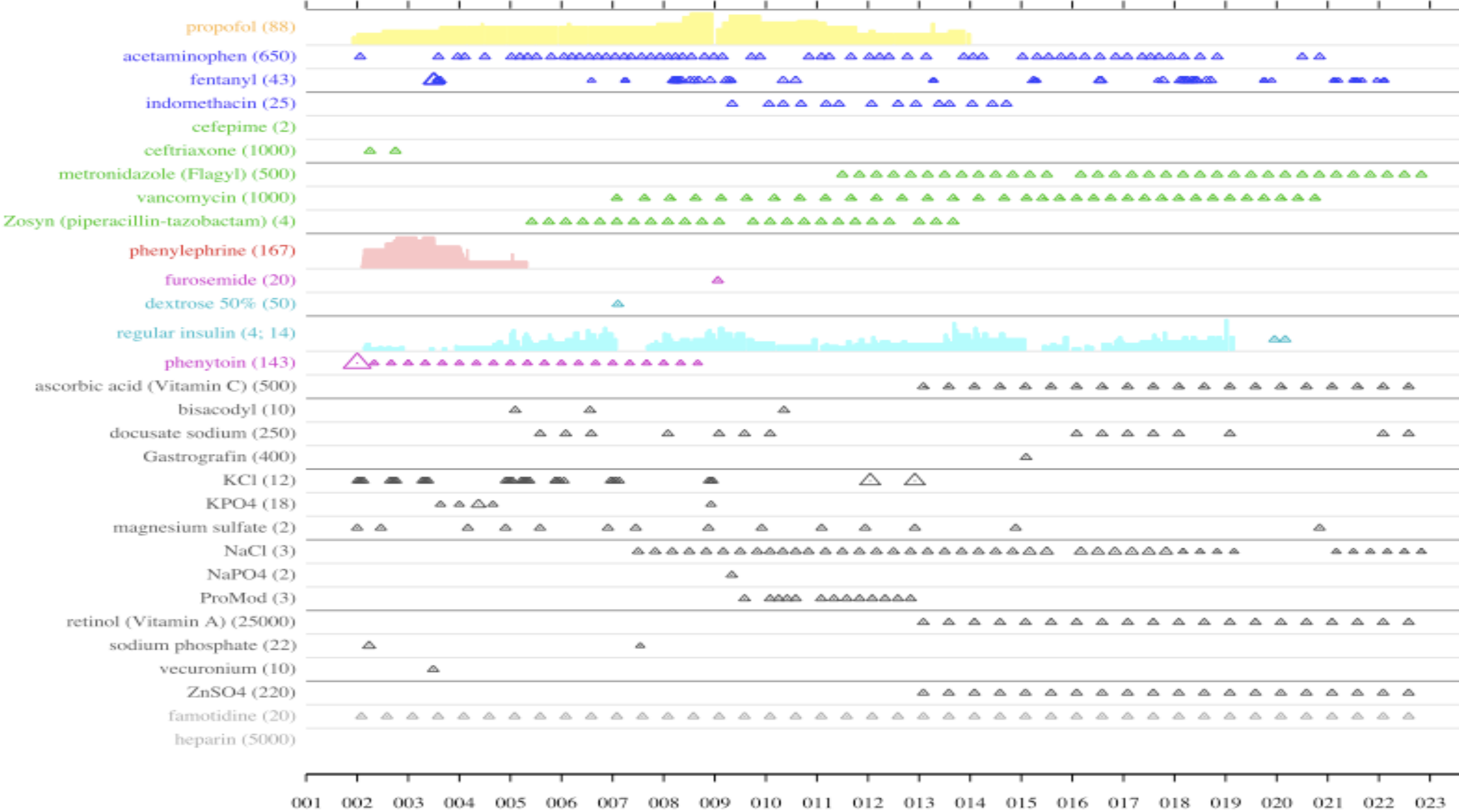


The enhanced heart-rate DBN's inferences on data from a healthy 40-year-old man



ICU data: 22 variables, 1min avg.




propofol (88)
acetaminophen (650)
fentanyl (43)
indomethacin (25)
cefepime (2)
ceftriaxone (1000)
metronidazole (Flagyl) (500)
vancomycin (1000)
Zosyn (piperacillin-tazobactam) (4)
phenylephrine (167)
furosemide (20)
dextrose 50% (50)
regular insulin (4; 14)
phenytoin (143)
ascorbic acid (Vitamin C) (500)
bisacodyl (10)
docusate sodium (250)
Gastrografin (400)
KCl (12)
KPO4 (18)
magnesium sulfate (2)
NaCl (3)
NaPO4 (2)
ProMod (3)
retinol (Vitamin A) (25000)
sodium phosphate (22)
vecuronium (10)
ZnSO4 (220)
famotidine (20)
heparin (5000)
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