Introduction to Digital Data Acquisition:

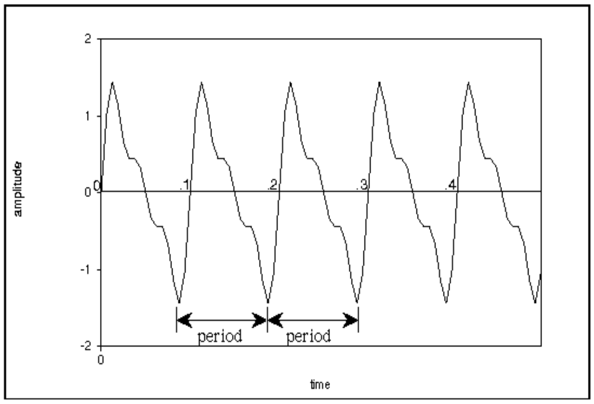
Sampling

Physical world is analog

- Digital systems need to
  - Measure analog quantities
    - Switch inputs, speech waveforms, etc
  - Control analog systems
    - Computer monitors, automotive engine control, etc
- Analog-to-digital: A/D converter (ADC)
  - Example: CD recording
- Digital-to-analog: D/A converter (DAC)
  - Example: CD playback

A little background

- For periodic waveforms, the duration of the waveform before it repeats is called the period of the waveform



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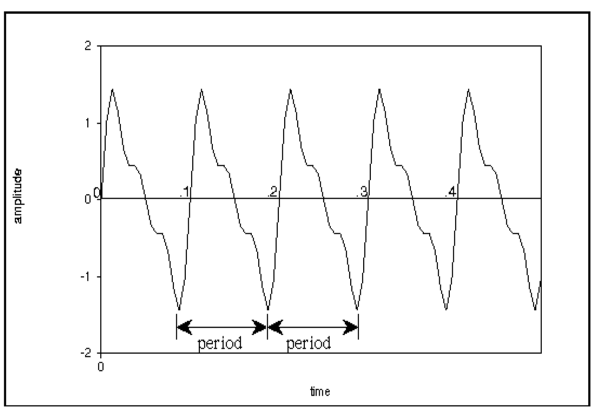
Frequency

- the rate at which a regular vibration pattern repeats itself (frequency = 1/period)



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Frequency of a Waveform

- The unit for frequency is cycles/second, also called Hertz (Hz).
- The frequency of a waveform is equal to the reciprocal of the period.

frequency = 1/period

Frequency of a Waveform

- Examples:

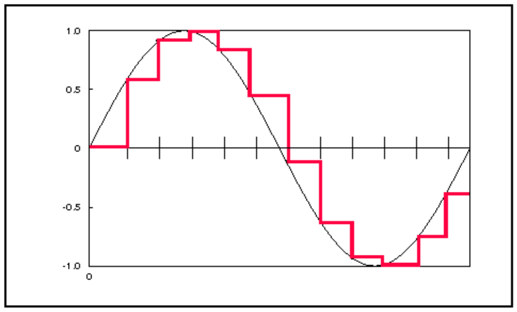
  frequency = 10 Hz
  period = .1 (1/10) seconds

  frequency = 100 Hz
  period = .01 (1/100) seconds

  frequency = 261.6 Hz (middle C)
  period = .0038226 (1/ 261.6) seconds

Waveform Sampling

- To represent waveforms in digital systems, we need to digitize or sample the waveform.



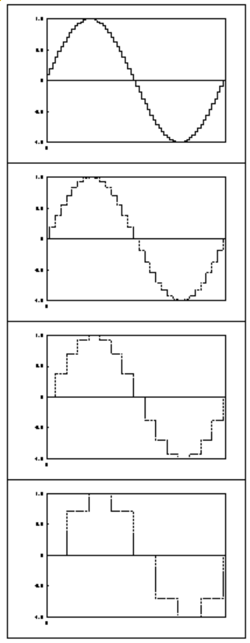
- side effects of digitization:
  - introduces some noise
  - limits the maximum upper frequency range

Sampling Rate

- The sampling rate (SR) is the rate at which amplitude values are digitized from the original waveform.
  - CD sampling rate (high-quality): SR = 44,100 samples/second
  - medium-quality sampling rate: SR = 22,050 samples/second
  - phone sampling rate (low-quality): SR = 8,192 samples/second

Sampling Rate

- Higher sampling rates allow the waveform to be more accurately represented



64 samples/cycle

32 samples/cycle

16 samples/cycle

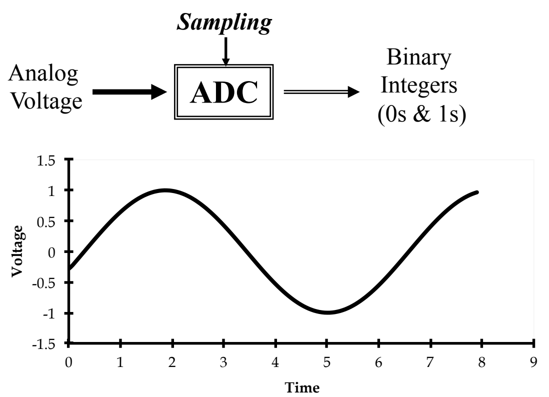
8 samples/cycle

Digital Data Acquisition

- Data Representation - *Digital vs. Analog*
- Analog-to-Digital Conversion
- Number Systems
  - Binary Numbers
  - Binary Arithmetic
- Sampling & Aliasing

Analog-to-Digital Conversion

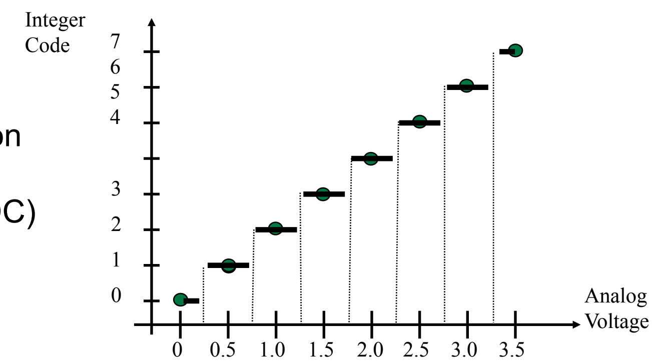
- Converts analog voltages to binary integers.



Analog-to-Digital Conversion

- **ADC calibration**

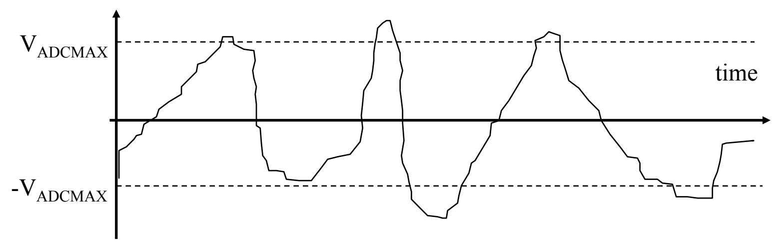
Calibration Curve
(3 bit ADC)



Analog-to-Digital Conversion

- <u>Input Range</u>
  - Unipolar: (0, V_{ADCMAX})
  - Bipolar: ($-V_{ADCMAX}$, $+V_{ADCMAX}$) (Nominal Range)
  - **Clipping**:

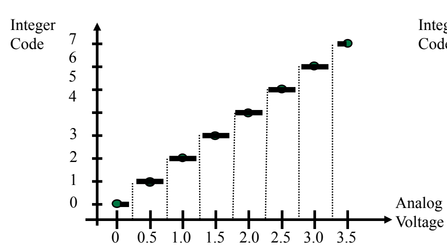
    If $|V_{IN}| > |V_{ADCMAX}|$, then $|V_{OUT}| = |V_{ADCMAX}|$

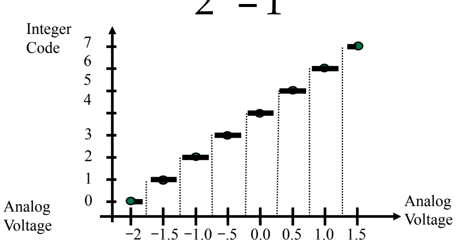


Analog-to-Digital Conversion

- <u>Quantization Interval</u> (*Q*)
  - *n* bit ADC, the input range is divided into 2^n-1 intervals.
  - 3 bit ADC:

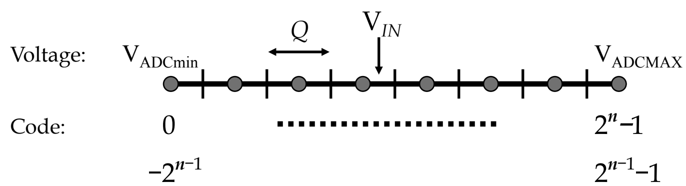
$$Q = \frac{V_{ADCMAX} - V_{ADC\min}}{2^n - 1}$$





Analog-to-Digital Conversion

- <u>Voltage to Integer Code</u>
  - *n* bit ADC



Positive Coding:

$$\text{Code} = \text{Round}\left[\frac{V_{IN} - V_{ADC\,min}}{Q}\right]$$

Positive and Negative Coding:

$$\text{Code} = \text{Round}\left[\frac{V_{IN}}{Q}\right]$$

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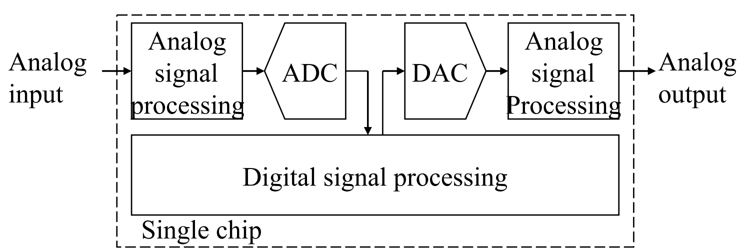
Why A/D-conversion?



- Signals are analog by nature
- ADC necessary for DSP
- Digital signal processing provides:
  - Close to infinite SNR
  - Low system cost
  - Repetitive system

- ADC bottle necks:
  - Dynamic range
  - Conversion speed
  - Power consumption

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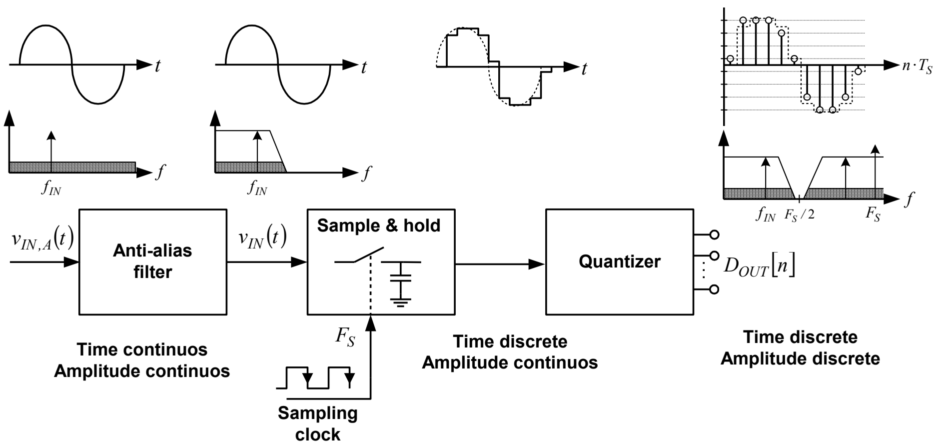
A/D-converter basics

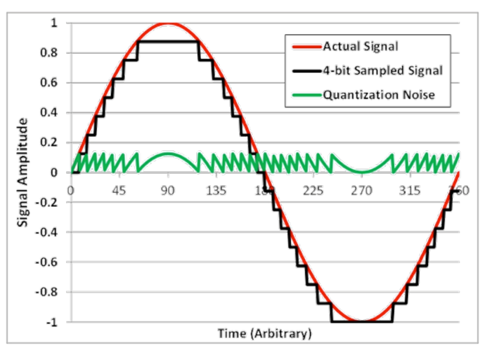


$$D_{OUT}^{ideal}[n] = G_{ideal} \cdot v_{IN}(n \cdot Ts) + q(n)$$

$$D_{OUT}^{real}[n] = G_{ideal} \cdot (1 + \varepsilon) \cdot v_{IN}(n \cdot Ts) + q(n) + e_{offset}(n) + e_{noise}(n) + e_{jitter}(n) + e_{distortion}(n)$$

The Theory

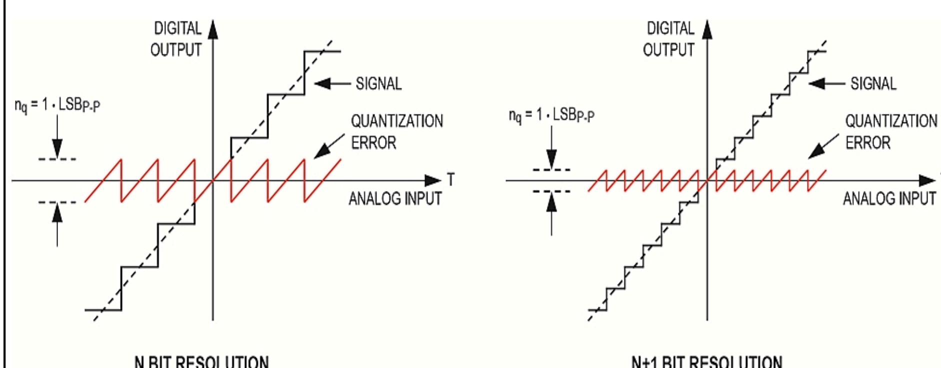
- Sampling theory is a subset of communications theory
  - Same basic math
    - Want to record signal, not noise
  - **Quantization**: Conversion from analog to discrete values
  - **Coding**: Assigning a digital word to each discrete value
    - Thermometer code, Gray code...
- Quantization adds noise
  - Analog signal is continuous
  - Digital representation is approximate
  - Difference (error) is noiselike



Some terminology

- Resolution (n): Number of states in bits
  - Example: A 3-bit A/D
- Full-scale range (FSR): The input or output voltage range
  - Example: ADC inputs outside the FSR are always 111 or 000
- Step size (Q): $\frac{\text{FSR}}{2^n}$
- RMS quantization error: $\frac{Q}{\sqrt{12}}$
  - RMS value of triangle wave
  - http://www.analog.com/media/en/training-seminars/tutorials/MT-001.pdf

Quantization noise



•N-bit converter: $\delta = \frac{V_{FSR}}{2^N}$

Quantization noise

- Noise energy:

$$V_{Q(RMS)} = \sqrt{\frac{1}{\delta}\int_{-\delta/2}^{\delta/2} V_Q^2 dV_Q} = \sqrt{\frac{\delta^2}{12}}$$

- Signal energy:

$$V_{in(RMS)} = \frac{\delta \cdot 2^N}{2\sqrt{2}}$$

- SNR for ideal ADC:

$$SNR = 20\log(\frac{V_{in(RMS)}}{V_{Q(RMS)}})$$

$$SNR = 20\log(2^N \cdot \sqrt{\frac{3}{2}})$$

$$SNR = 6.02 \times N + 1.76[dB]$$

CSE/EE474 21

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$$SNR = 6.02 \times N + 1.76[dB]$$

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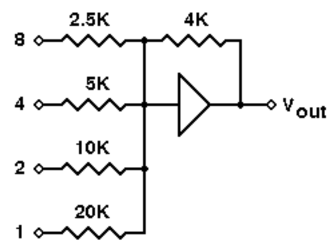
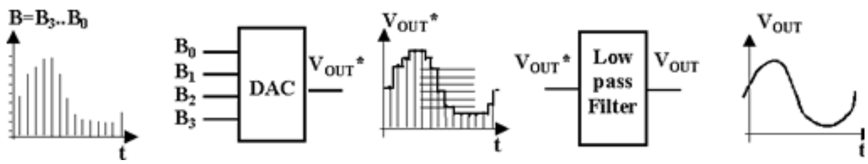
D/A converters

- Easier to design and use than A/Ds
- Types
  - Weighted current source DAC
  - R–2R DAC
  - Multiplying DAC
- Need to smooth the output

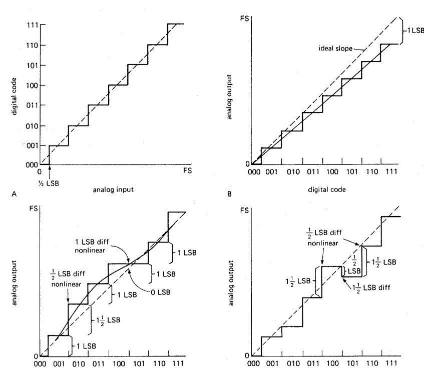


Digital to Analog Converter converts a digital signal to an analog output



You will use DACs

- DAC specs are tricky!
  - Check the errors
  - Check the settling
- Vendors use deceptive advertising
  - 16-bit DAC!!!
    - But errors may give only 12-bit accuracy
    - You have to figure this out from the specs



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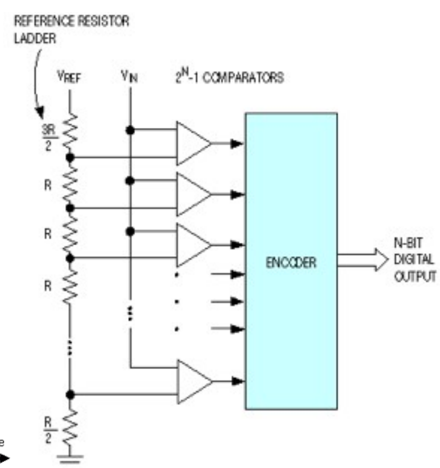
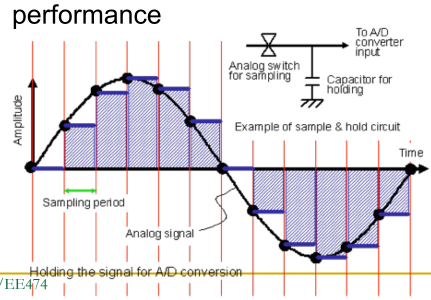
A/D converters

- ***Hard*** to design
- Contain digital parts
  - Encoders
  - FSMs
- Many types
  - Successive approximation
  - Flash
  - Pipelined-flash
  - Integrating
  - Sigma-delta
  - Charge balanced
  - Folding
  - Others

Example: Flash A/D

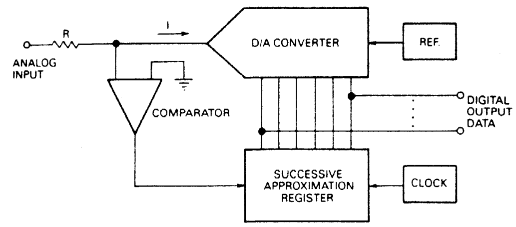
- Advantages
  - Ultra-fast
- Disadvantages
  - High power
  - Low resolution
  - Metastability
- Sample/hold improves performance



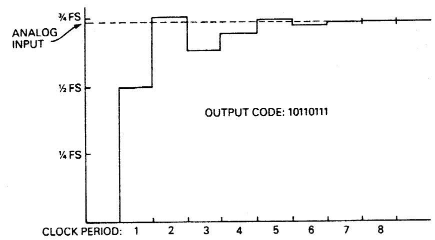


Example: Successive approximation ADC

- Advantages
  - Low power
  - High resolution
- Disadvantages
  - Slow
- Problem: DAC must settle to LSB accuracy at every step



Successive Approximation A/D Converter



D/A Output for 8-Bit Successive Approximation Conversion

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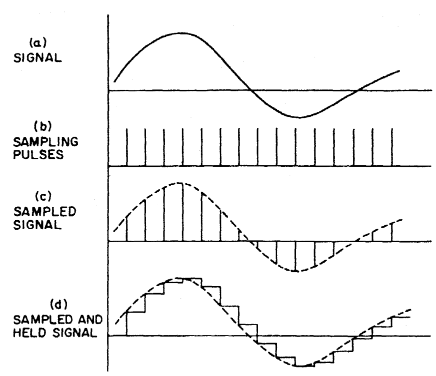
Sampling

- Quantizing a signal
  1) We sample it
  2) We encode the samples
- Questions:
  - How fast do we sample?
  - How do we so this in hardware?
  - What resolution do we need?



Signal Sampling

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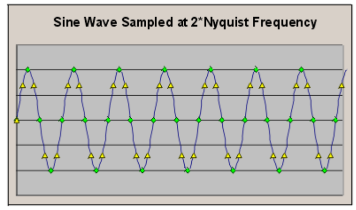
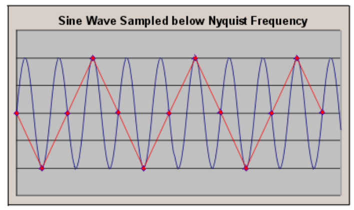
Shannon's sampling theorem

If a continuous, band-limited signal contains no frequency components higher than f_c, then we can recover the original signal without distortion if we sample at a rate of at least $2f_c$ samples/second

- $2f_c$ is called the Nyquist rate
- Real life
  - Sample at $2.5f_c$ or faster
  - Sample clock should not be coherent with the input signal

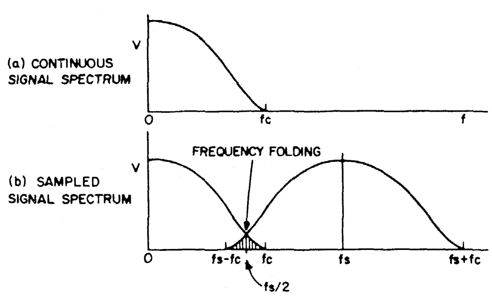




http://www.videomicroscopy.com/vancouverlecture/nyquist.htm

Frequency domain analysis

- Take the Fourier Transform of the signal
  - Shows a signal's *frequency* components
- Undersampled frequency components fold back!



Frequency Spectra Demonstrating the Sampling Theorem

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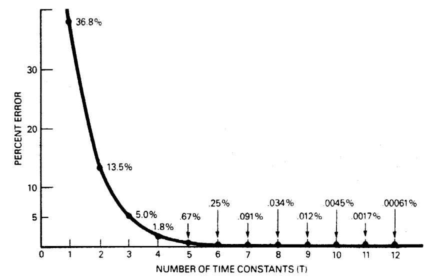
Sampling speed versus bit resolution

- Hardware issues
  - Sampling speed depends on bit resolution
    - Think time constants
    - $\text{Settling error} = e^{-\frac{t}{\tau}}$
- Examples:
  - 8-bit resolution takes 5.5τ
  - 12-bit resolution takes 8.3τ
  - 16-bit resolution takes 11τ



Output Settling Error as a Function of Number of Time Constants

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Nyquist–Shannon sampling theorem

- A theorem, developed by Harry Nyquist, and proven by Claude Shannon, which states that an analog signal waveform may be uniquely reconstructed, without error, from samples taken at equal time intervals.

CSE/EE474 32

Nyquist–Shannon sampling theorem

- The sampling rate must be equal to, or greater than, twice the highest frequency component in the analog signal.
- Stated differently:
- The highest frequency which can be accurately represented is one-half of the sampling rate.

Nyquist Theorem and Aliasing

- Nyquist Theorem:
  We can digitally represent only frequencies up to half the sampling rate.
  - Example:
    CD: SR=44,100 Hz
    Nyquist Frequency = SR/2 = 22,050 Hz
  - Example:
    SR=22,050 Hz
    Nyquist Frequency = SR/2 = 11,025 Hz

Nyquist Theorem and Aliasing

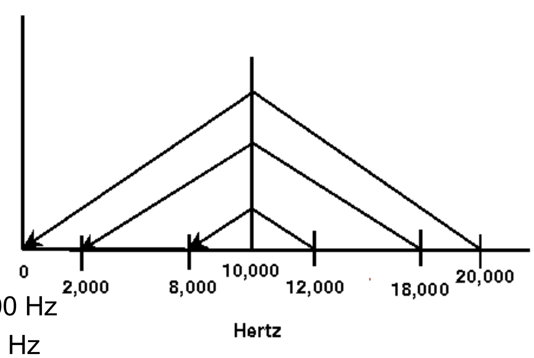
- Frequencies above Nyquist frequency "fold over" to sound like lower frequencies.
  - This foldover is called *aliasing*.
- Aliased frequency f in range [SR/2, SR] becomes f':

$$f' = |f - SR/2|$$

Nyquist Theorem and Aliasing

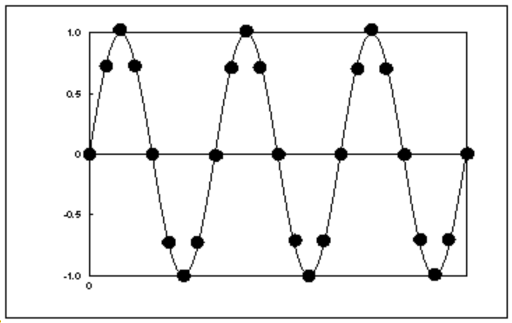
$$f' = |f - SR/2|$$

- <u>Example</u>:
  - SR = 20,000 Hz
  - Nyquist Frequency = 10,000 Hz
  - f = 12,000 Hz --> f' = 8,000 Hz
  - f = 18,000 Hz --> f' = 2,000 Hz
  - f = 20,000 Hz --> f' = 0 Hz



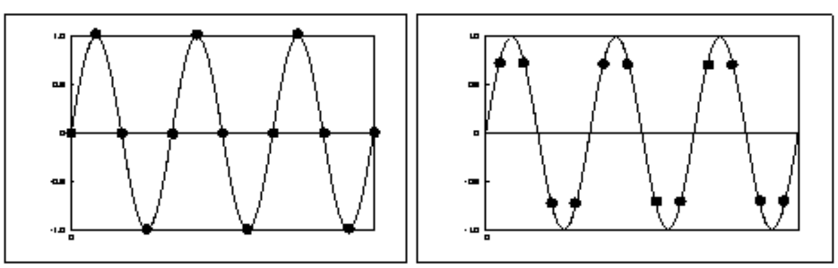
Nyquist Theorem and Aliasing

- Graphical Example 1a:
  - SR = 20,000 Hz
  - Nyquist Frequency = 10,000 Hz
  - f = 2,500 Hz (no aliasing)



Nyquist Theorem and Aliasing

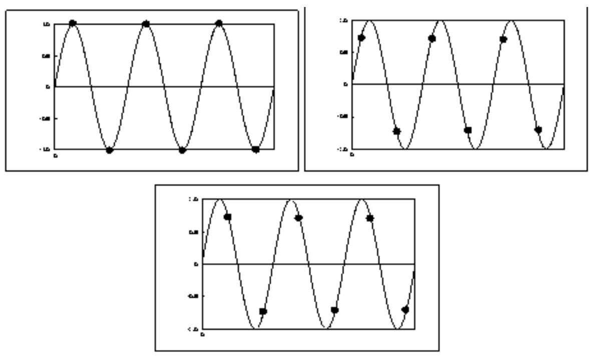
- Graphical Example 1b:
  - SR = 20,000 Hz
  - Nyquist Frequency = 10,000 Hz
  - f = 5,000 Hz (no aliasing)



(left and right figures have same frequency, but have different sampling points)

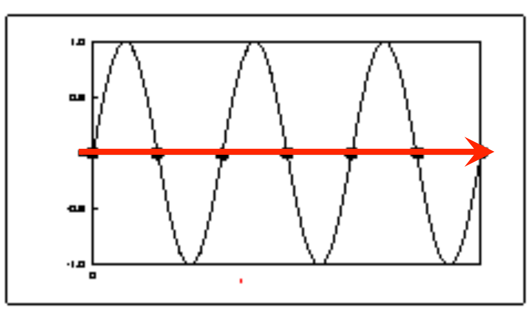
Nyquist Theorem and Aliasing

- Graphical Example 2:
  - SR = 20,000 Hz
  - Nyquist Frequency = 10,000 Hz
  - f = 10,000 Hz (no aliasing)



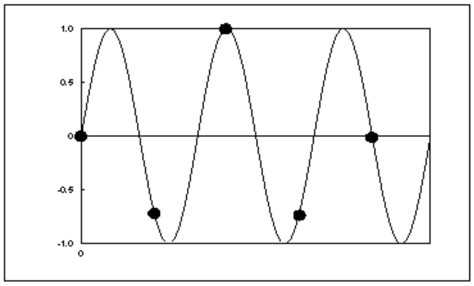
Nyquist Theorem and Aliasing

- Graphical Example 2:
  - BUT, if sample points fall on zero-crossings the sound is completely cancelled out



Nyquist Theorem and Aliasing

- Graphical Example 3:
  - SR = 20,000 Hz
  - Nyquist Frequency = 10,000 Hz
  - f = 12,500 Hz, f' = 7,500



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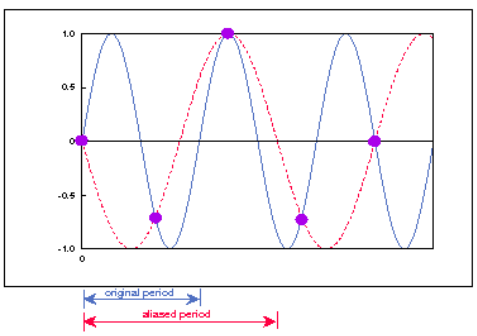
Nyquist Theorem and Aliasing

- Graphical Example 3:
  - Fitting the simplest sine wave to the sampled points gives an aliased waveform (dotted line below):



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Method to reduce aliasing noise

Input voltage = V

Use low pass filter to remove high frequency before sampling