MapReduce Algorithms

CSE 490H

Algorithms for MapReduce

- Sorting
- Searching
- TF-IDF
- BFS
- PageRank
- More advanced algorithms

MapReduce Jobs

- Tend to be very short, code-wise
  - IdentityReducer is very common
- "Utility" jobs can be composed
- Represent a *data flow*, more so than a procedure

Sort: Inputs

- A set of files, one value per line.
- Mapper key is file name, line number
- Mapper value is the contents of the line

Sort Algorithm

- Takes advantage of reducer properties: (key, value) pairs are processed in order by key; reducers are themselves ordered

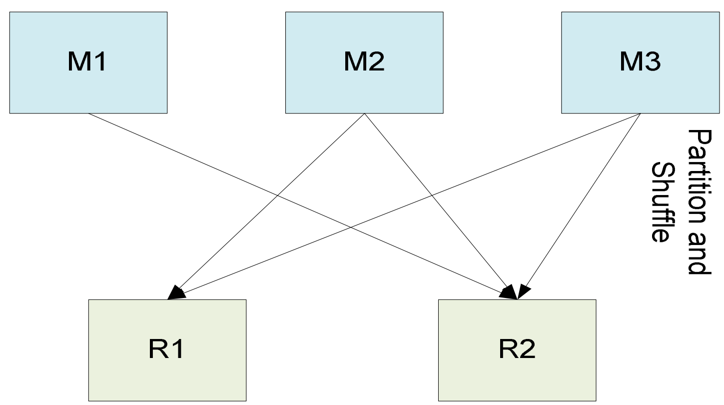
- Mapper: Identity function for value

$$(k, v) \rightarrow (v, \_)$$

- Reducer: Identity function (k', _) -> (k', "")

Sort: The Trick

- (key, value) pairs from mappers are sent to a particular reducer based on hash(key)
- Must pick the hash function for your data such that $k_1 < k_2 \Rightarrow hash(k_1) < hash(k_2)$



Final Thoughts on Sort

- Used as a test of Hadoop's raw speed
- Essentially "IO drag race"
- Highlights utility of GFS

Search: Inputs

- A set of files containing lines of text
- A search pattern to find

- Mapper key is file name, line number
- Mapper value is the contents of the line
- Search pattern sent as special parameter

Search Algorithm

- Mapper:
  - Given (filename, some text) and “pattern”, if “text” matches “pattern” output (filename, _)
- Reducer:
  - Identity function

Search: An Optimization

- Once a file is found to be interesting, we only need to mark it that way once
- Use *Combiner* function to fold redundant (filename, _) pairs into a single one
  - Reduces network I/O

TF-IDF

- Term Frequency – Inverse Document Frequency
  - Relevant to text processing
  - Common web analysis algorithm

The Algorithm, Formally

$$\mathrm{tf_i} = \frac{n_i}{\sum_k n_k}$$

$$\mathrm{idf_i} = \log \frac{|D|}{|\{d : t_i \in d\}|}$$

$$\mathrm{tfidf} = \mathrm{tf} \cdot \mathrm{idf}$$

- $|D|$: total number of documents in the corpus
- $|\{d : t_i \in d\}|$: number of documents where the term t_i appears (that is $n_i \neq 0$).

Information We Need

- Number of times term X appears in a given document
- Number of terms in each document
- Number of documents X appears in
- Total number of documents

Job 1: Word Frequency in Doc

- Mapper
  - Input: (docname, contents)
  - Output: ((word, docname), 1)
- Reducer
  - Sums counts for word in document
  - Outputs ((word, docname), *n*)
- Combiner is same as Reducer

Job 2: Word Counts For Docs

- Mapper
  - Input: ((word, docname), *n*)
  - Output: (docname, (word, *n*))
- Reducer
  - Sums frequency of individual *n*'s in same doc
  - Feeds original data through
  - Outputs ((word, docname), (*n*, *N*))

Job 3: Word Frequency In Corpus

- Mapper
  - Input: ((word, docname), (*n, N*))
  - Output: (word, (docname, *n*, *N*, 1))
- Reducer
  - Sums counts for word in corpus
  - Outputs ((word, docname), (*n*, *N*, *m*))

Job 4: Calculate TF-IDF

- Mapper
  - Input: ((word, docname), (n, N, m))
  - Assume D is known (or, easy MR to find it)
  - Output ((word, docname), TF*IDF)
- Reducer
  - Just the identity function

Working At Scale

- Buffering (doc, *n*, *N*) counts while summing 1's into *m* may not fit in memory
  - How many documents does the word "the" occur in?
- Possible solutions
  - Ignore very-high-frequency words
  - Write out intermediate data to a file
  - Use another MR pass

Final Thoughts on TF-IDF

- Several small jobs add up to full algorithm
- Lots of code reuse possible
  - Stock classes exist for aggregation, identity
- Jobs 3 and 4 can really be done at once in same reducer, saving a write/read cycle
- Very easy to handle medium-large scale, but must take care to ensure flat memory usage for largest scale

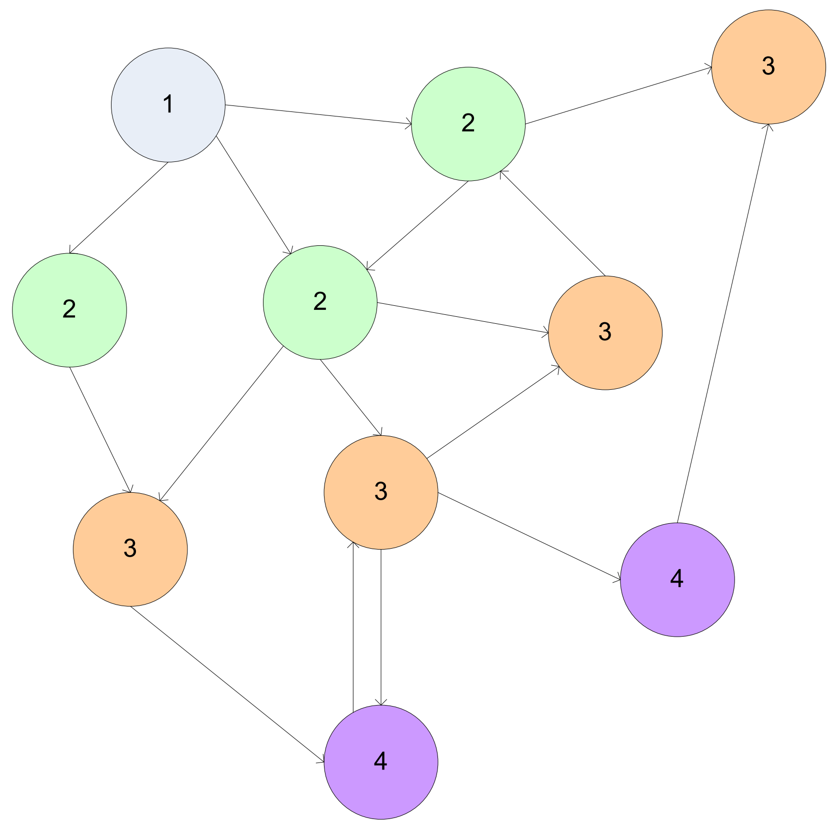
BFS: Motivating Concepts

- Performing computation on a graph data structure requires processing at each node
- Each node contains node-specific data as well as links (edges) to other nodes
- Computation must traverse the graph and perform the computation step

- *How do we traverse a graph in MapReduce? How do we represent the graph for this?*

Breadth-First Search

- Breadth-First Search is an *iterated* algorithm over graphs
- Frontier advances from origin by one level with each pass



Breadth-First Search & MapReduce

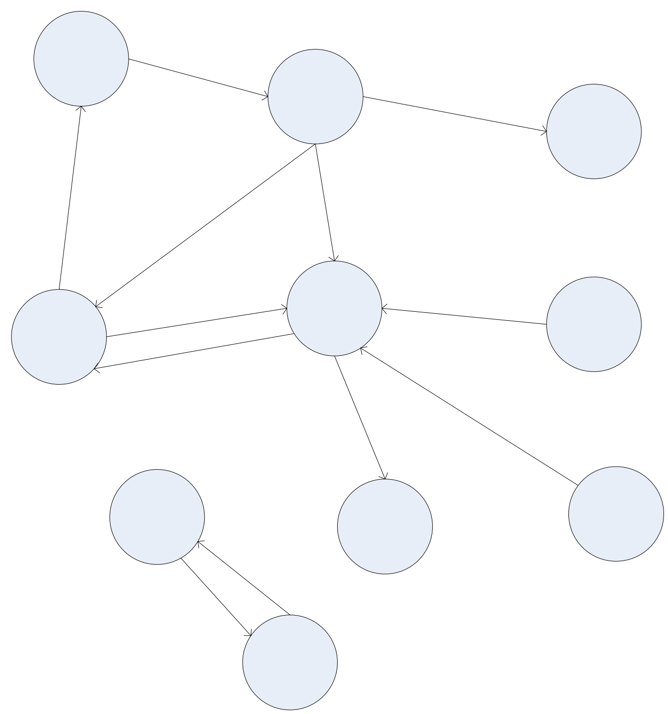
- Problem: This doesn't “fit” into MapReduce
- Solution: Iterated passes through MapReduce – map some nodes, result includes additional nodes which are fed into successive MapReduce passes

Breadth-First Search & MapReduce

- Problem: Sending the entire graph to a map task (or hundreds/thousands of map tasks) involves an enormous amount of memory
- Solution: Carefully consider how we represent graphs

Graph Representations

- The most straightforward representation of graphs uses references from each node to its neighbors



Direct References

- Structure is inherent to object
- Iteration requires linked list “threaded through” graph
- Requires common view of shared memory (synchronization!)
- Not easily serializable

```
class GraphNode
{
  Object data;
  Vector<GraphNode>
    out_edges;
  GraphNode
    iter_next;
}
```

Adjacency Matrices

- Another classic graph representation. M[i][j]= '1' implies a link from node *i to j.*
- Naturally encapsulates iteration over nodes

	1	2	3	4
1	0	1	0	1
2	1	0	1	1
3	0	1	0	0
4	1	0	1	0

Adjacency Matrices: Sparse Representation

- Adjacency matrix for most large graphs (e.g., the web) will be overwhelmingly full of zeros.
- Each row of the graph is absurdly long
- Sparse matrices only include non-zero elements

Sparse Matrix Representation

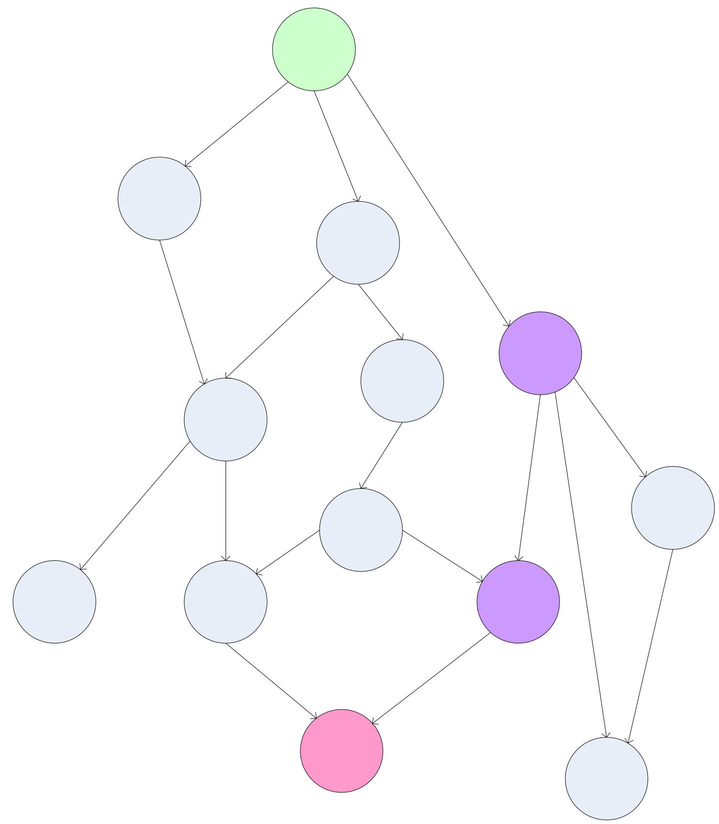
1: (3, 1), (18, 1), (200, 1)
2: (6, 1), (12, 1), (80, 1), (400, 1)
3: (1, 1), (14, 1)
…

Sparse Matrix Representation

1: 3, 18, 200
2: 6, 12, 80, 400
3: 1, 14
…

Finding the Shortest Path

- A common graph search application is finding the shortest path from a start node to one or more target nodes
- Commonly done on a single machine with *Dijkstra's Algorithm*
- Can we use BFS to find the shortest path via MapReduce?



This is called the single-source shortest path problem. (a.k.a. SSSP)

Finding the Shortest Path: Intuition

- We can define the solution to this problem inductively:
  - DistanceTo(startNode) = 0
  - For all nodes *n* directly reachable from startNode, DistanceTo(n) = 1
  - For all nodes *n* reachable from some other set of nodes *S*,

    DistanceTo(n) = 1 + min(DistanceTo(m), m $\in$ S)

From Intuition to Algorithm

- A map task receives a node *n as* a key, and *(D, points-to)* as its value
  - *D* is the distance to the node from the start
  - *points-to* is a list of nodes reachable from *n*
  - $\forall p \in$ points-to, emit (p, D+1)

- Reduce task gathers possible distances to a given *p* and selects the minimum one

What This Gives Us

- This MapReduce task can advance the known frontier by one hop
- To perform the whole BFS, a non-MapReduce component then feeds the output of this step back into the MapReduce task for another iteration
  - Problem: Where'd the points-to list go?
  - Solution: Mapper emits (n, points-to) as well

Blow-up and Termination

- This algorithm starts from one node
- Subsequent iterations include many more nodes of the graph as frontier advances
- Does this ever terminate?
  - Yes! Eventually, routes between nodes will stop being discovered and no better distances will be found. When distance is the same, we stop
  - Mapper should emit *(n, D)* to ensure that "current distance" is carried into the reducer

Adding weights

- Weighted-edge shortest path is more useful than cost==1 approach
- Simple change: points-to list in map task includes a weight 'w' for each pointed-to node
  - emit (p, $D+w_p$) instead of (p, D+1) for each node p
  - Works for positive-weighted graph

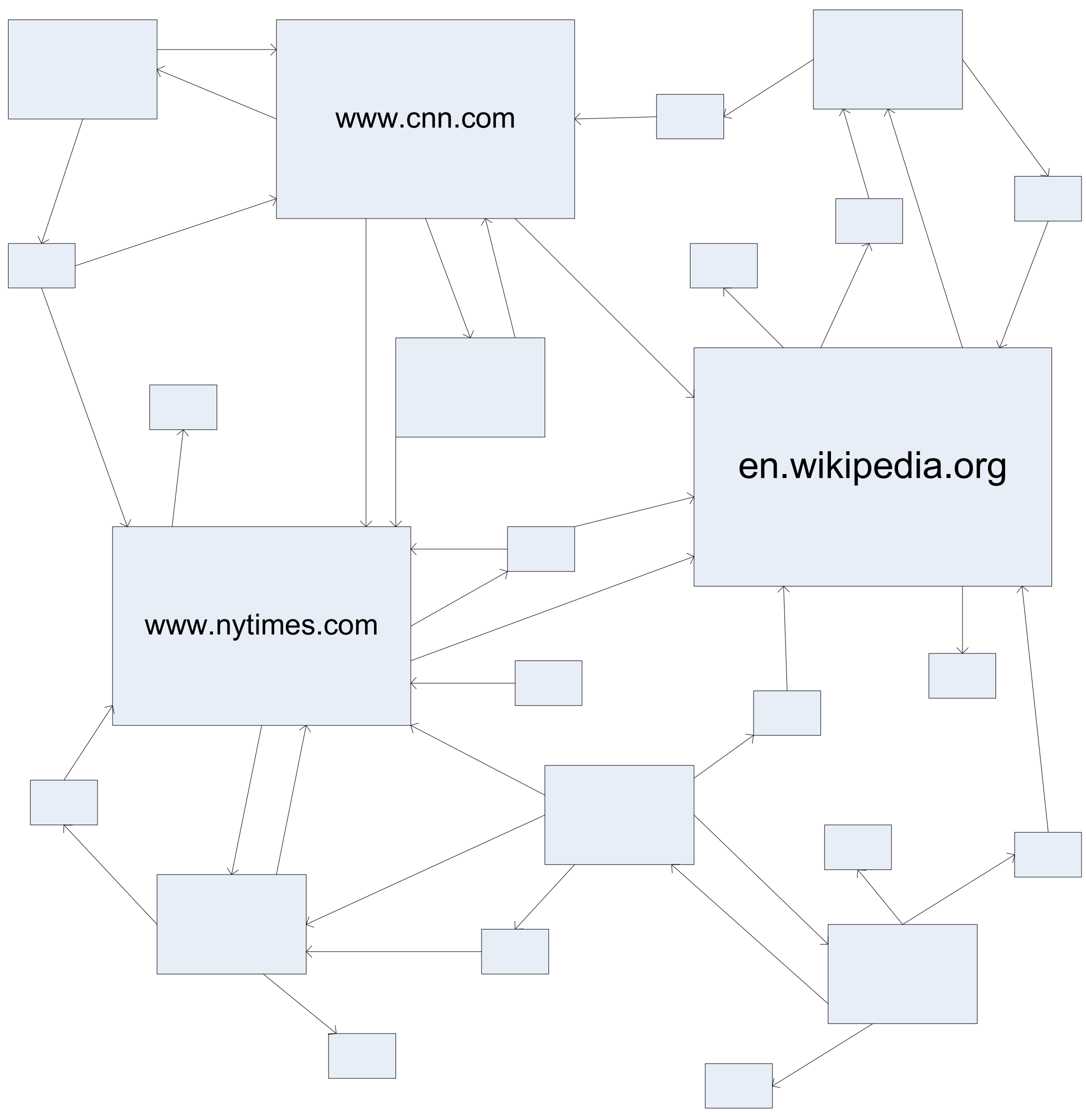
Comparison to Dijkstra

- Dijkstra's algorithm is more efficient because at any step it only pursues edges from the minimum-cost path inside the frontier
- MapReduce version explores all paths in parallel; not as efficient overall, but the architecture is more scalable
- Equivalent to Dijkstra for weight=1 case

PageRank: Random Walks Over The Web

- If a user starts at a random web page and surfs by clicking links and randomly entering new URLs, what is the probability that s/he will arrive at a given page?
- The *PageRank* of a page captures this notion
  - More “popular” or “worthwhile” pages get a higher rank

PageRank: Visually



PageRank: Formula

Given page A, and pages T_1 through T_n linking to A, PageRank is defined as:

$$PR(A) = (1\text{-}d) + d\,(PR(T_1)/C(T_1) + \ldots + PR(T_n)/C(T_n))$$

C(P) is the cardinality (out-degree) of page P

d is the damping (“random URL”) factor

PageRank: Intuition

- Calculation is iterative: PR_{i+1} is based on PR_i
- Each page distributes its PR_i to all pages it links to. Linkees add up their awarded rank fragments to find their PR_{i+1}
- *d* is a tunable parameter (usually = 0.85) encapsulating the “random jump factor”

$$PR(A) = (1-d) + d\,(PR(T_1)/C(T_1) + ... + PR(T_n)/C(T_n))$$

PageRank: First Implementation

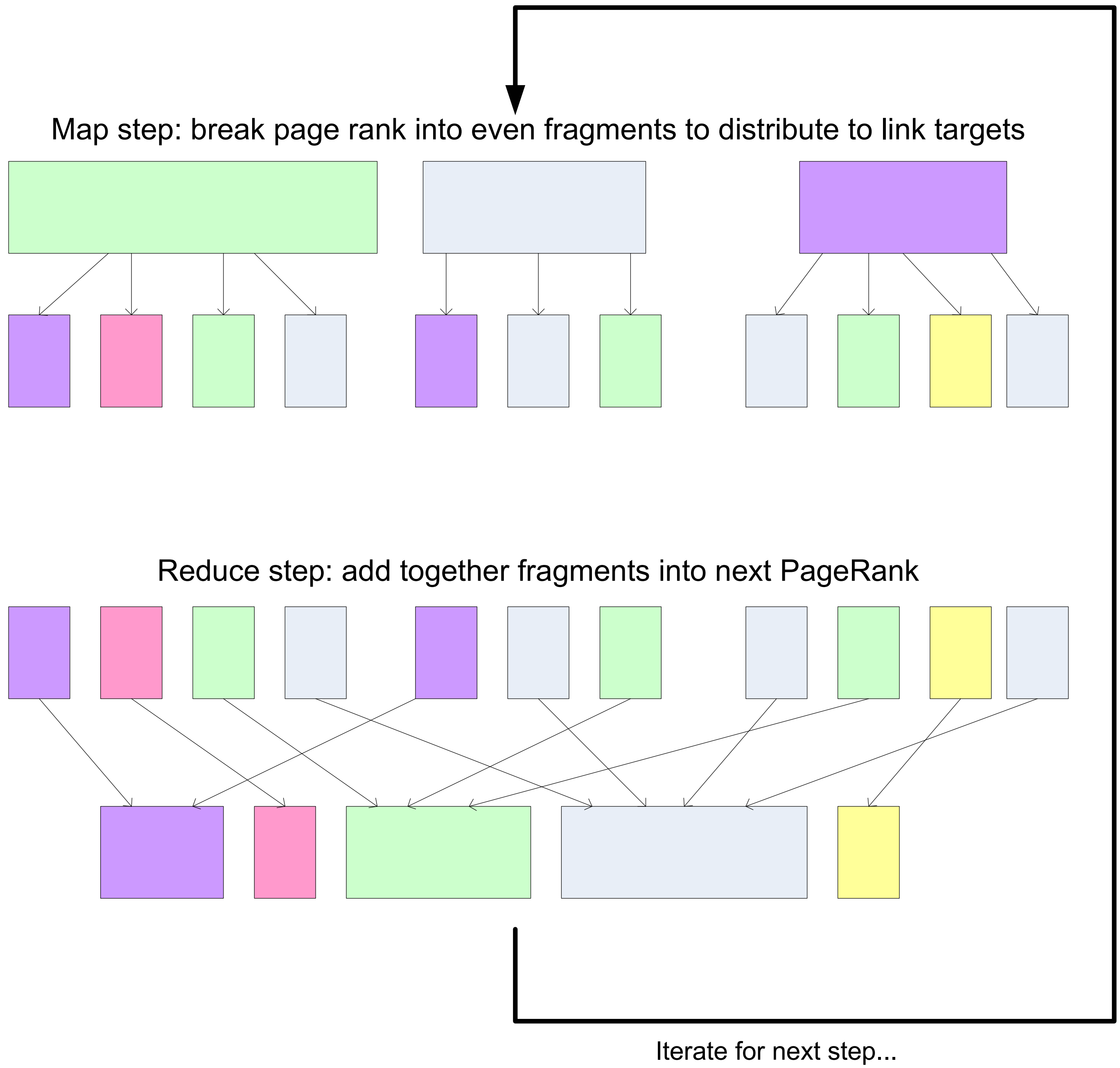
- Create two tables 'current' and 'next' holding the PageRank for each page. Seed 'current' with initial PR values
- Iterate over all pages in the graph, distributing PR from 'current' into 'next' of linkees
- current := next; next := fresh_table();
- Go back to iteration step or end if converged

Distribution of the Algorithm

- Key insights allowing parallelization:
  - The 'next' table depends on 'current', but not on any other rows of 'next'
  - Individual rows of the adjacency matrix can be processed in parallel
  - Sparse matrix rows are relatively small

Distribution of the Algorithm

- Consequences of insights:
  - We can *map* each row of 'current' to a list of PageRank “fragments” to assign to linkees
  - These fragments can be *reduced* into a single PageRank value for a page by summing
  - Graph representation can be even more compact; since each element is simply 0 or 1, only transmit column numbers where it's 1


Map step: break page rank into even fragments to distribute to link targets
Reduce step: add together fragments into next PageRank
Iterate for next step...

Phase 1: Parse HTML

- Map task takes (URL, page content) pairs and maps them to (URL, (PR_{init}, list-of-urls))
  - PR_{init} is the "seed" PageRank for URL
  - list-of-urls contains all pages pointed to by URL
- Reduce task is just the identity function

Phase 2: PageRank Distribution

- Map task takes (URL, (cur_rank, url_list))
  - For each *u* in url_list, emit (*u*, cur_rank/|url_list|)
  - Emit (URL, url_list) to carry the points-to list along through iterations

$$PR(A) = (1\text{-}d) + d\,(PR(T_1)/C(T_1) + \ldots + PR(T_n)/C(T_n))$$

Phase 2: PageRank Distribution

- Reduce task gets (URL, url_list) and many (URL, *val*) values
  - Sum *val*s and fix up with *d*
  - Emit (URL, (new_rank, url_list))

$$PR(A) = (1-d) + d\,(PR(T_1)/C(T_1) + \ldots + PR(T_n)/C(T_n))$$

Finishing up...

- A subsequent component determines whether convergence has been achieved (Fixed number of iterations? Comparison of key values?)
- If so, write out the PageRank lists - done!
- Otherwise, feed output of Phase 2 into another Phase 2 iteration

PageRank Conclusions

- MapReduce runs the “heavy lifting” in iterated computation
- Key element in parallelization is independent PageRank computations in a given step
- Parallelization requires thinking about minimum data partitions to transmit (e.g., compact representations of graph rows)
  - Even the implementation shown today doesn't actually scale to the whole Internet; but it works for intermediate-sized graphs