CSE 521: Design and Analysis of Algorithms
Assignment #6
May 24, 2002
Due: Wednesday, June 5

Reading Assignment: All the stuff I've been handing out.

Problems:

1. Give a linear program for the following optimal pipeline problem: A D bit data object must be sent through a pipeline of n stages. The time it takes an x bit packet to get through stage i is $o_i + \frac{x}{b_i}$, where o_i is the overhead of the i-th stage and b_i is the bandwidth of the i-th stage (in bits/sec). The values o_i and b_i are constants, part of the input. Your linear program should determine how the data should be broken up into k pieces, not necessarily of equal size, so as to minimize the time through the pipeline.

2. Drawing graphs nicely is a problem that arises constantly in applications. Consider the problem of drawing a tree. Some characteristics that would be desirable in the drawing are:

   - All nodes on the same level in the tree should line up horizontally.
   - The vertical distance between a node and it's children in the tree should not be less than some minimum value m.
   - All nodes should lie within a certain window on the screen.
   - The parent of a set of nodes should be centered over those nodes in the horizontal direction.
   - The height and width of the tree drawing should be small.

   How would you formulate the problem of placing the tree nodes in the drawing using linear programming? (The problem statement has purposefully been left somewhat vague. It is up to you to formalize both the problem and the solution.)

3. We discussed the problem of finding a maximum $s-t$ flow in a flow network $G = (V, E)$, where $c_{v,w}$ is the capacity on edge $(v, w) \in E$.

   - Formulate this problem as a linear programming problem. (For purposes of making the rest of this problem easier, I recommend that you introduce a fictitious edge of infinite capacity from t to s and insist on conservation of flow at s and t as well; the objective now is to maximize the flow on this special edge from t to s.)
   - What is the dual program?

- To develop an intuitive understanding of the dual program, consider the integer version of it, where each variable is required to take either value 0 or value 1. Explain why this version of the program is equivalent to minding an $s-t$ cut of minimum value.
- **Extra Credit** We have discussed the fact that optima of linear programs occur at vertices of the feasible set (sometimes referred to as extreme points). Prove that every extreme point solution of the dual to the max flow linear program (i.e. the LP relaxation of the integer program you considered in the previous part of the problem) is integer (i.e., 0/1).

4. Prove that the first-fit algorithm for binpacking uses at most twice the optimal number of bins. (*The binpacking problem:* Let x_1, x_2,...,x_n be the set of real numbers each between 0 and 1. Partition the numbers into as few subsets (bins) as possible so that the sum of numbers in each subset is at most one. *The first-fit algorithm:* Put x_1 into the first bin, and then, for each i, put x_i in the first bin that has room for it, or start a new bin if there is no room in any of the used bins and put x_i in it.)

5. Give an approximation algorithm using randomized rounding with an $O(\log n)$ approximation ratio for the set multicover problem: Given a universe U of n elements, a covering factor r_e for each element $e \in U$, a collection of subsets of U: $\mathcal{S} = \{S_1, \ldots, S_m\}$, and a cost function $c(S_i)$ for each $S_i \in \mathcal{S}$, find a minimum cost subcollection of $\mathcal{S}$ (with repetition allowed) that covers element e at least r_e times, for each $e \in U$. The cost of picking a set k times is k times its cost.