CSE 521: Design and Analysis of Algorithms
Assignment #2
January 12, 2005
Due: Wednesday, January 19

Reading Assignment: K&T, Section 5.1, 5.2, 5.4, 5.6, 5.7, 5.9

Questions:

1. (Kleinberg and Tardos, Chapter 4, Problem 5) Let's consider a long, quiet country road with houses scattered very sparsely along it. (We can picture the road as a long line segment, with an eastern endpoint and a western endpoint.) Further, let's suppose that despite the bucolic setting, the residents of all these houses are avid cell phone users. You want to place cell phone base stations at certain points along the road, so that every house is within 4 miles of one of the base stations.

   Give an efficient algorithm that achieves this goal, using as few base stations as possible.

2. (Kleinberg and Tardos, Chapter 4, Problem 7) Your friend is working as a camp counselor, and he is in charge of organizing activities for a set of junior-high-school-age campers. One of his plans is the following minitriathalon exercise: each contestant must swim 20 laps of a pool, then bike 10 miles, then run 3 miles. The plan is to send the contestants out in a staggered fashion, via the following rule: the contestants must use the pool one at a time. In other words, first one contestant swims the 20 laps, gets out, and starts biking. As soon as this first person is out of the pool, a second contestant begins swimming the 20 laps; as soon as he/she's out and starts biking, a third contestant begins swimming ... and so on.)

   Each contestant has a projected *swimming time* (the expected time it will take him or her to complete the 20 laps), a projected *biking time* (the expected time it will take him or her to complete the 10 miles of bicycling), and a projected *running time* (the time it will take him or her to complete the 3 miles of running). Your friend wants to decide on a *schedule* for the triathalon: an order in which to sequence the starts of the contestants. Let's say that the *completion time* of a schedule is the earliest time at which all contestants will be finished with all three legs of the triathalon, assuming they each spend exactly their projected swimming, biking, and running times on the three parts. (Again, note that participants can bike and run simultaneously, but at most one person can be in the pool at any time.) What's the best order for sending people out, if one wants the whole competition to be over as early as possible? More precisely, give an efficient algorithm that produces a schedule whose completion time is as small as possible.

3. (Kleinberg and Tardos, Chapter 4, Problem 9) One of the basic motivations behind the minimum spanning tree problem is the goal of designing a spanning network for a set of nodes with minimum *total* cost. Here, we explore another type of objective: designing a spanning network for which the *most expensive* edge is as cheap as possible.

   Specifically, let $G = (V, E)$ be a connected graph with n vertices, m edges, and positive edge weights that you may assume are all distinct. Let $T = (V, E')$ be a spanning tree of G; we define the *bottleneck edge* of T to be the edge of T with the greatest weight.

   A spanning tree T of G is a *minimum bottleneck spanning tree* if there is no spanning tree T' of G with a lighter bottleneck edge.

   (a) Is every minimum bottleneck tree of G a minimum spanning tree of G? Prove or give a counter-example.

   (b) Is every minimum spanning tree of G a minimum bottleneck tree of G? Prove or give a counter-example.

4. (Kleinberg and Tardos, Chapter 4, Problem 31) Given a list of n natural numbers $d_1, d_2, \ldots, d_n$, show how to decide in polynomial time whether there exists an undirected graph $G = (V, E)$ whose node degrees are precisely the numbers $d_1, d_2, \ldots, d_n$. (That is, if $V = \{v_1, v_2, \ldots, v_n\}$, then the degree of v_i should be exactly d_i.) G should not contain multiple edges between the same pair of nodes, or "loop" edges with both endpoints equal to the same node.

5. Let $M = (E, \mathcal{F})$ be a matroid, and let $\mathcal{B}$ be the collection of bases of M.

   (a) Prove: If $B_1 \in \mathcal{B}$ and $B_2 \in \mathcal{B}$ and $x \in B_2 \setminus B_1$, then there is an element y in $B_1 \setminus B_2$ such that $B_1 \setminus \{y\} \cup \{x\}$ is a basis.

   (b) Prove that $(E, \{E \setminus B | B \in \mathcal{B}\})$ is a matroid.