CSE 521: Design and Analysis of Algorithms
Assignment #7
Due: Wednesday, Feb 23

Problems:

1. `QuickSelect` is the following simple algorithm for finding the k-th smallest element in an unsorted set S.

   `QuickSelect`(S, k):

   (a) Pick a pivot element p uniformly at random from S.

   (b) By comparing p to each element of S, split S into two pieces: $S_1 = \{x \in S | x < p\}$ and $S_2 = \{x \in S | x > p\}$

   (c) If $|S_1| = k - 1$ then output p
   If $|S_1| > k - 1$, then output `QuickSelect`(S_1, k)
   If $|S_1| < k - 1$, then output `QuickSelect`(S_2, $k - |S_1| - 1$)

   Prove the best bound you can on the expected number of comparisons made by `QuickSelect` on a set S of size n. You may assume that initially $k = n/2$ (i.e., we are trying to find the median of S) which is the worst case.

2. Generalizing the notion of a cut-set, we define an r-way cut-set in a graph as a set of edges whose removal breaks the graph into r or more connected components. Explain how the basic randomized min-cut algorithm (not the recursive version) can be used to find minimum r-way cut sets, and bound the probability it succeeds in one iteration. How many repetitions of a complete iteration would be needed to reduce the probability of error to $1/n$.

3. Think about and explain as best you can the following design decisions from the linear time randomized minimum spanning tree algorithm:

   - the decision to do two Boruvka steps at the beginning (as opposed to say 0, 1 or more than 2 Boruvka steps).
   - the decision to sample half the edges as opposed to a fraction p of the edges for some choice of $p \neq 1/2$.