CSE 521: Design and Analysis of Algorithms **Fall 2019**

Problem Set 5

Deadline: December 6th (at 6:00PM) in canvas

1) You are given data containing grades in different courses for 5 students; say $G_{i,j}$ is the grade of student i in course j. (Of course, $G_{i,j}$ is not defined for all i, j since each student has only taken a few courses.) We are trying to "explain" the grades as a linear function of the student's innate aptitude, the easiness of the course and some error term.

$$G_{i,j} = \text{aptitude}_i + \text{easiness}_j + \epsilon_{i,j},$$

where $\epsilon_{i,j}$ is an error term of the linear model. We want to find the best model that minimizes the sum of the $|\epsilon_{i,j}|$'s.

a) Write a linear program to find aptitude_i and easiness_j for all i, j minimizing $\sum_{i,j} |\epsilon_{i,j}|$.

b) Use any standard package for linear programming (Matlab/CVX, Freemat, Sci-Python, Excel etc.; we recommend CVX on matlab) to fit the best model to this data. Include a printout of your code, the objective value of the optimum, $\sum_{i,j} |\epsilon_{i,j}|$, and the calculated easiness values of all the courses and the aptitudes of all the students.

	MAT	CHE	ANT	REL	POL	ECO	COS
Alice	B		A	B+	A-	C	
Bob	B+	A-		A-		B+	B
Chris	B	B+			A	A-	B+
David	A+		B-	A		A+	B-
Evan		B-	D	B+	B	B	C

Assume $A = 4, B = 3$ and so on. Also, let $B- = 2.66$, $B+ = 3.33$ and $A- = 3.66$, $A+ = 4.33$ and so on.

2) Write the dual of the LP relaxation of the vertex cover problem. Recall for a graph $G = (V, E)$, the LP relaxation of the vertex cover is as follows:

$$\begin{aligned} \min \quad & \sum_v c_v x_v \\ \text{s.t.}, \quad & x_u + x_v \geq 1 \quad \forall u \sim v, \\ & x_v \geq 0 \qquad \forall v \in V. \end{aligned} \tag{5.1}$$