CSE 521: The Design and Analysis of Algorithms
Assignment #1
January 14, 1999
due: Thursday, January 21

1. Someone asked in class whether there was any way other than Strassen's to multiply two matrices in $M_2(R)$ using 7 multiplications in the underlying ring R. (This problem will teach you not to ask questions like that.) Show that there is a way that first computes r and u using 4 multiplications, and then computes s and t with 3 additional multiplications. For ease of discovery and grading, show both the graphical devices used on pages 741–743 (the 4×4 "dot matrices") and the actual ring operations on $a, b, \ldots, h$.

2. (a) Let m_c be the minimum number of multiplications to multiply two $c \times c$ matrices. (For instance, the "standard" method shows $m_2 \leq 8$, and Strassen showed $m_2 \leq 7$.) Determine the asymptotic complexity of $n \times n$ matrix multiplication, in terms of n, c, and m_c, which occurs if the matrices are decomposed into c^2 submatrices each of size $n/c \times n/c$, and the m_c-multiplication algorithm is used recursively. You may assume that $m_c > c^2$.

   (b) Exercise 31.2-3 on page 744. (For your interest, Laderman has shown that $m_3 \leq 23$.)

   (c) Exercise 31.2-4 on page 744.

3. Exercise 31.3-8 on page 749. (Hint: Consider the Boolean matrix A with entries

$$A_{ij} = \begin{cases} 1, & \text{if } i = j \text{ or } (i,j) \text{ is an edge} \\ 0, & \text{otherwise} \end{cases} .$$

   What is the (i,j) entry of A^t, where this tth power is computed via Boolean matrix multiplication?)