CSE 521: The Design and Analysis of Algorithms
Assignment #3
February 4, 1999
due: Thursday, February 11

1. Exercise 16.3-6 on page 319. Support your algorithm with a sketch of correctness and time analysis. (If you cannot solve this exercise, substitute 16.3-5.)

2. In this problem you will devise an algorithm for a variant of alignment called "local alignment" that often concerns biologists. Given two strings S and T, the local alignment problem is to find (contiguous) substrings A of S and B of T such that the optimal (ordinary) alignment of A and B has the maximum value, over all such substrings A and B. For instance, if a match scores $+2$ and a mismatch, insertion, or deletion scores -1, then an optimal local alignment of abcxdex and xxxcde is $\begin{matrix} \text{c} & \text{x} & \text{d} & \text{e} \\ \text{c} & \text{-} & \text{d} & \text{e} \end{matrix}$.

   Devise an $O(nm)$ time algorithm, where $n = |S|$ and $m = |T|$. You may assume that $\sigma(x, -) \leq 0$ and $\sigma(-, x) \leq 0$. (Hint: let $v(i, j)$ be the maximum value of an optimal (ordinary) alignment of α and β, over all suffixes α of $S[1..i]$ and all suffixes β of $T[1..j]$.)

3. Show how to solve the fractional knapsack problem in $O(n)$ time. You will need the result of Section 10.3, that the median of n numbers can be found in $O(n)$ time.

4. Exercise 17.4-4 on page 350.