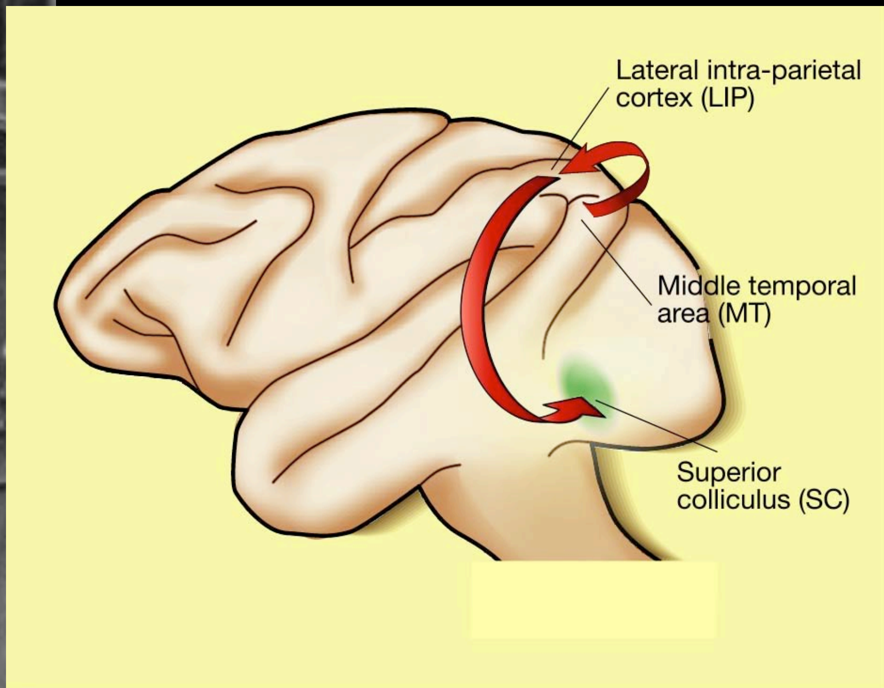
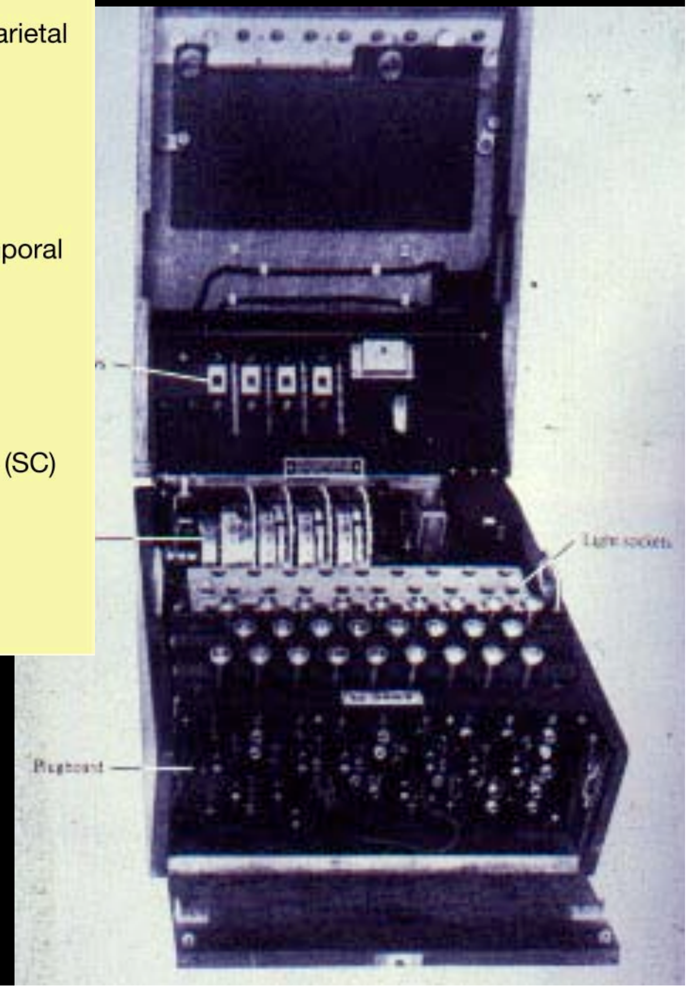
A Neural Mechanism for Decision Making

K C Y W D K D O P E D B A I Q S D F M K C N F A E O I E N C V N S

E N C H P D N C O E N A S H Q E N D N C K R N D N Q I O M Z C P Q





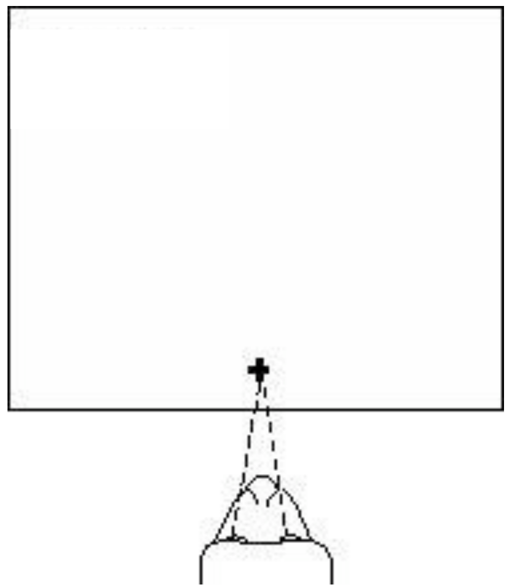
What is a decision?

- A commitment to a proposition or selection of an action
- Based on
  - evidence
  - prior knowledge
  - payoff

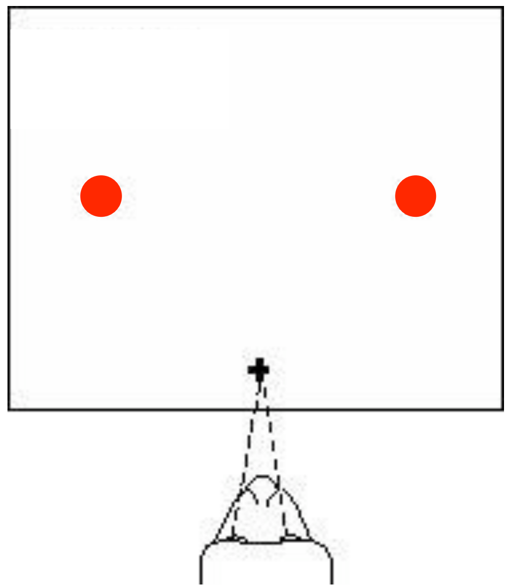
Why study decisions?

- They are a model of higher brain function
- They are experimentally tractable
  - Combined behavior and physiology in rhesus monkeys

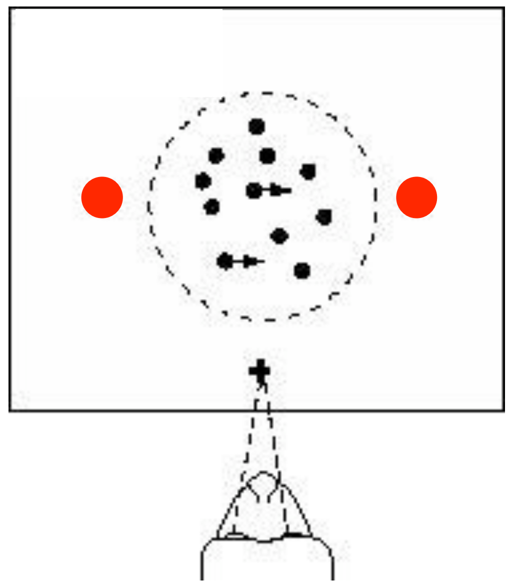
Direction-Discrimination Task
Reaction-time version

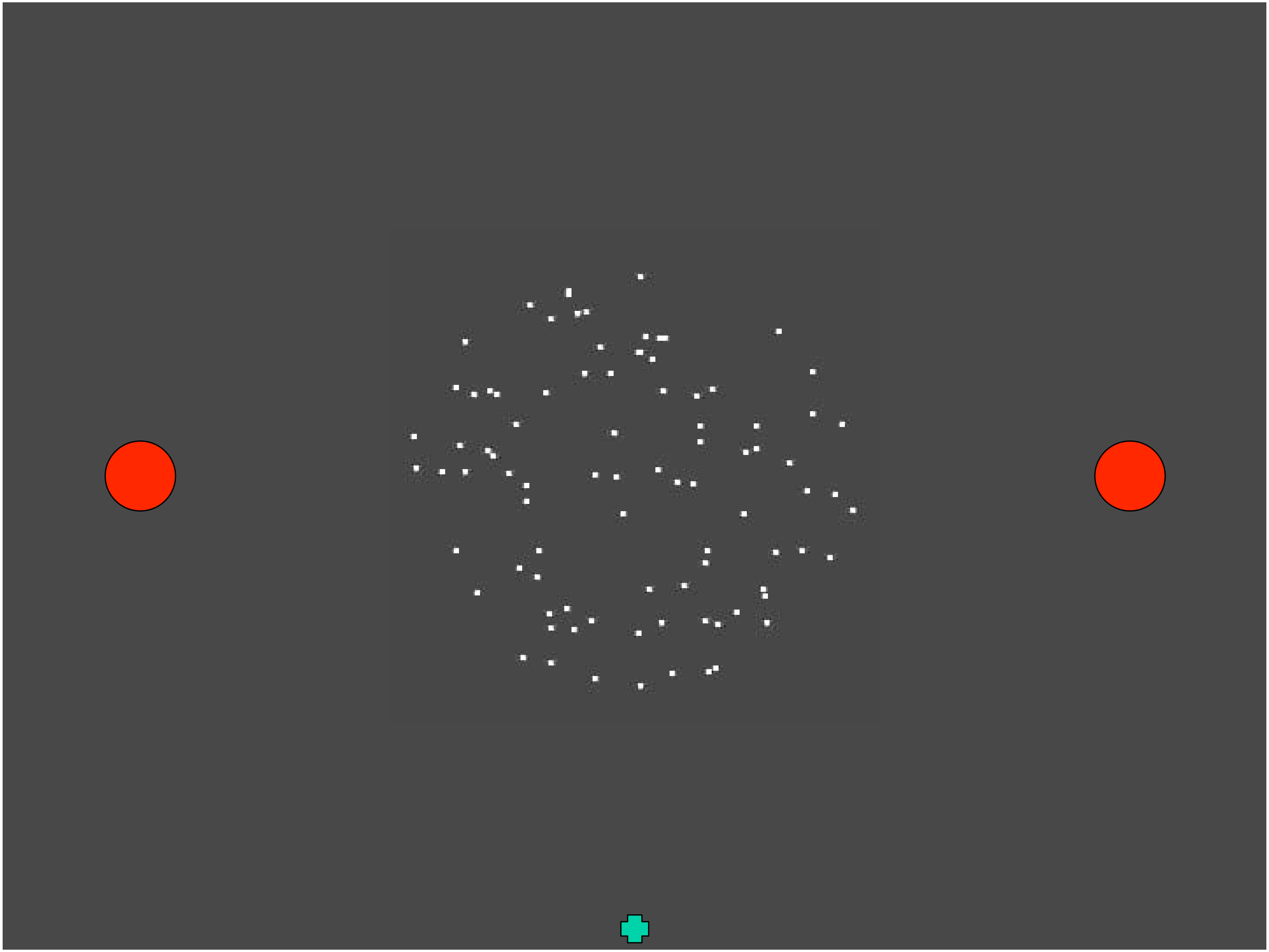


Direction-Discrimination Task
Reaction-time version

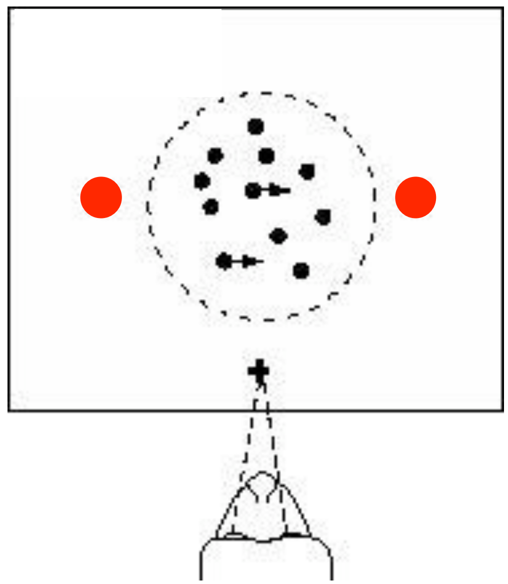


Direction-Discrimination Task
Reaction-time version



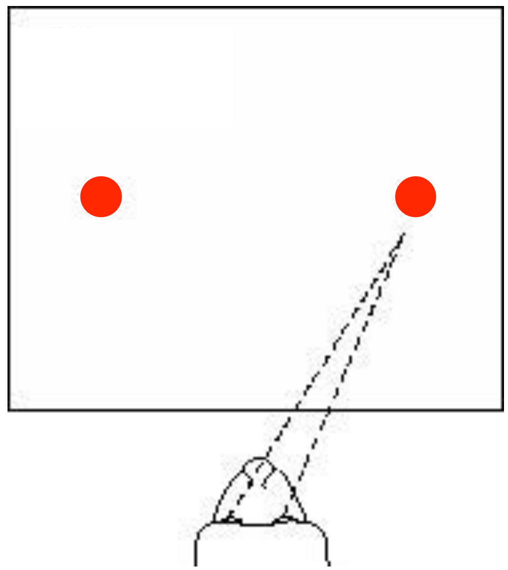


Direction-Discrimination Task
Reaction-time version

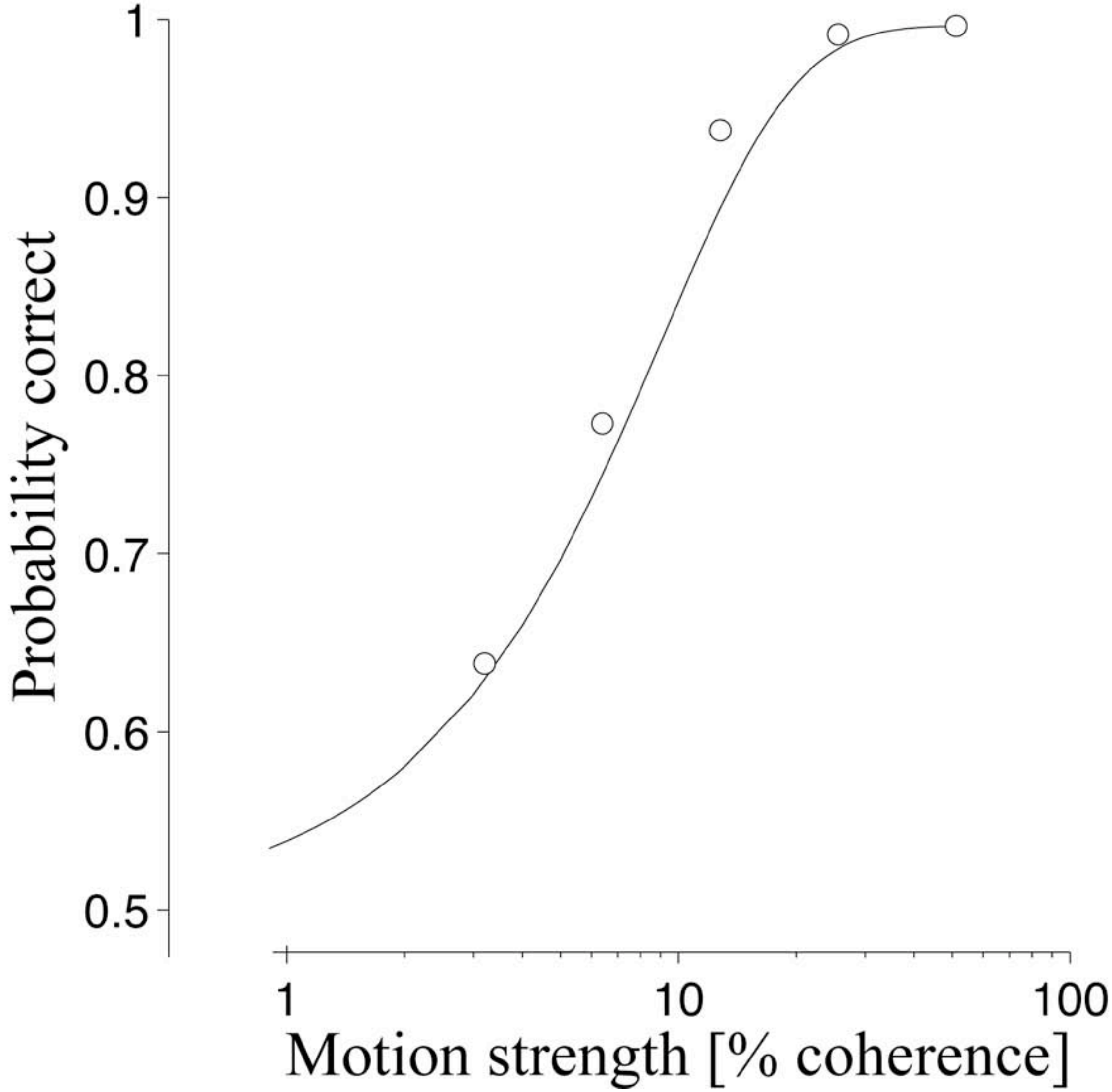


Direction-Discrimination Task
Reaction-time version

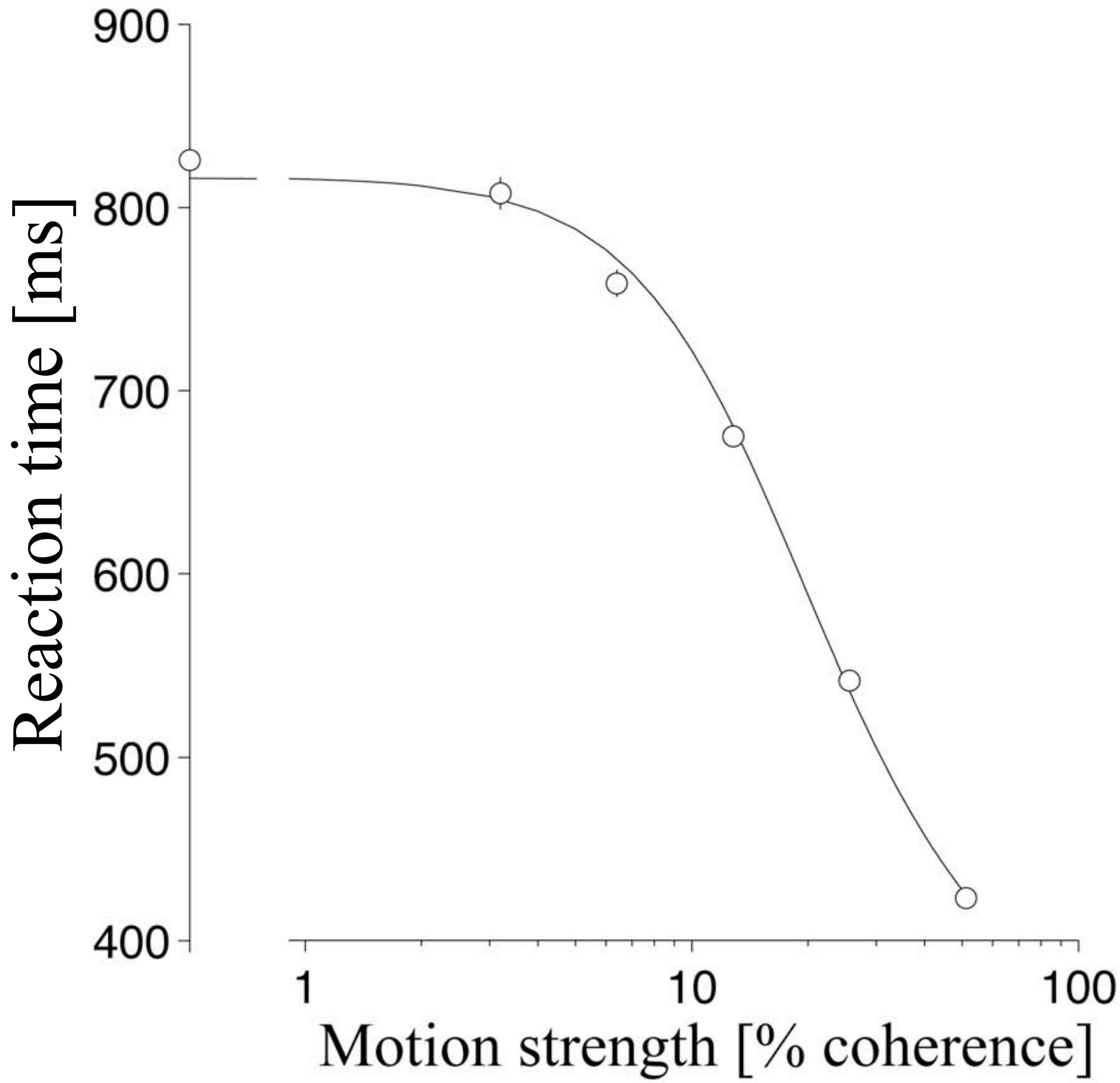
Reward for correct choice

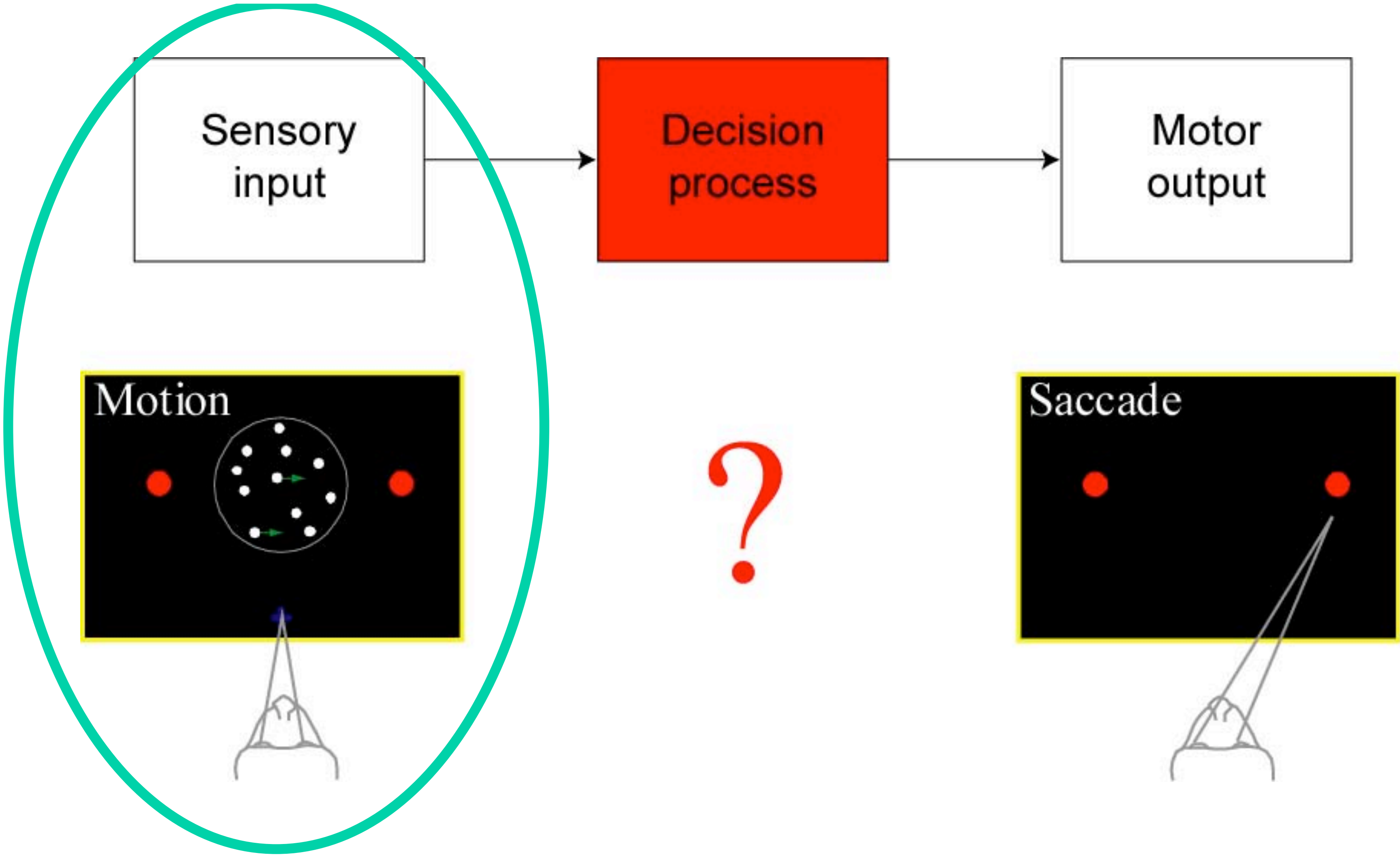


Psychometric function: Accuracy

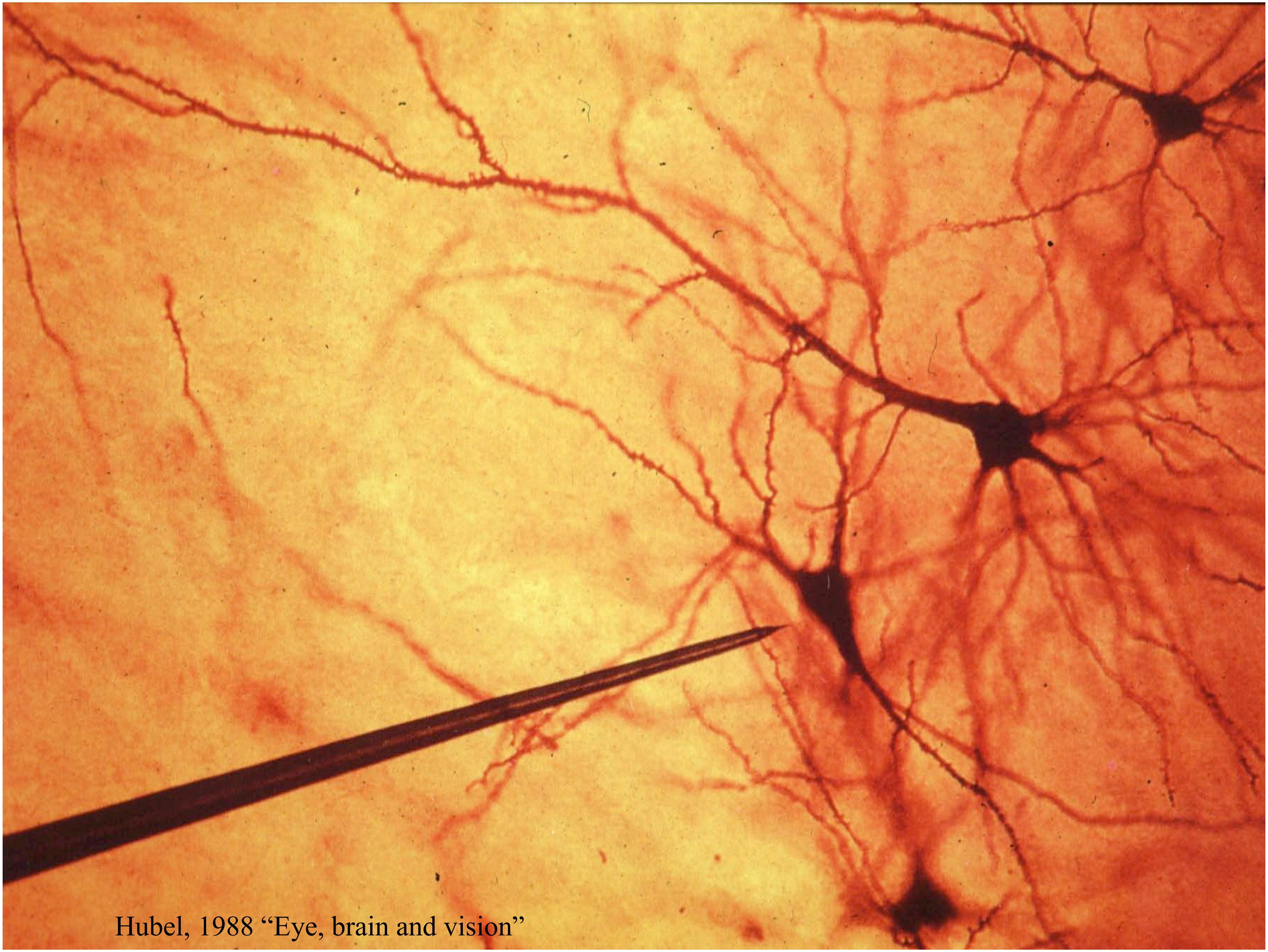


Chronometric function: Speed



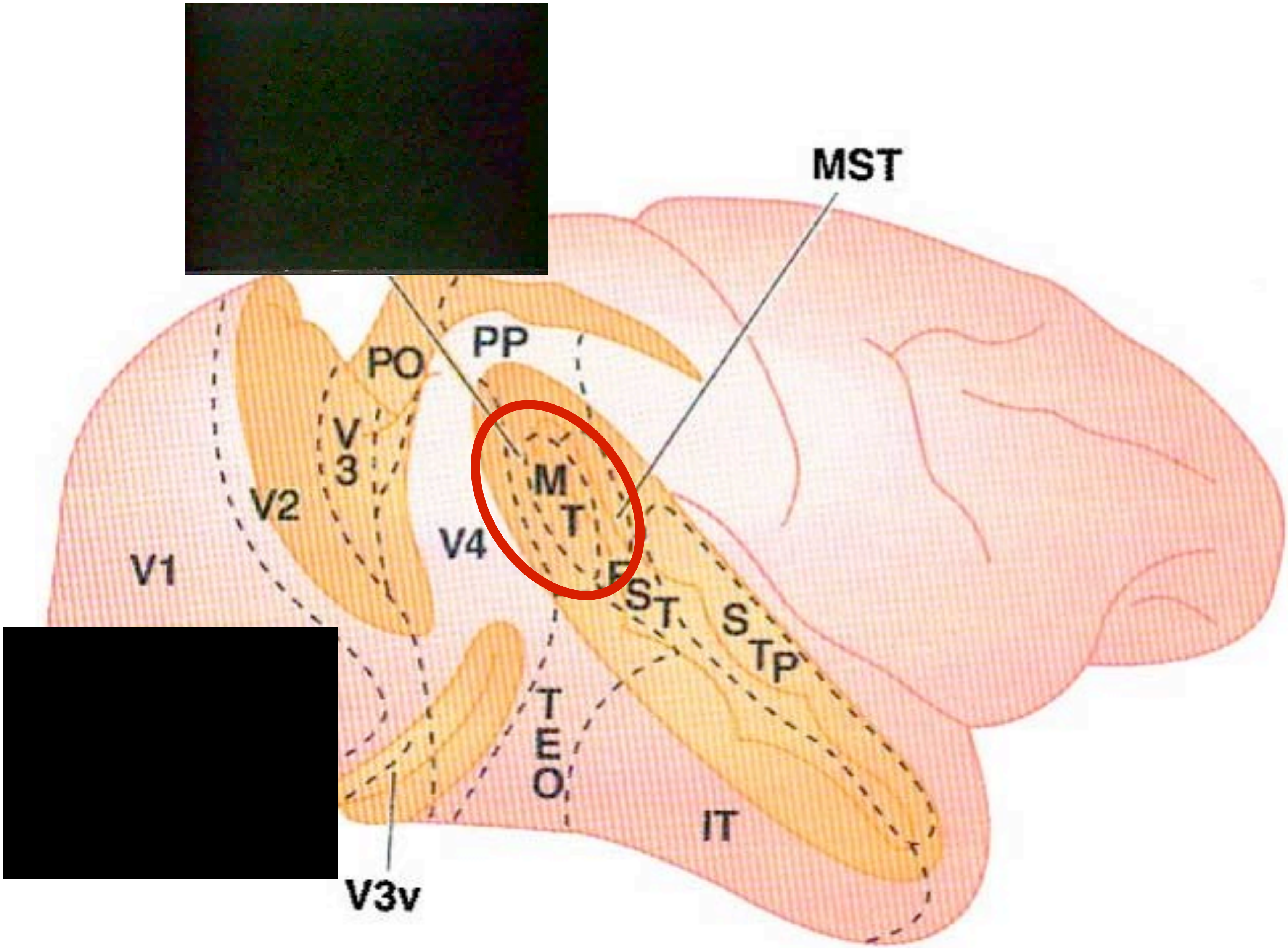

Sensory input
Decision process
Motor output
Motion
?
Saccade

Information is coded by spikes



Hubel, 1988 “Eye, brain and vision”

Sensory "Evidence"



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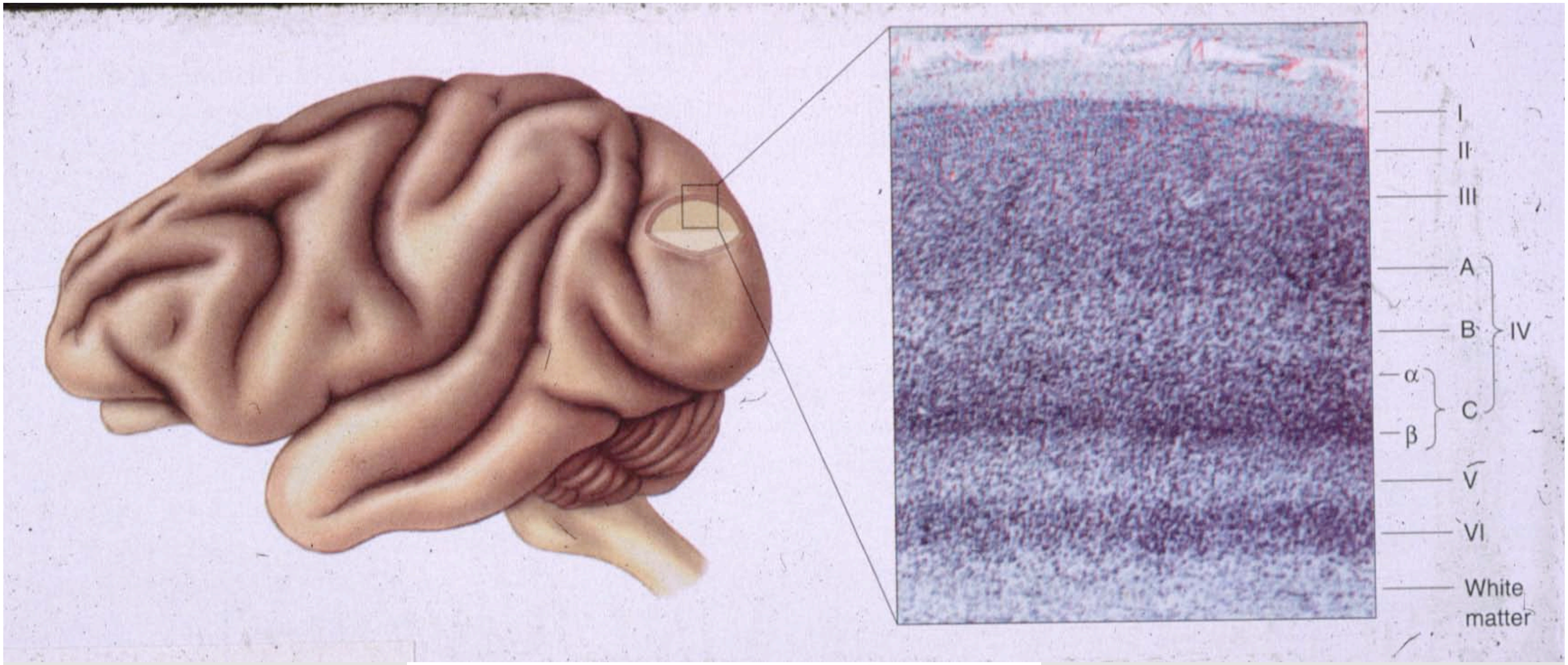
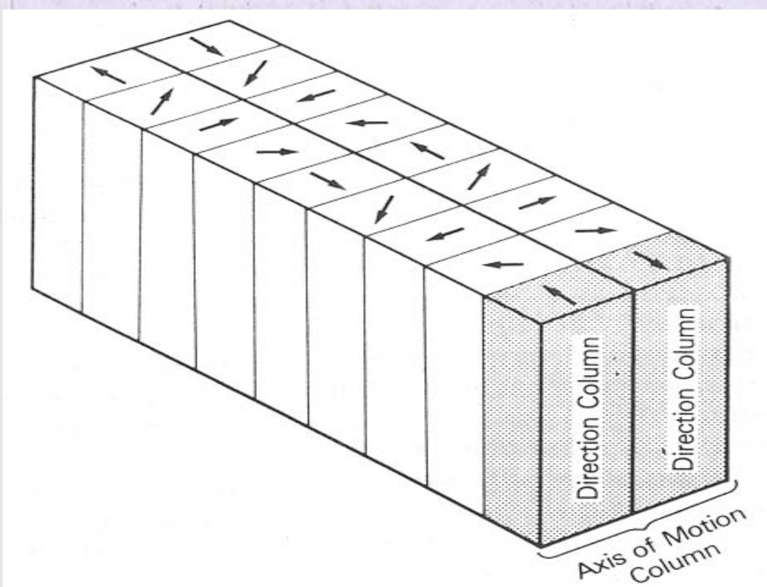
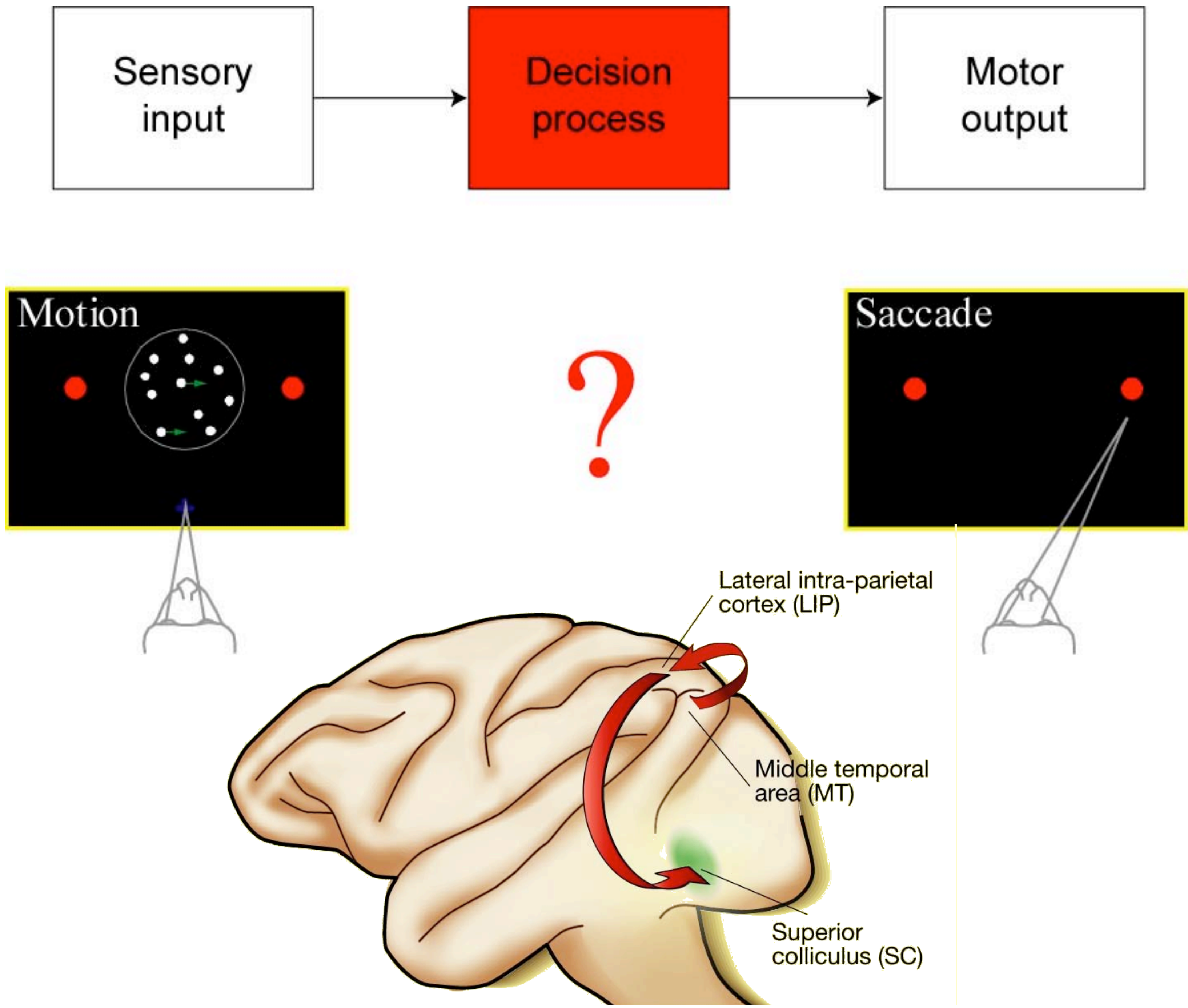
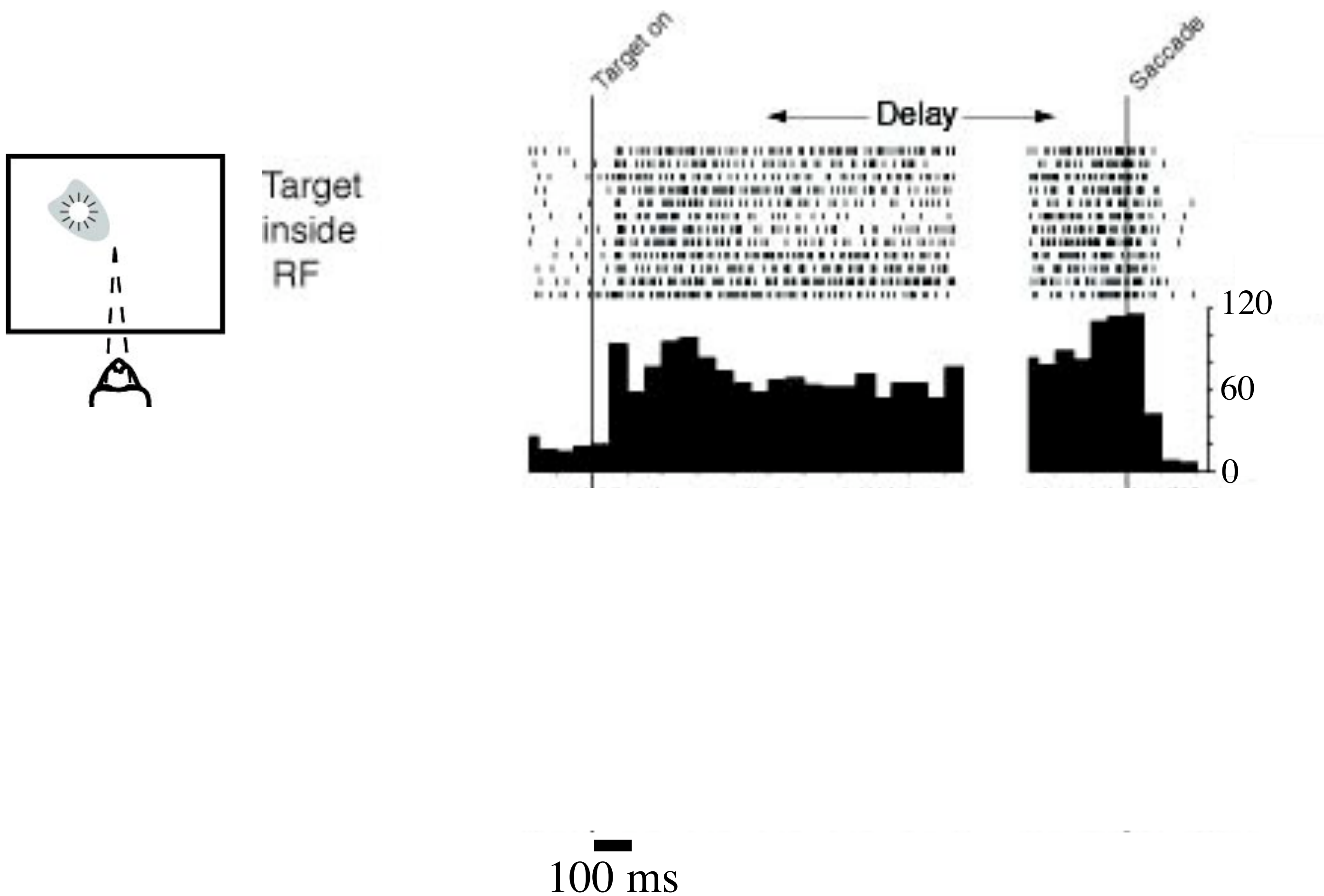




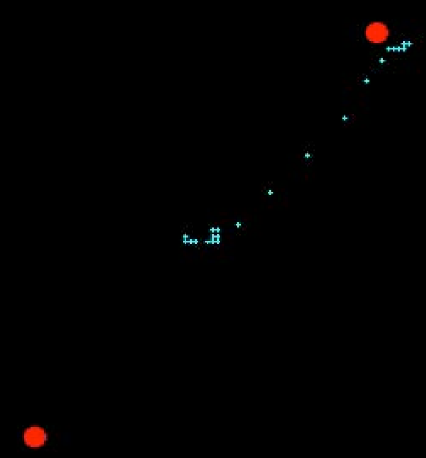
Albright et al., 1984 J. Neurophysiol.


Sensory input
Decision process
Motor output
Motion
?
Saccade
Lateral intra-parietal cortex (LIP)
Middle temporal area (MT)
Superior colliculus (SC)

Spatially-selective, eye movement-related, persistent activity in area LIP



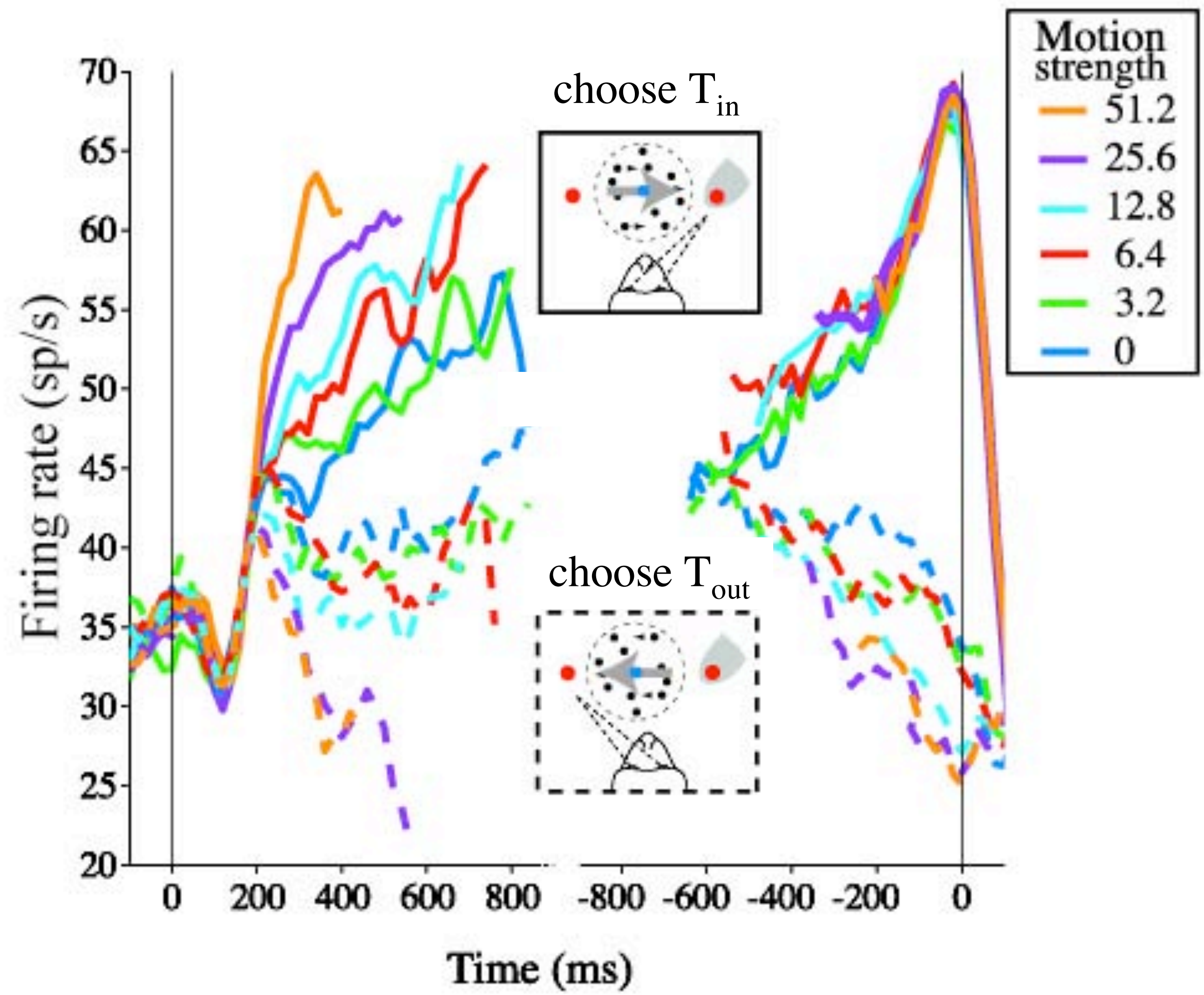
LIP activity during direction discrimination task



LIP activity during direction discrimination task

LIP activity during direction discrimination task

Average LIP activity in RT motion task



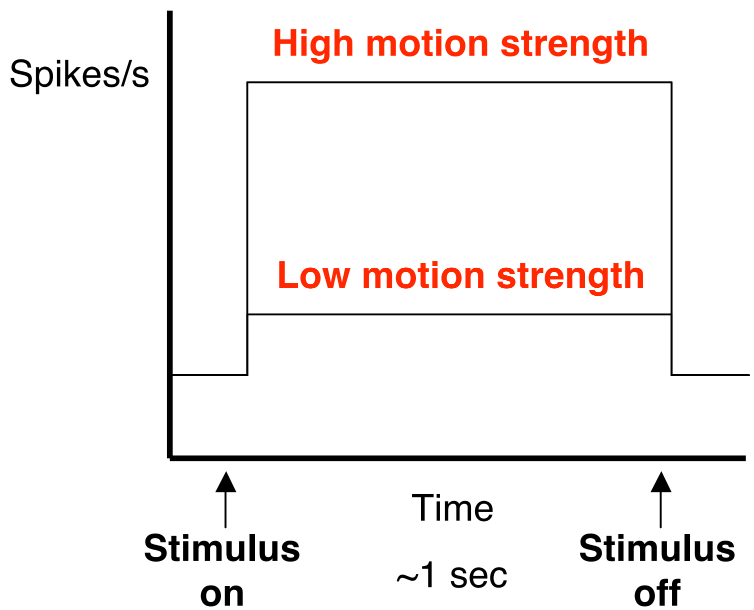
Roitman & Shadlen, 2002 J. Neurosci.

A Neural Integrator for Decisions?

<u>MT: Sensory Evidence</u>

Motion energy

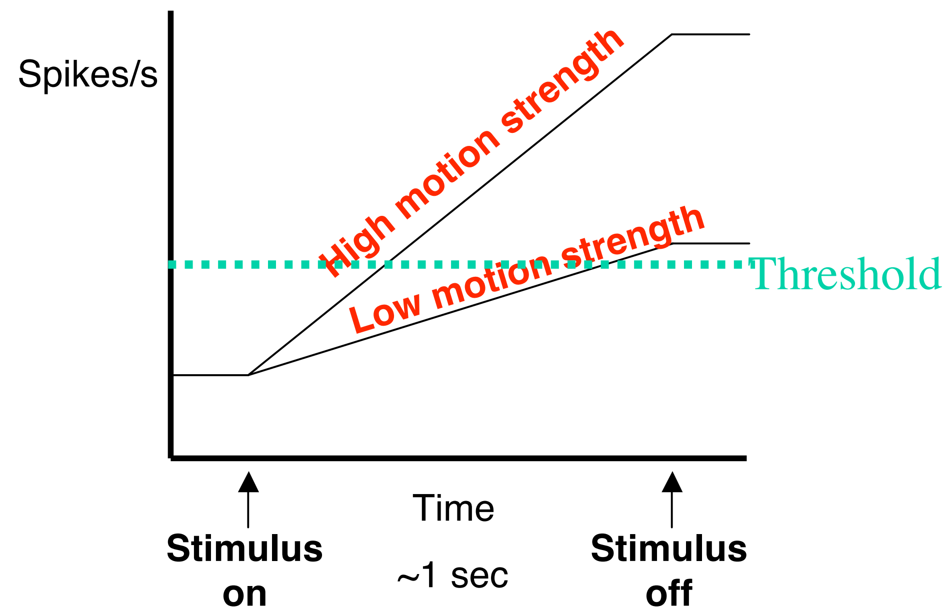
"step"



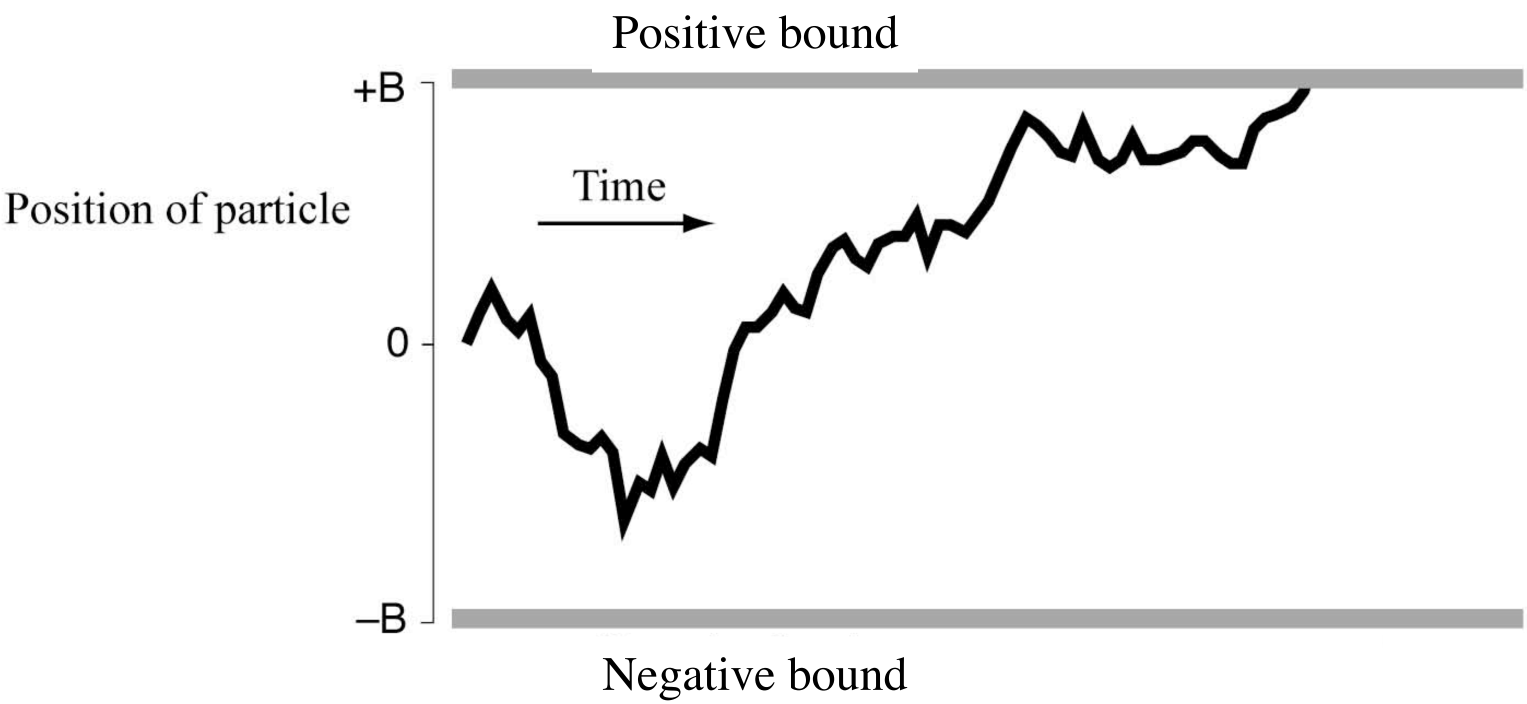
<u>LIP: Decision Formation</u>

Accumulation of evidence

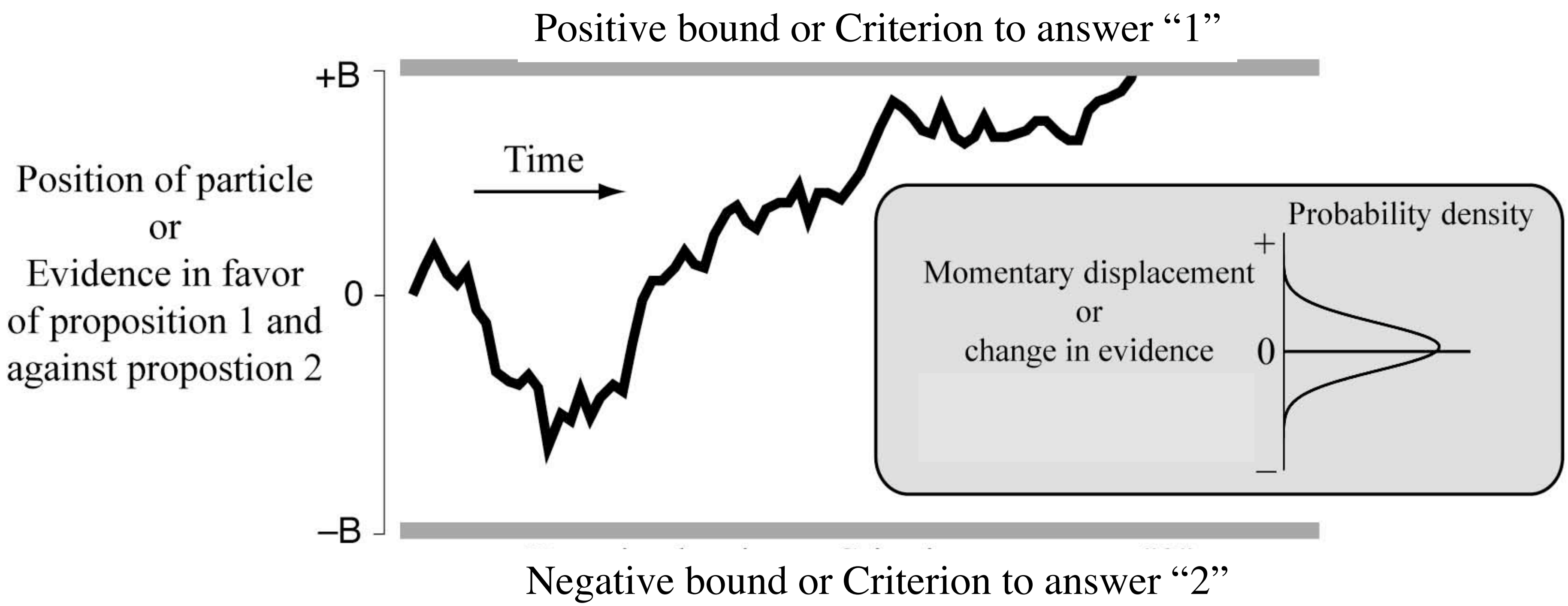
"ramp"



Diffusion to bound model

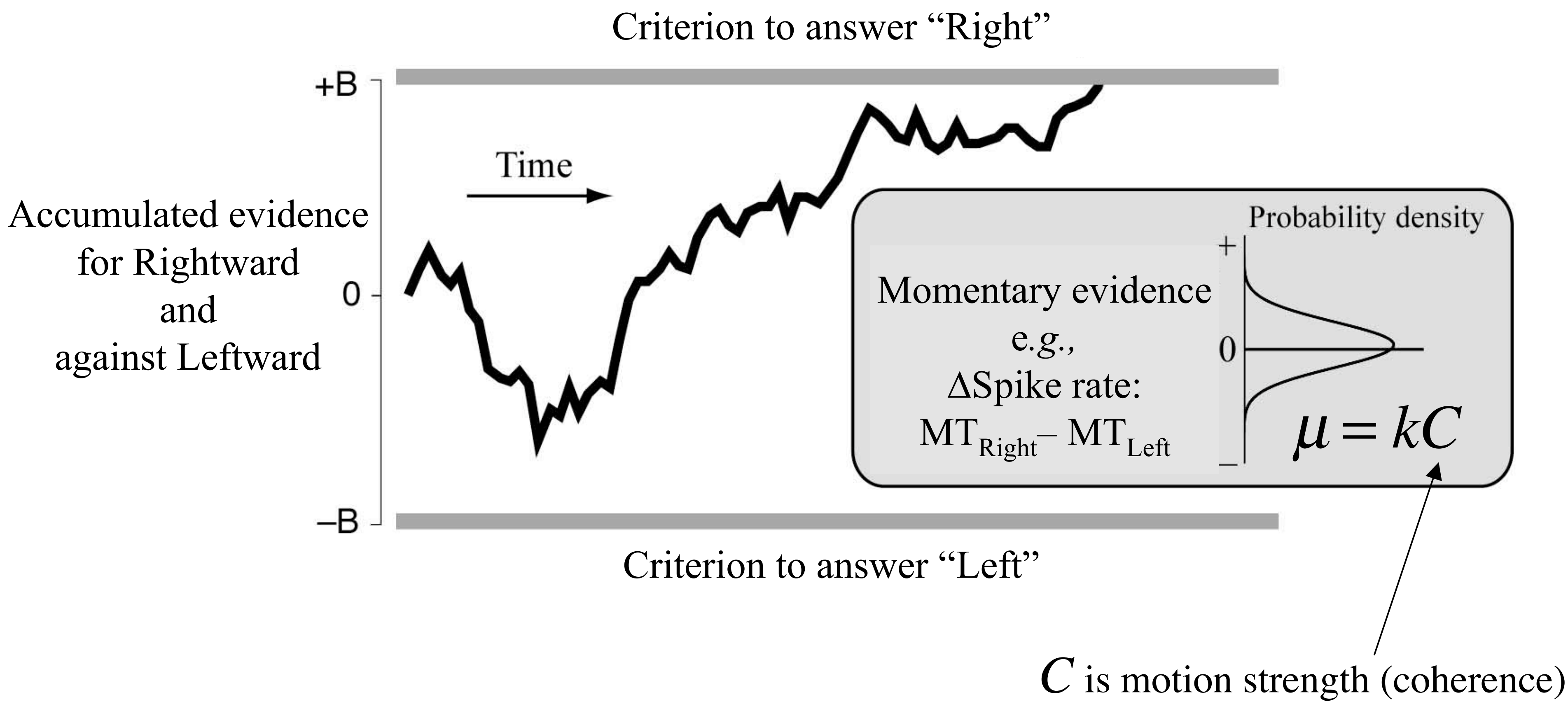


Diffusion to bound model



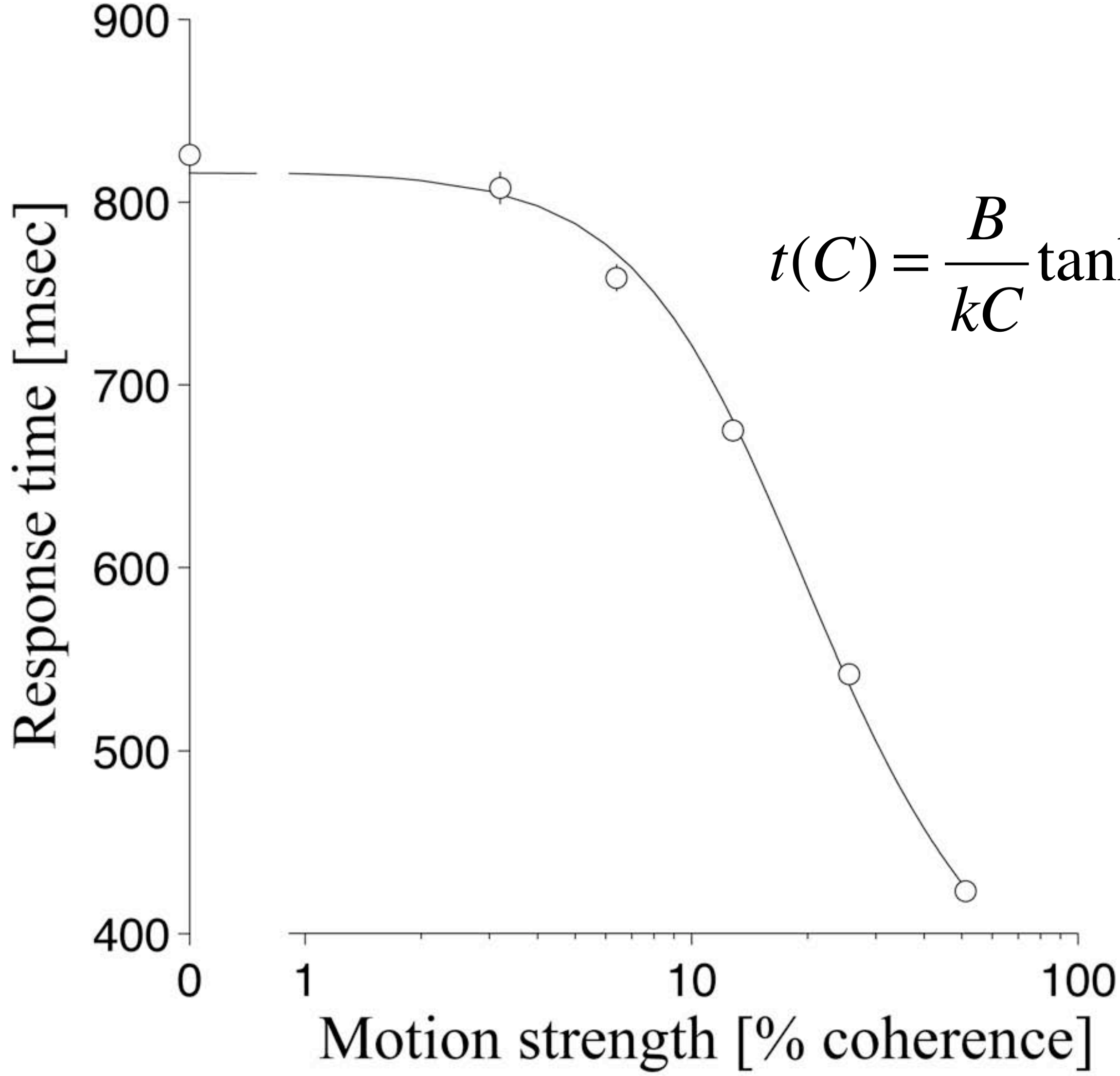
Proposed by Wald, 1947 and Turing (WW II, classified);
Stone, 1960; then Laming, Link, Ratcliff, Smith, . . .

Diffusion to bound model



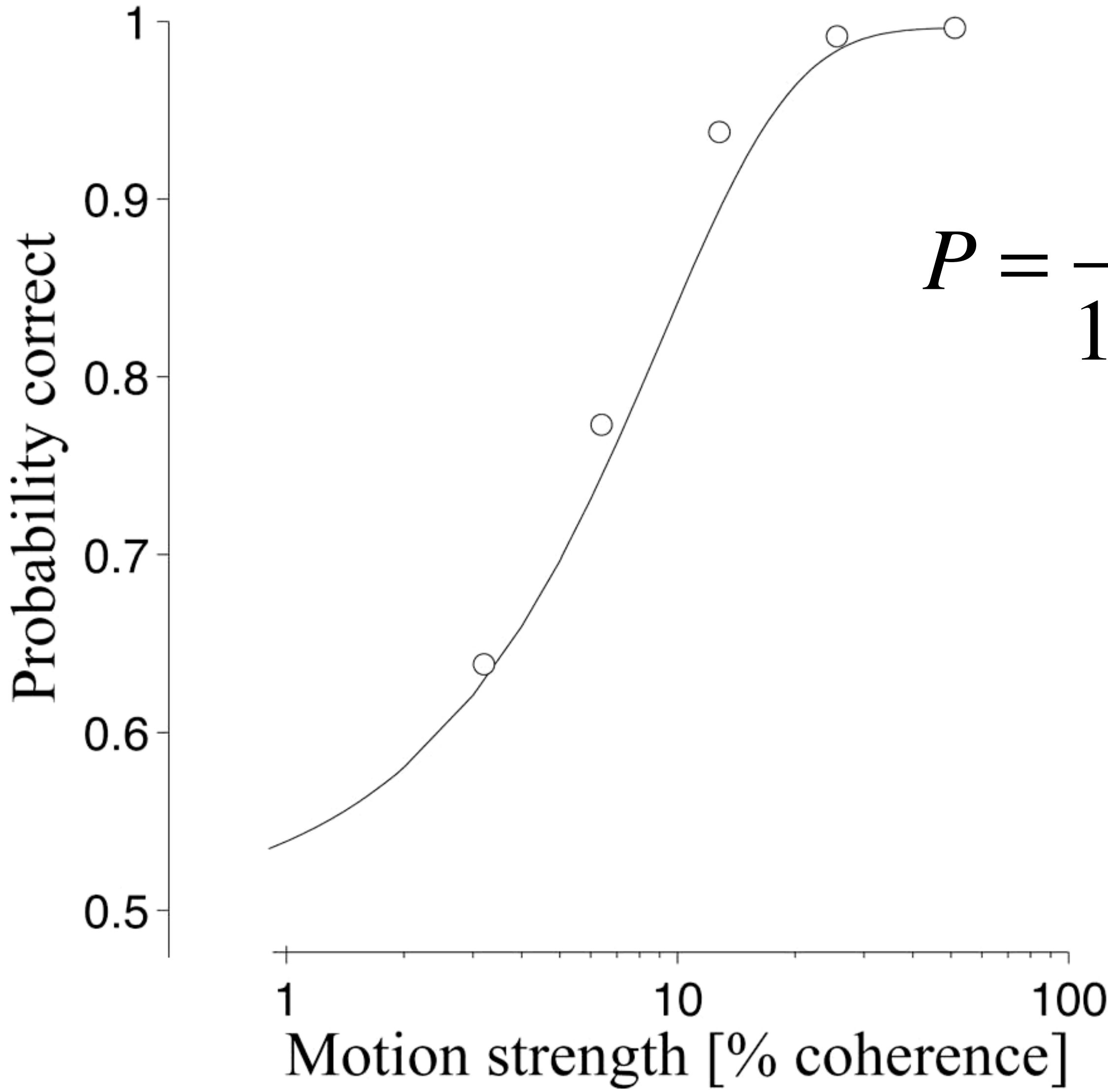
Shadlen & Gold (2004)
Palmer et al (in press)

Best fitting chronometric function
“Diffusion to bound”

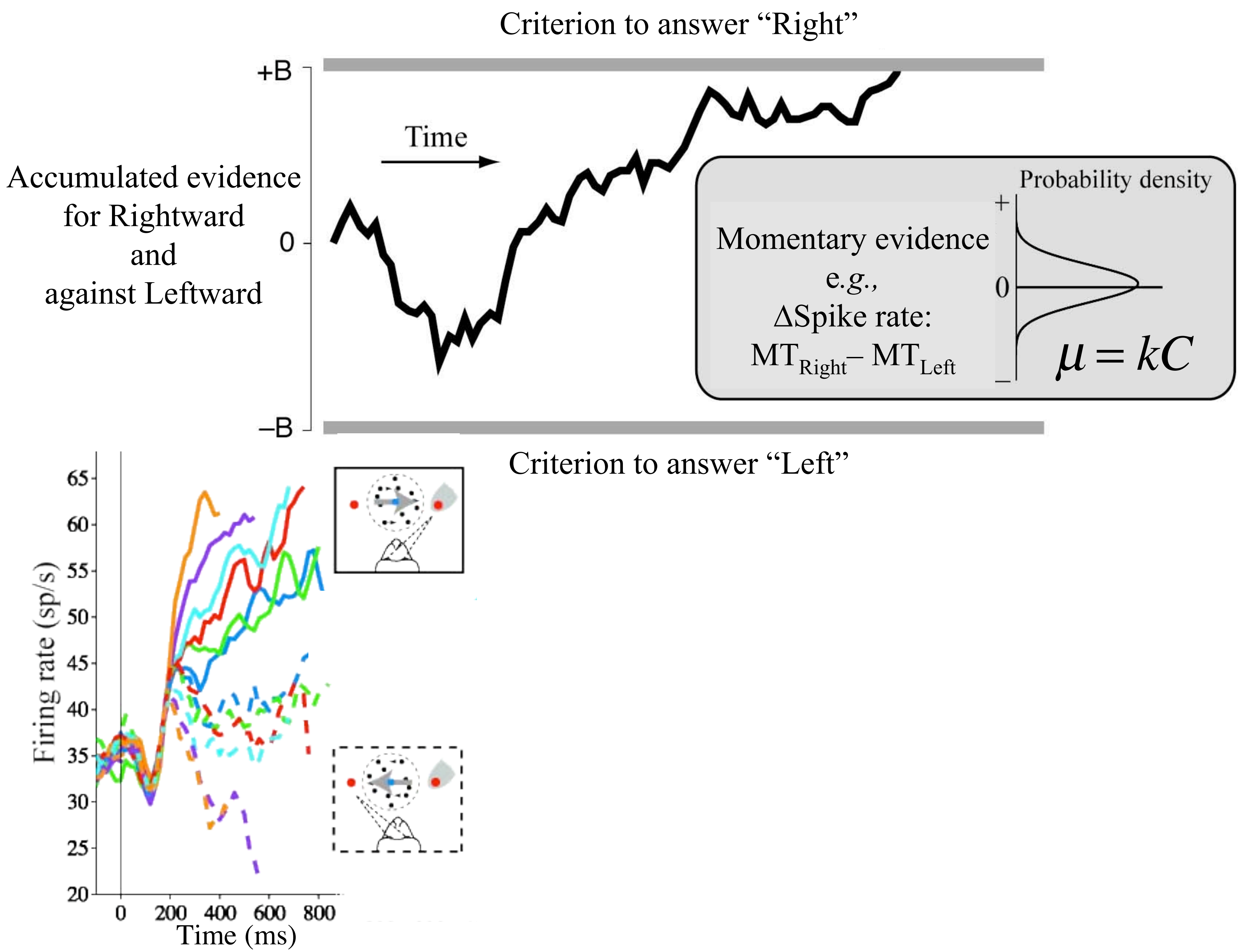


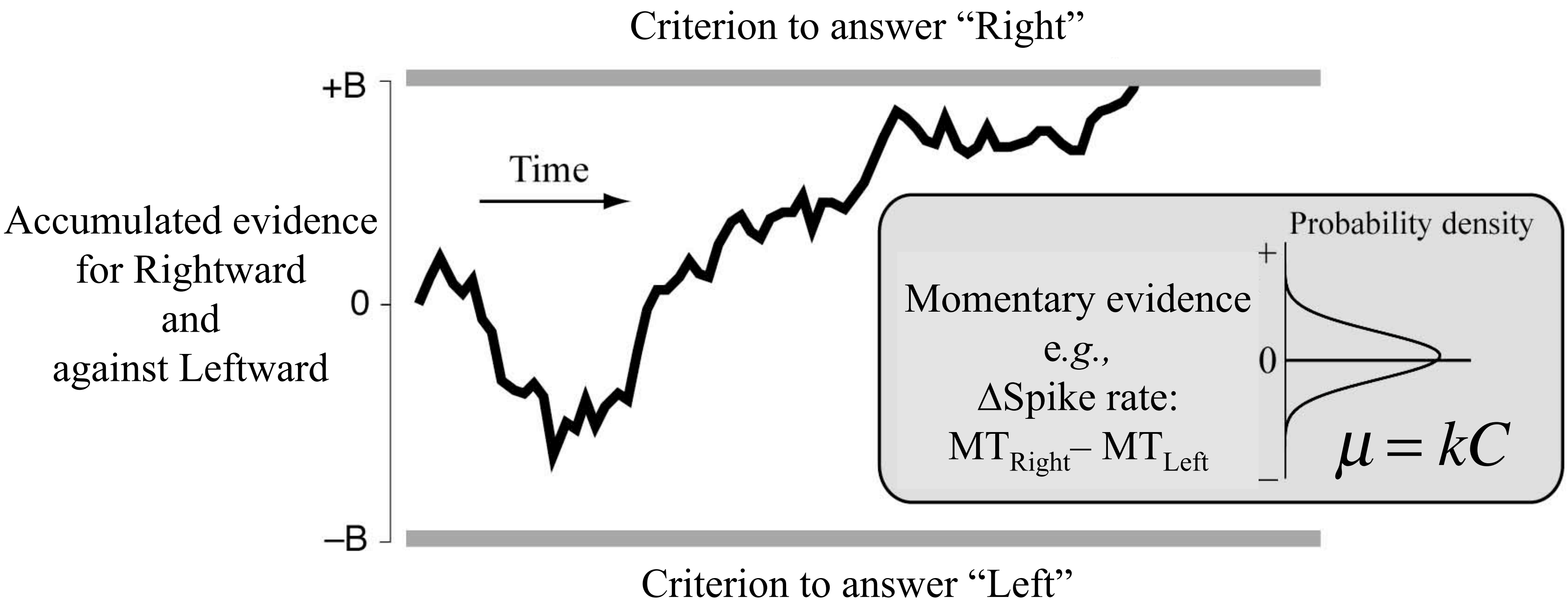
$$t(C) = \frac{B}{kC}\tanh(BkC) + t_{nd}$$

Predicted psychometric function
"Diffusion to bound"



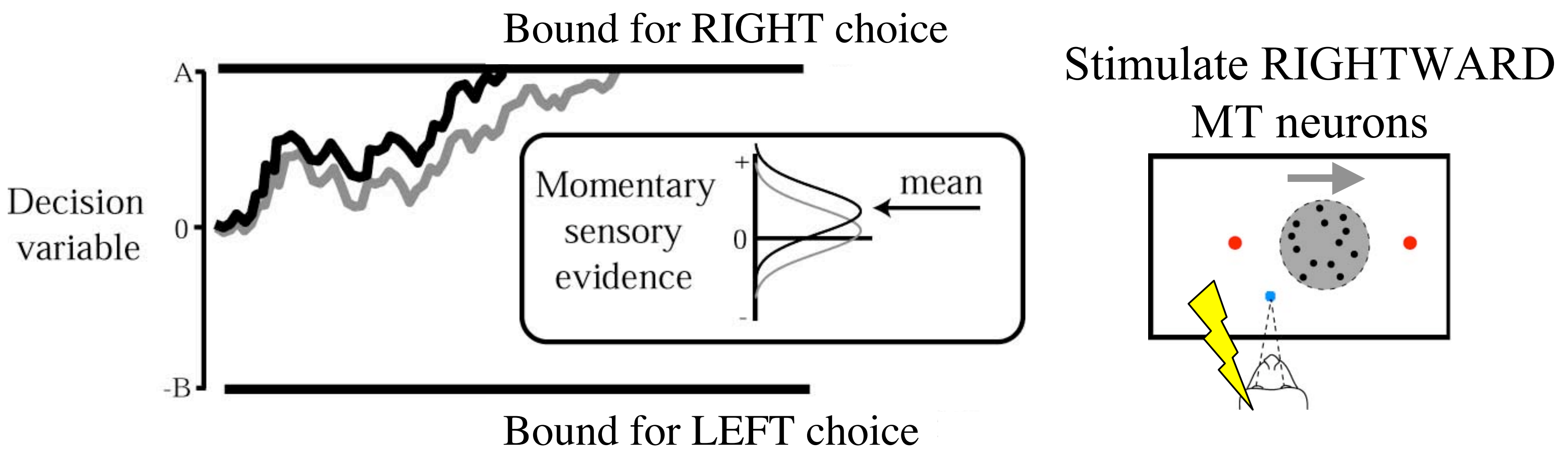
$$P = \frac{1}{1 + e^{-2k|C|B}}$$


Criterion to answer “Right”
+B
0
–B
Time
Accumulated evidence
for Rightward
and
against Leftward
Probability density
Momentary evidence
e.g.,
ΔSpike rate:
$MT_{Right} - MT_{Left}$
+
0
–
$\mu = kC$
Criterion to answer “Left”
Firing rate (sp/s)
65
60
55
50
45
40
35
30
25
20
0
200
400
600
800
Time (ms)

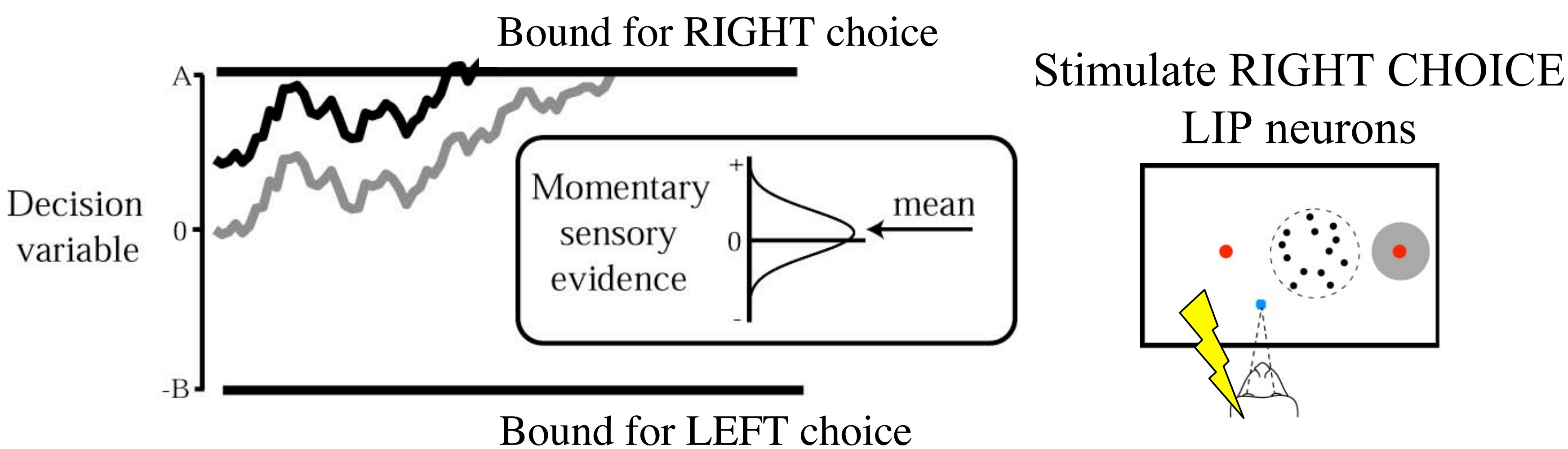


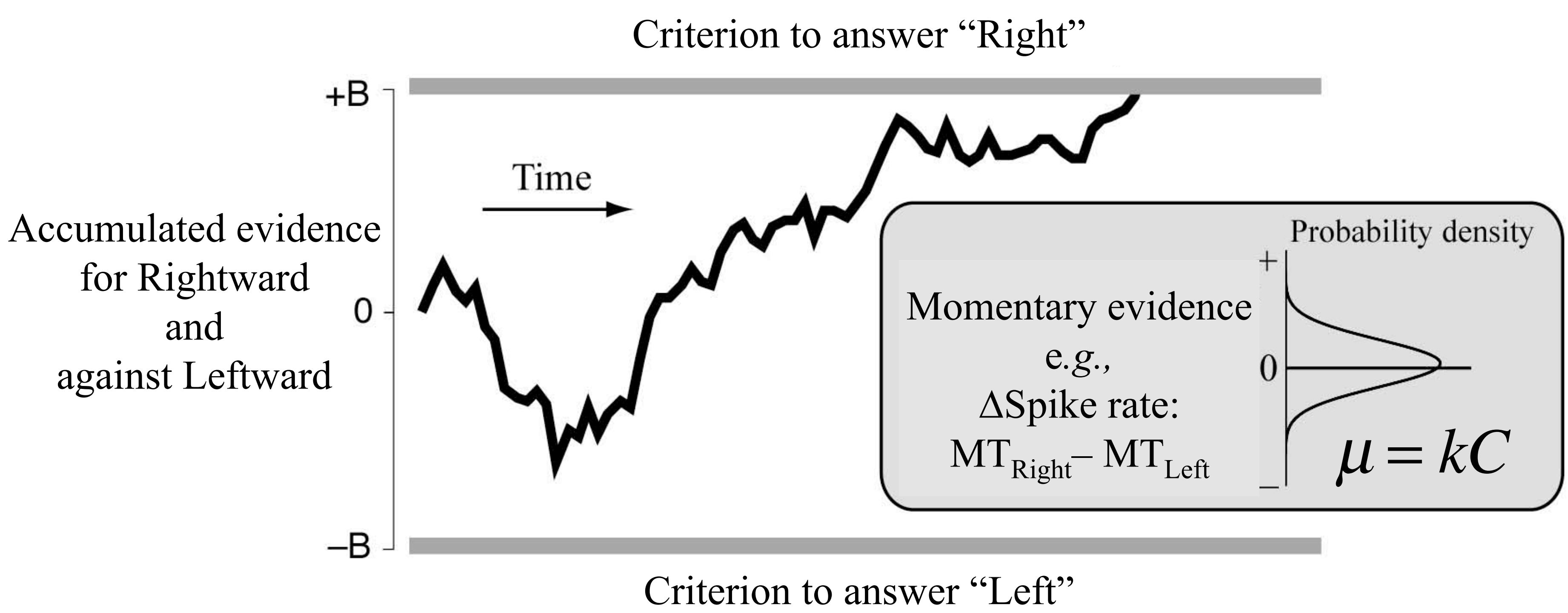
- LIP represents $\int dt$ of momentary motion evidence
- Momentary evidence is a spike rate difference from area MT
- The accumulated evidence used by the monkey is in area LIP

The momentary evidence is a Δ between opposite direction signals in area MT



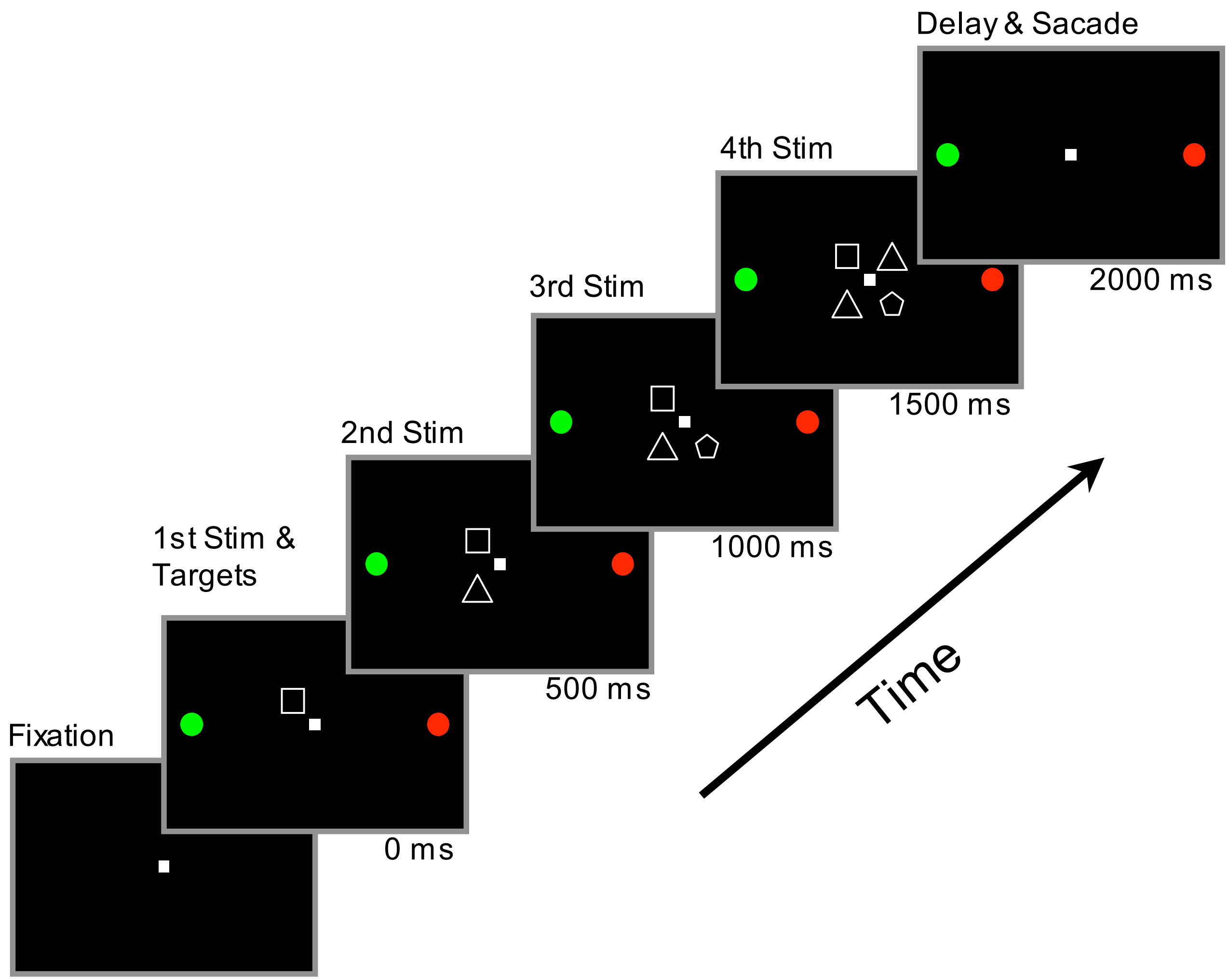
The accumulated evidence used by the monkey is in area LIP



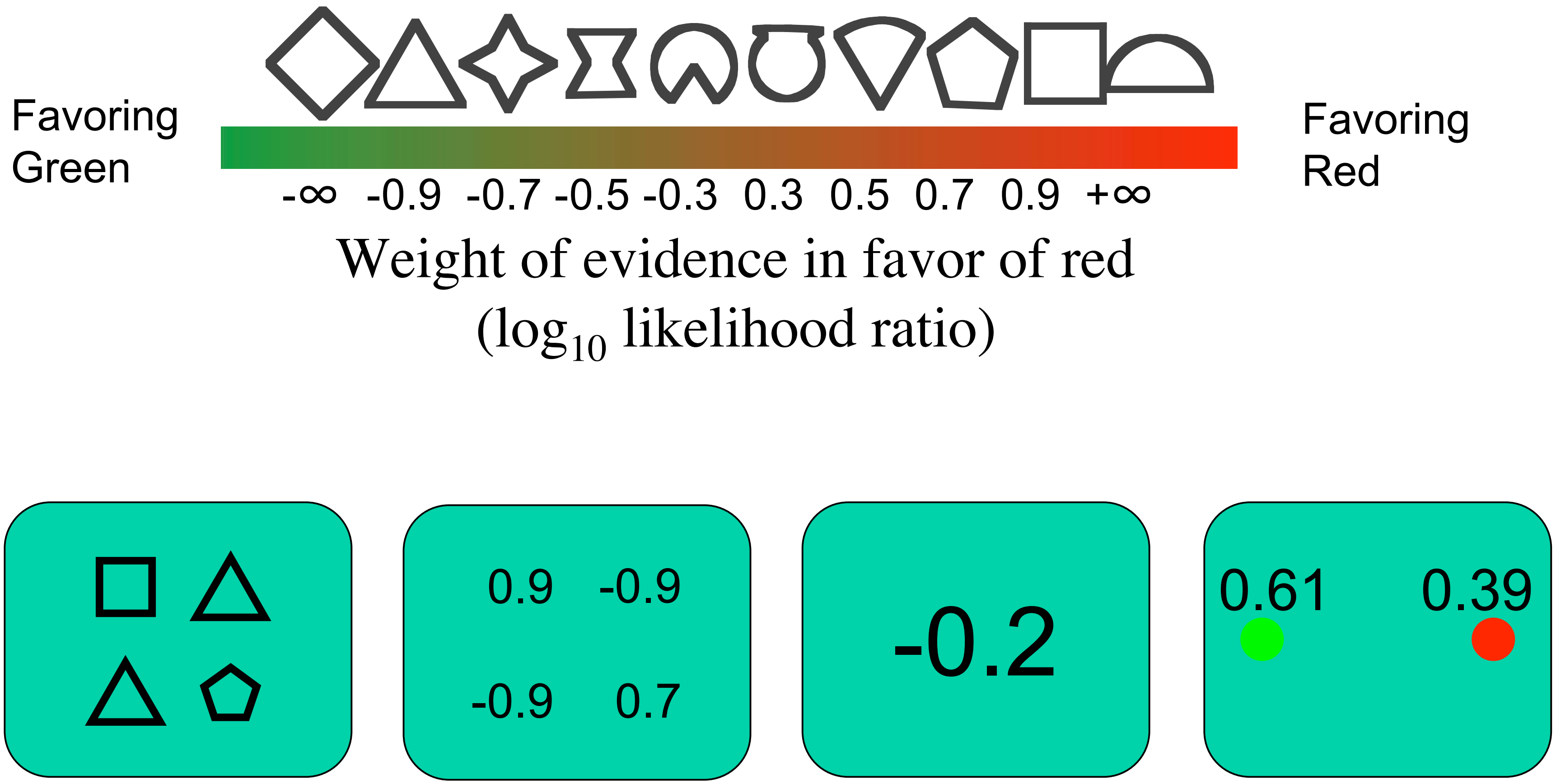


- LIP represents $\int dt$ of momentary motion evidence
- Momentary evidence is a spike rate difference from area MT
- The accumulated evidence used by the monkey is in area LIP
- How and where is the integral computed?
- How is the bound set?
- How is a bound crossing detected?

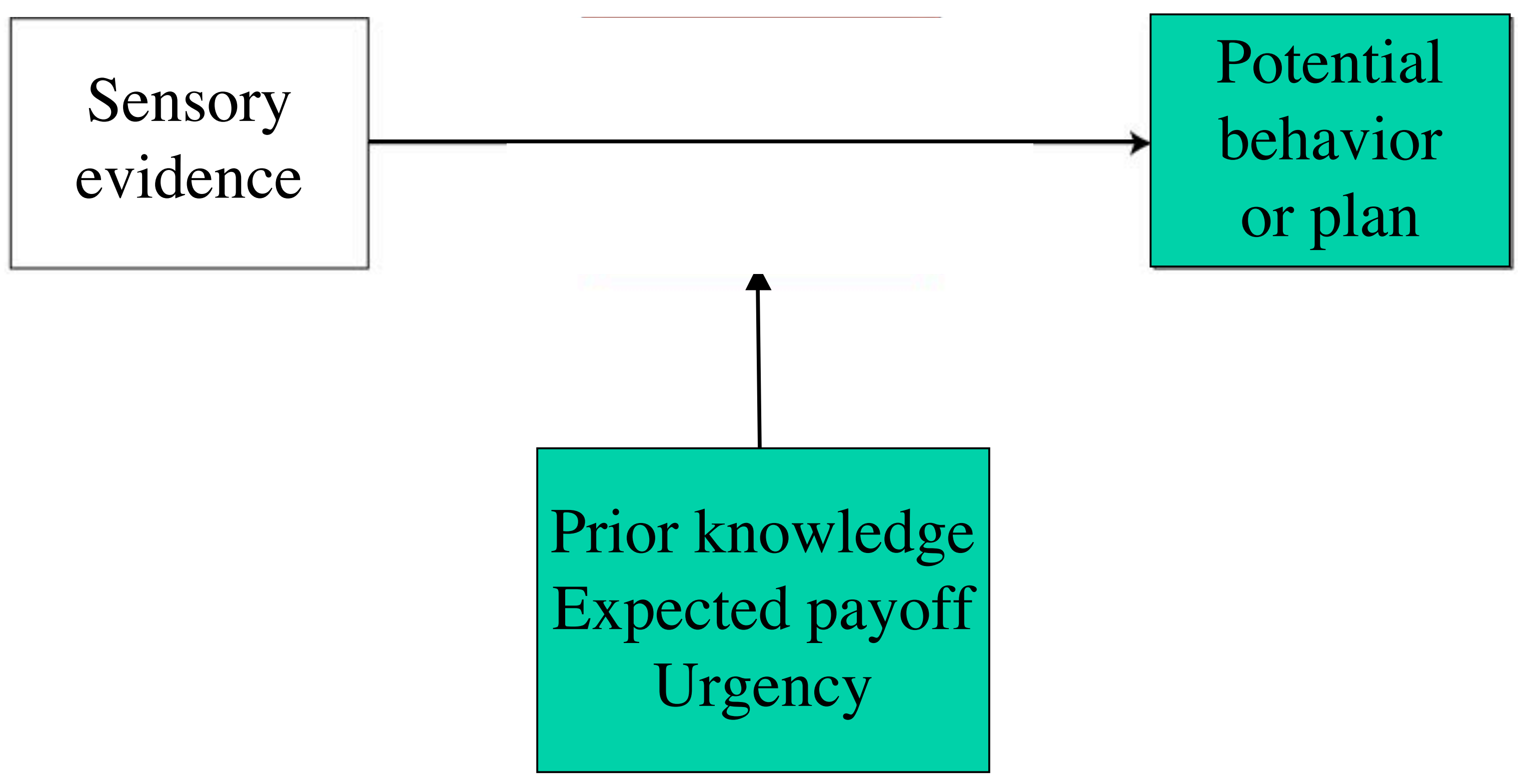
Probabilistic categorization task: 4-card stud



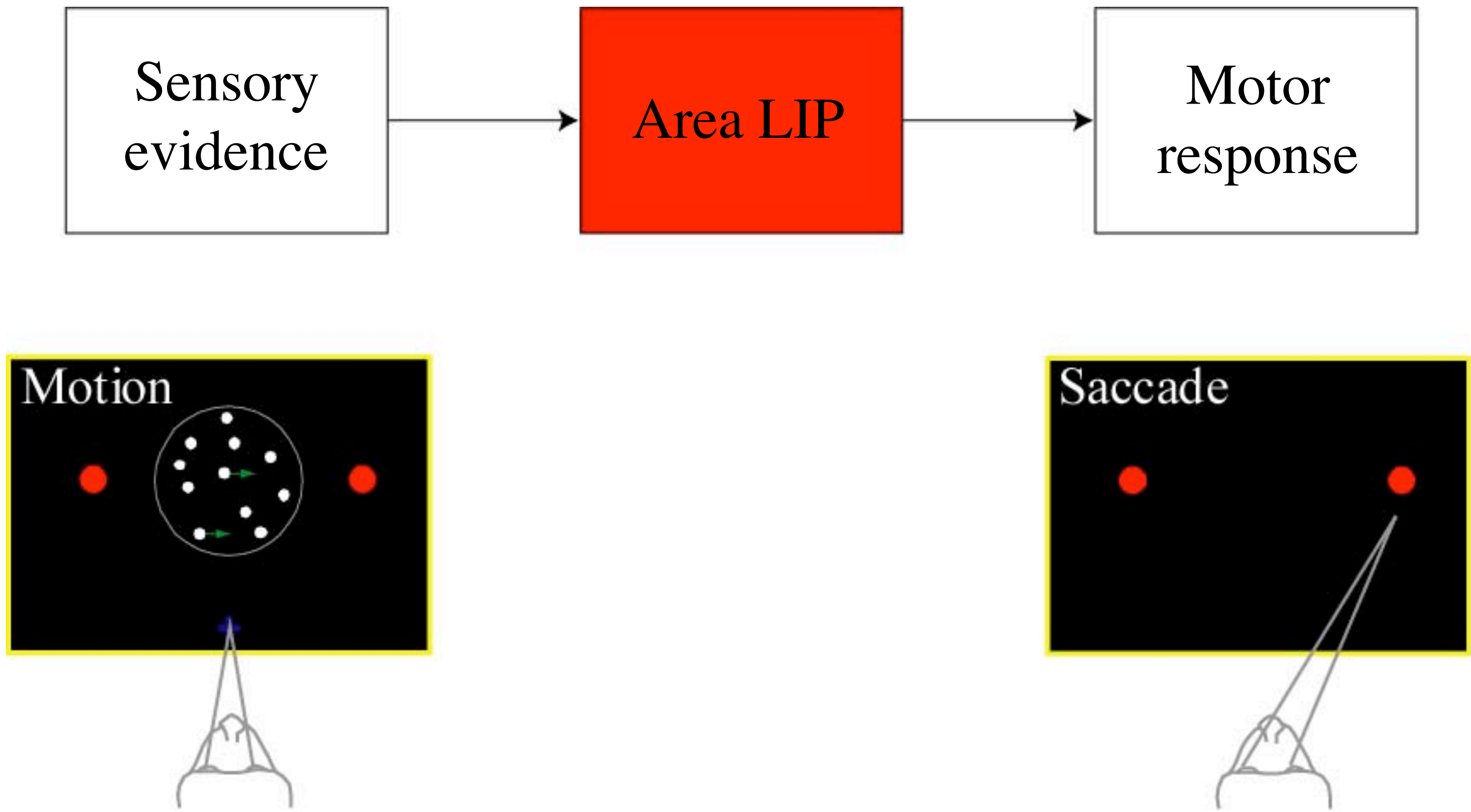
Tianming Yang


Favoring Green
Favoring Red
-∞ -0.9 -0.7 -0.5 -0.3 0.3 0.5 0.7 0.9 +∞
Weight of evidence in favor of red
(log10 likelihood ratio)
0.9 -0.9
-0.9 0.7
-0.2
0.61 0.39

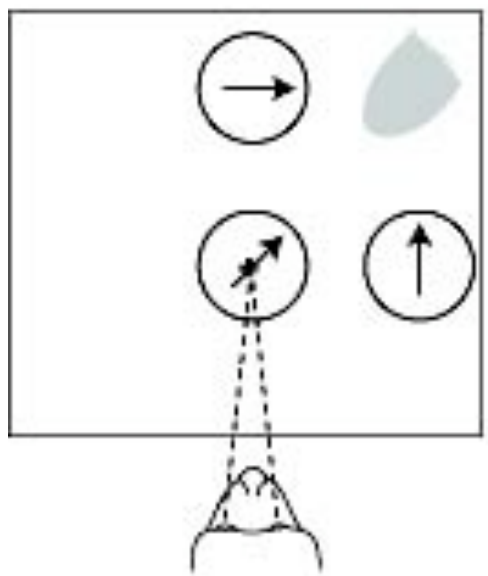
From sensorimotor integration to cognition and its disorders



From sensorimotor integration to cognition and its disorders



From sensorimotor integration to cognition and its disorders



From sensorimotor integration to cognition and its disorders

Evanescent sensory stream $\xrightarrow{\int dt}$ Plans for the future

Leaky integration ⇨ confusion

Turing’s strategy: sequential analysis

***Hypothesis**: Messages encrypted by Enigma devices in same state*

Weight of evidence
in favor of
common rotor setting = { match

K C Y W D K D O P E D B A I Q S D F M K C N F A E O I E N C V N S D F N

E N C H P D N C O E N A S H Q E N D N C K R N D N Q I O M Z F J K C P Q

Turing's strategy: sequential analysis

Hypothesis*: Messages encrypted by Enigma devices in same state*

Weight of evidence in favor of common rotor setting = { match

K C Y W D K D O P E D B A I Q S D F M K C N F A E O I E N C V N S D F N

E N C H P D N C O E N A S H Q E N D N C K R N D N Q I O M Z F J K C P Q

Turing's strategy: sequential analysis

Weight of evidence in favor of common rotor setting

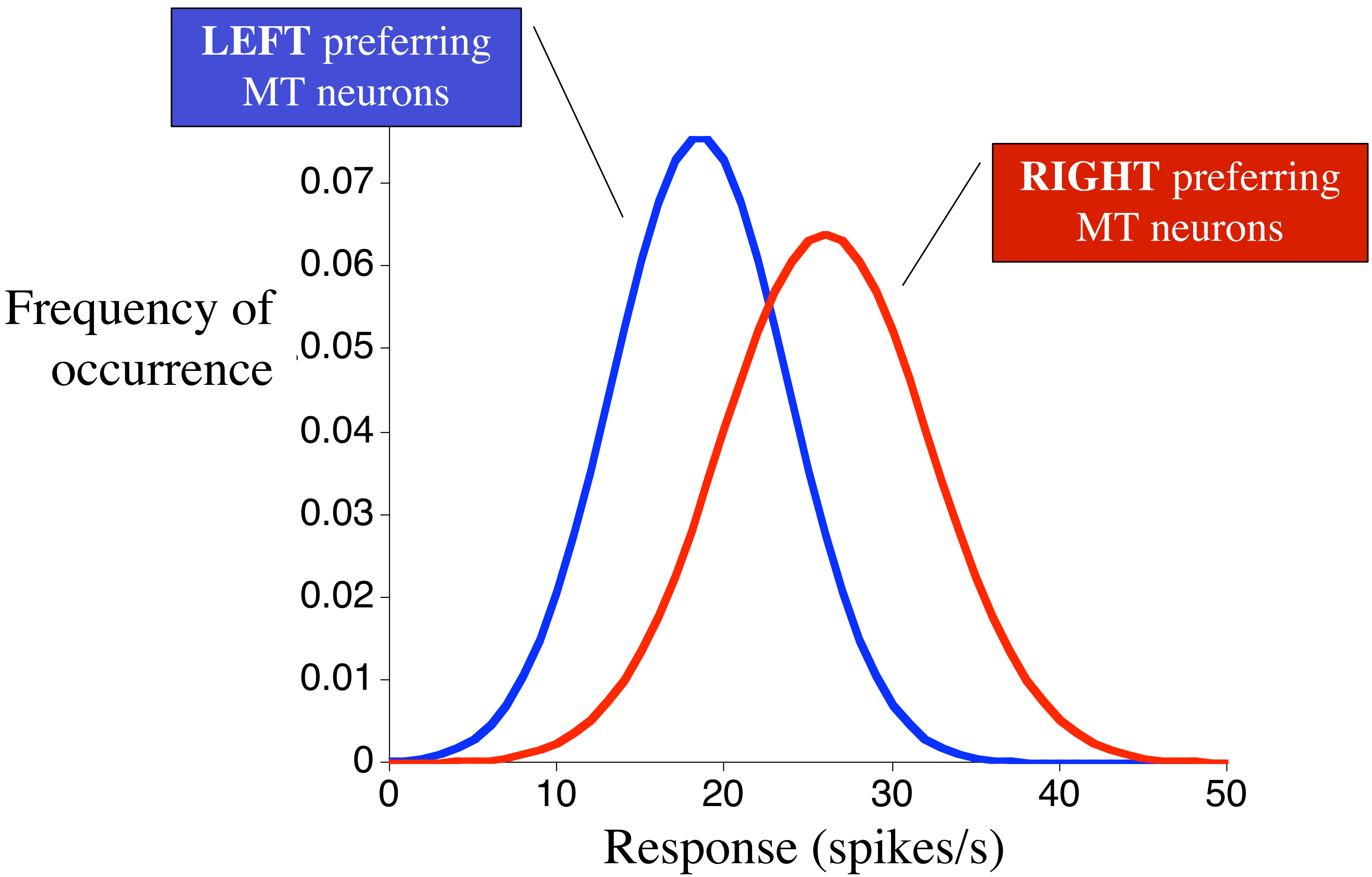
$$= \begin{cases} 10\log_{10}\left[\dfrac{\left(\frac{1}{13}\right)}{\left(\frac{1}{26}\right)}\right] = +3.0\ db & \text{match} \\ 10\log_{10}\left[\dfrac{\left(\frac{12}{13}\right)}{\left(\frac{25}{26}\right)}\right] = -0.17\ db & \text{non - match} \end{cases}$$

K C Y W D K D O P E D B A I Q S D F M K C N F A E O I E N C V N S D F N

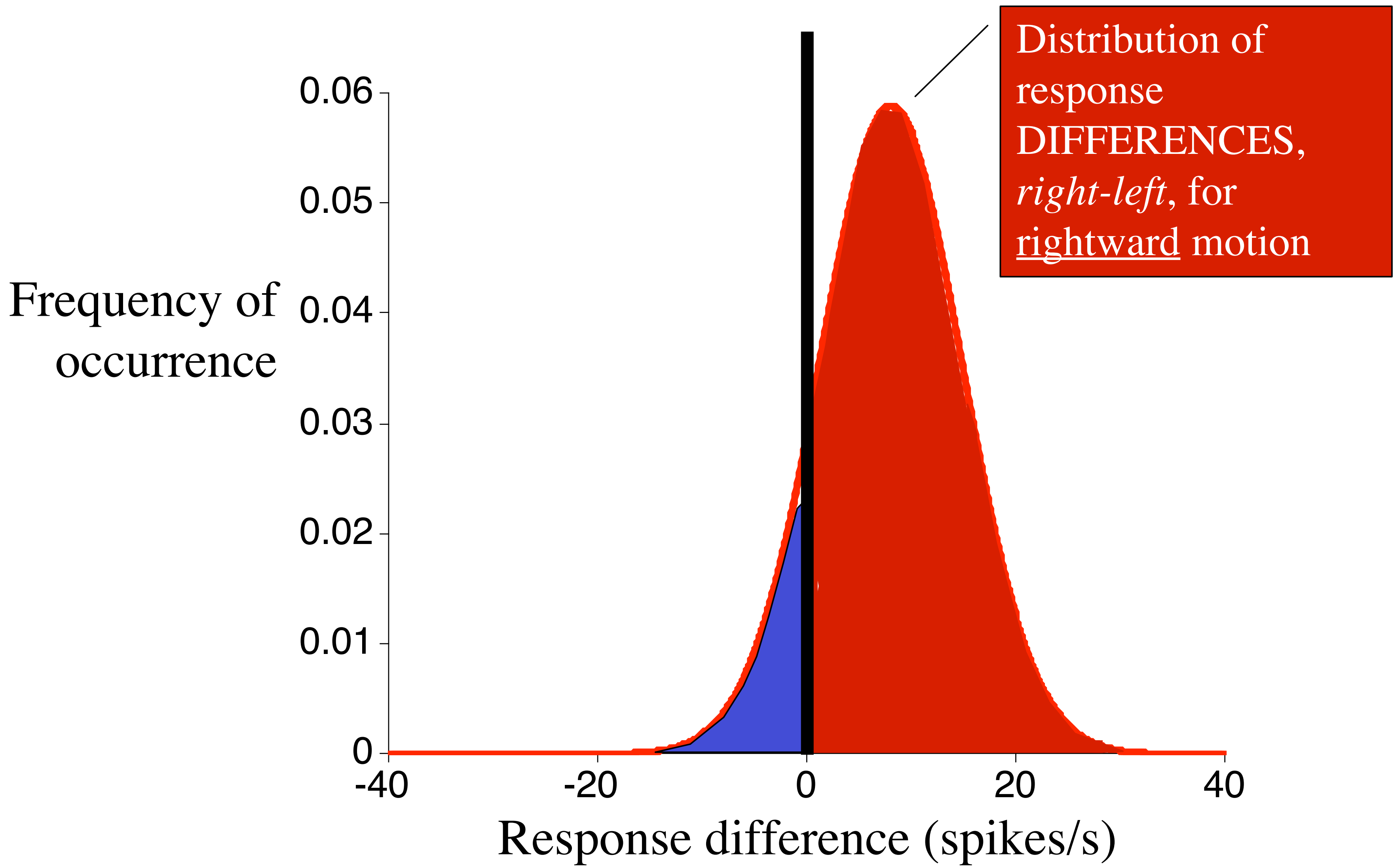
E N C H P D N C O E N A S H Q E N D N C K R N D N Q I O M Z F J K C P Q

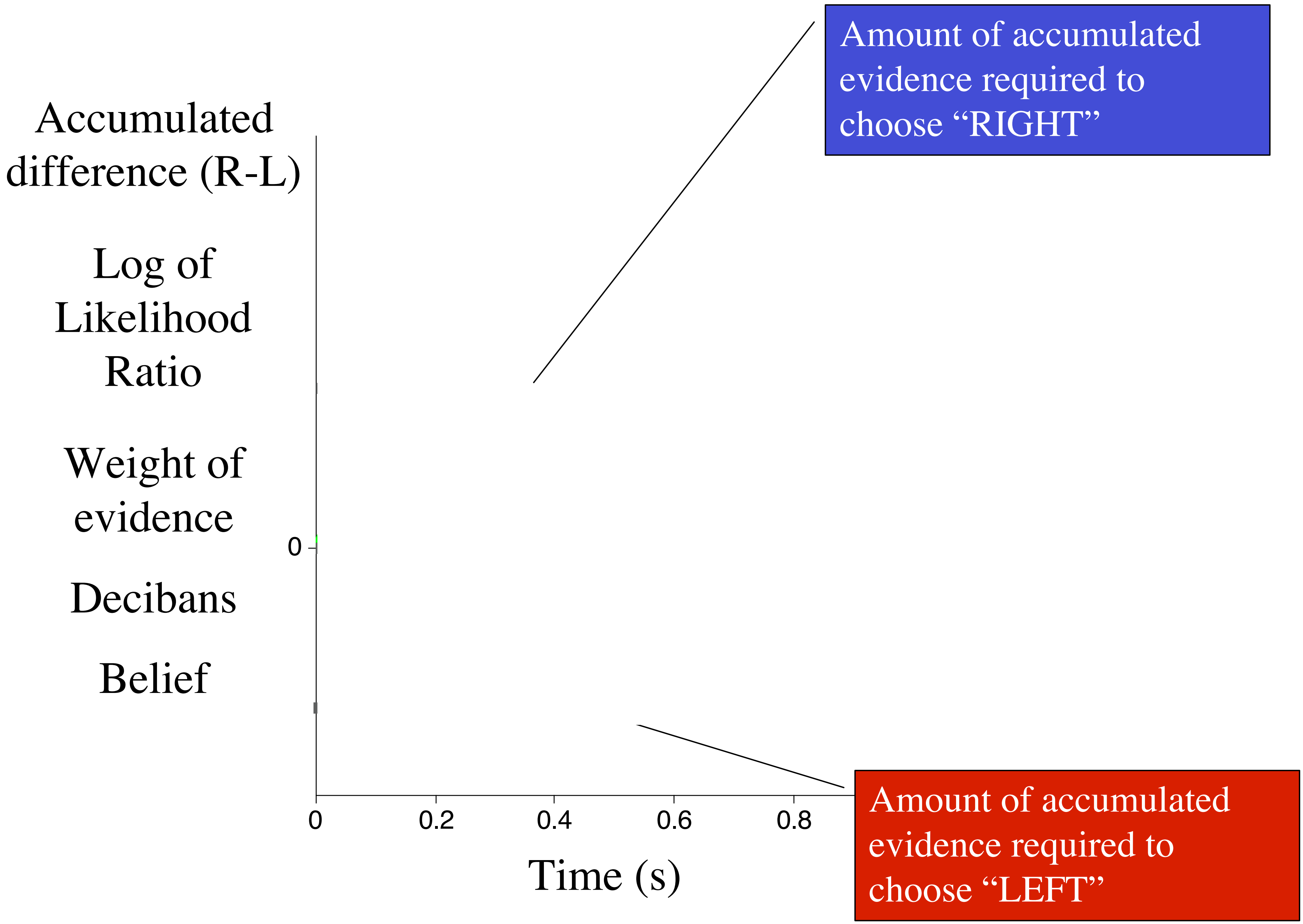
Weight of evidence in favor of common settings (decibans)

Variable response to weak RIGHTWARD motion



Difference in *spike rate* is proportional to the *logarithm of the likelihood ratio*




Accumulated difference (R-L)
Log of Likelihood Ratio
Weight of evidence
Decibans
Belief
0
0
0.2
0.4
0.6
0.8
Time (s)
Amount of accumulated evidence required to choose "RIGHT"
Amount of accumulated evidence required to choose "LEFT"

Random walk to bounds at ±A

$$Y_n = \sum_{i=1}^{n} X_i$$ random walk or diffusion

$$M_X(\theta) \equiv E\left[e^{\theta X}\right] = \int_{-\infty}^{\infty} f(x)e^{\theta x}dx$$ def. of MGF for X

$$M_{Y_n}(\theta) = M_X^n(\theta)$$ MGF for sums

$\tilde{Y} \equiv$ stopped accumulation

$$M_{\tilde{Y}}(\theta) = P_+ e^{\theta A} + (1 - P_+)e^{-\theta A}$$ MFG for $\tilde{Y}$

Stochastic processes: partial sums and Wald's martingale

Accumulation

$Y_0 = 0$

$+X_1$

Y_1

$+X_2$

Y_2

$\vdots \qquad \vdots$

Y_{n-1}

$+X_n$

$$Y_n = \sum_{i=1}^{n} X_n$$

Stochastic processes: partial sums and Wald's martingale

Accumulation		Wald's Martingale	
$Y_0 = 0$		$Z_0 = 1$	
	$+X_1$		$\times \dfrac{e^{\theta X_1}}{M(\theta)}$
Y_1		Z_1	
	$+X_2$		$\times \dfrac{e^{\theta X_2}}{M(\theta)}$
Y_2		Z_2	
$\vdots$	$\vdots$	$\vdots$	$\vdots$
Y_{n-1}		Z_{n-1}	
	$+X_n$		$\times \dfrac{e^{\theta X_n}}{M(\theta)}$
$Y_n = \sum_{i=1}^{n} X_n$		$Z_n = \dfrac{e^{\theta Y_n}}{M^n(\theta)}$	

Wald's martingale & identity

$$
\begin{aligned}
E\left[Z_{n+1}\middle|Y_1,Y_2,\ldots,Y_n\right] &= E\left[M_X^{-(n+1)}(\theta)e^{\theta Y_{n+1}}\middle|Y_1,Y_2,\ldots,Y_n\right] \\
&= E\left[M_X^{-(n+1)}(\theta)e^{\theta(Y_n+X_{n+1})}\right] \\
&= E[M_X^{-1}(\theta)M_X^{-n}(\theta)e^{\theta Y_n}e^{\theta X_{n+1}}] \\
&= E[Z_n M_X^{-1}(\theta)e^{\theta X_{n+1}}] \\
&= M_X^{-1}(\theta)Z_n E\left[e^{\theta X_{n+1}}\right] \\
&= Z_n
\end{aligned}
$$

$$
\begin{aligned}
E\left[Z_n\right] &= E\left[M_X^{-n}(\theta)e^{\theta Y_n}\right] \\
&= M_X^{-n}(\theta)E\left[e^{\theta Y_n}\right] \\
&= M_X^{-n}(\theta)M_{Y_n}(\theta) \\
&= 1
\end{aligned}
$$

Use Wald's martingale to simplify $M_{\tilde{Y}}(\theta)$

$$E\left[\tilde{Z}\right] = E\left[Z_n\right] = 1$$

$$E\left[M_X^{-n}(\theta) e^{\theta \tilde{Y}}\right] = 1$$

If there were a value for θ such that $M_X(\theta) = 1$, it no longer matters that n is a random number. At this special value, θ_1,

$$E\left[e^{\theta_1 \tilde{Y}}\right] = 1$$

E.g., for the Normal distribution, with mean μ and variance σ^2, $\theta_1 = -\frac{2\mu}{\sigma^2}$

$$M_{\tilde{Y}}(\theta_1) = P_+ e^{\theta_1 A} + (1 - P_+) e^{-\theta_1 A} = 1$$

$$P_+ = \frac{1}{1 + e^{\theta_1 A}}$$