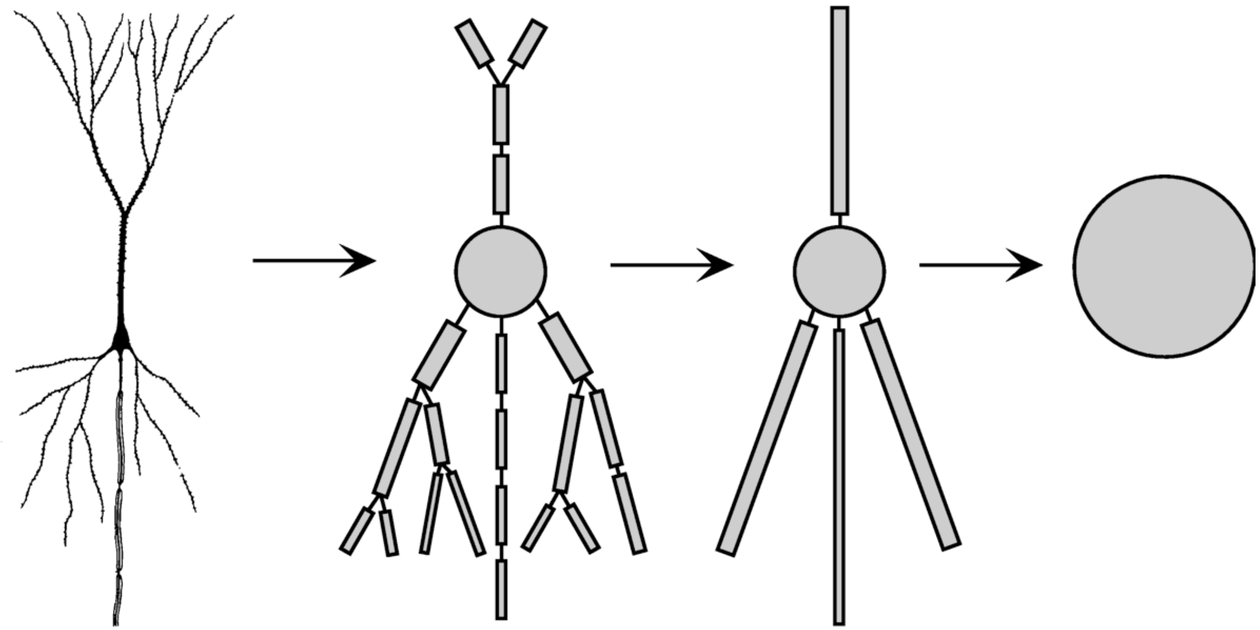
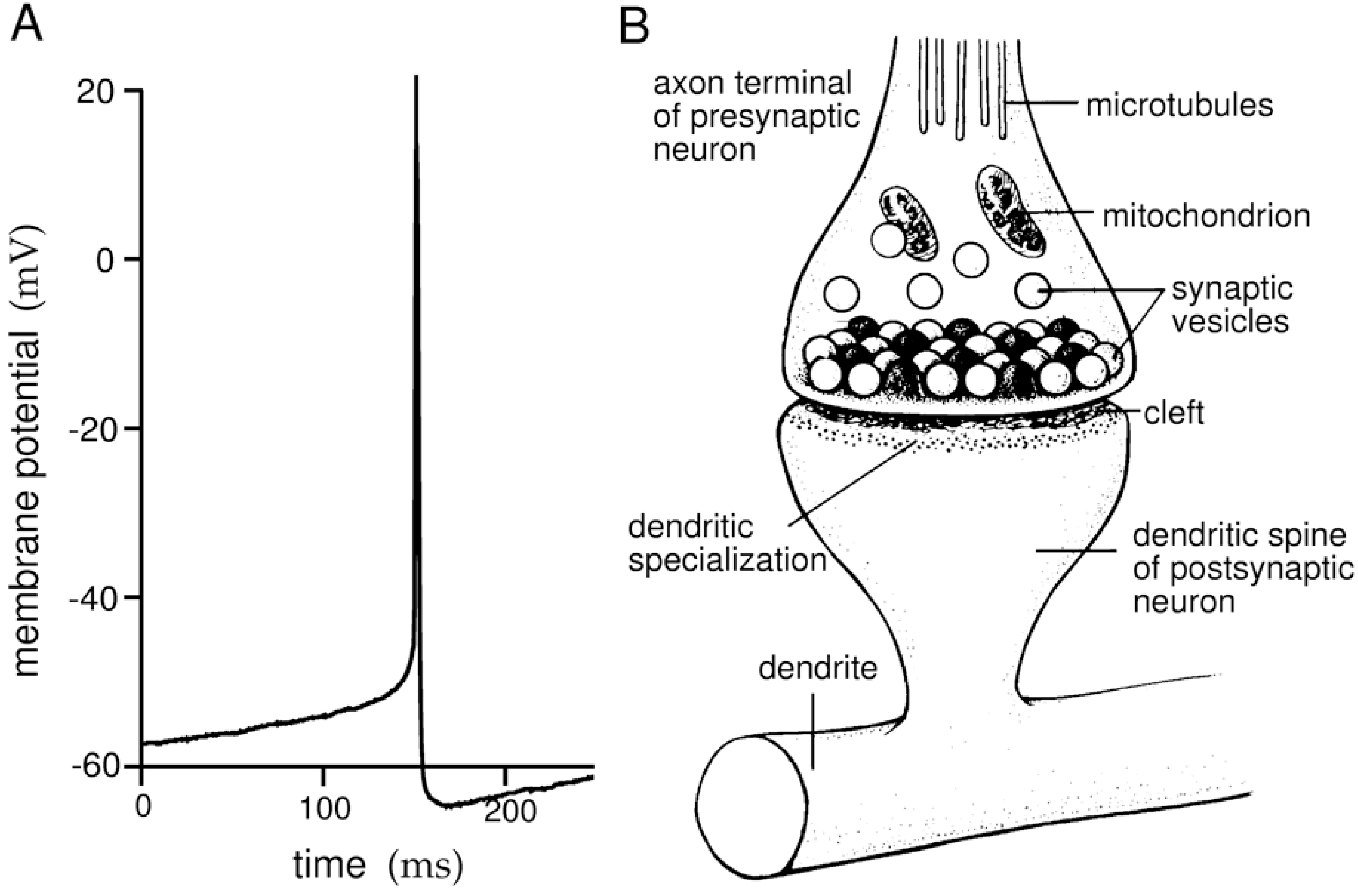
Biophysical Modeling

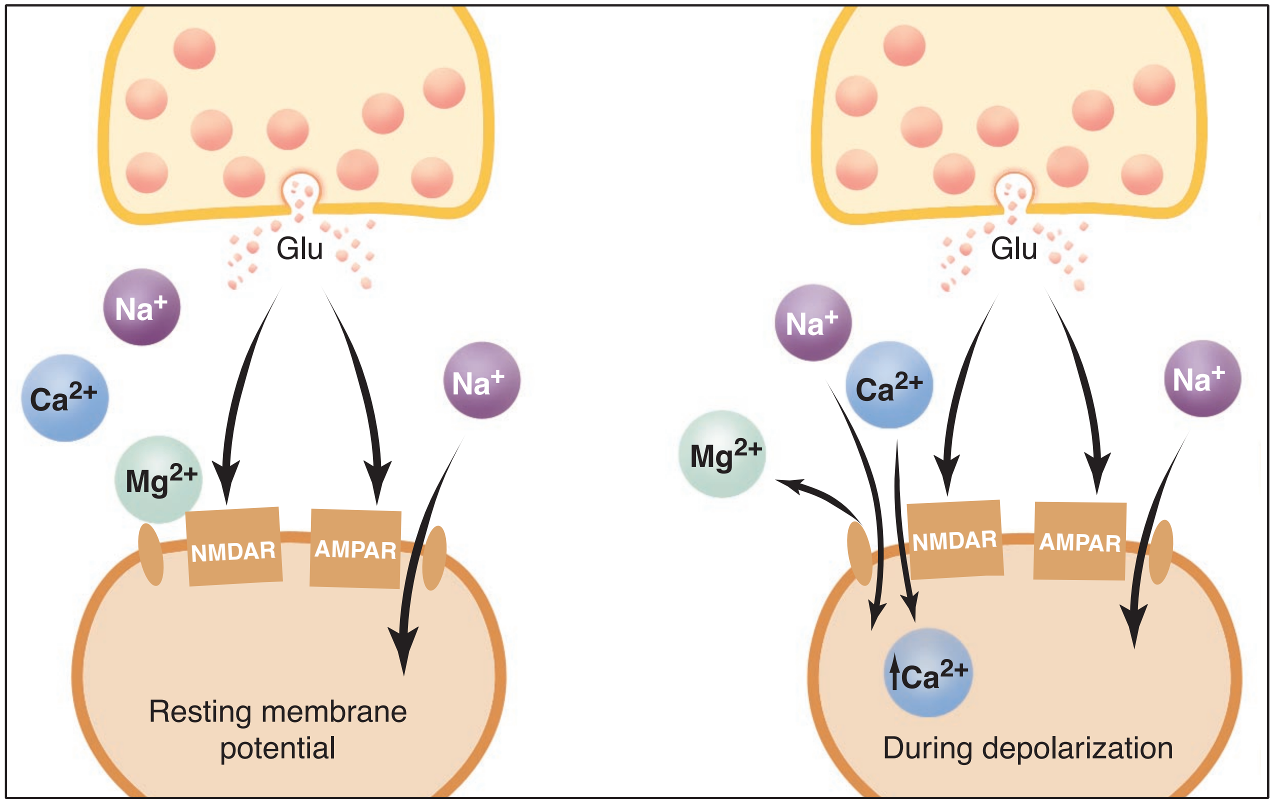


- Synapses
- Cable equation
- Multiple compartments
- Beyond Hodgkin-Huxley
- From dynamical systems

Chemical Synapses



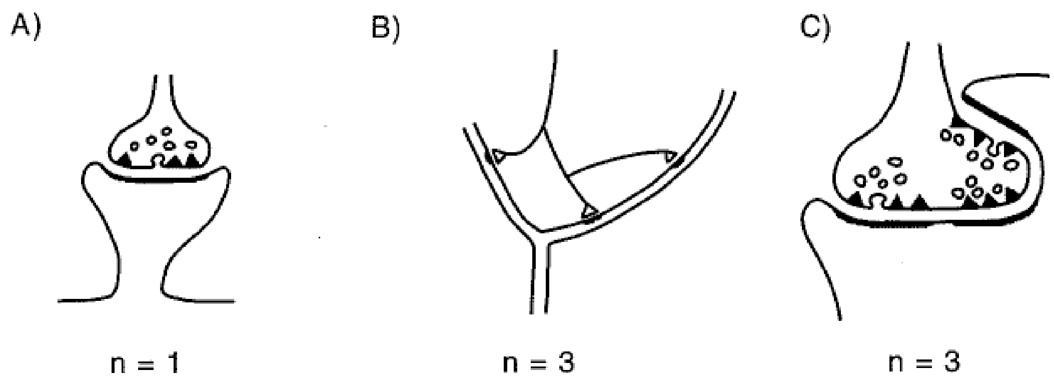
Chemical Synapses



Malenka and Nicoll, 1999

- Neurotransmitters
  - Glutamate (+)
    - AMPA
    - NMDA
  - GABA (-)
    - $GABA_A$

Quantal Hypothesis



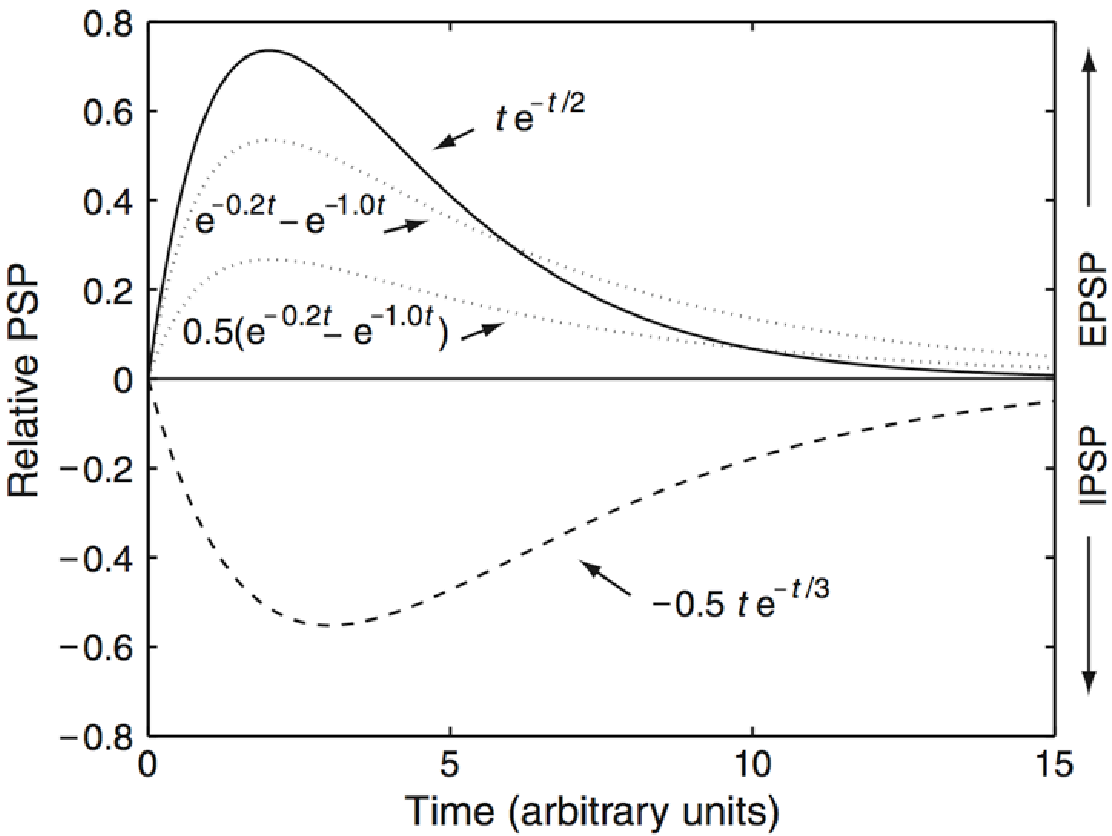
Koch, p. 312

binomial distribution:

$$f(k; n, p) = \binom{n}{k} p^k (1-p)^{n-k}$$

- Overall effect R = npq
  - n = number of release sites
    - drawn from some probability distribution (e.g. binomial)
    - at most, 1 vesicle is release per presynaptic spike
  - p = probability of release for release site
  - q = postsynaptic effect

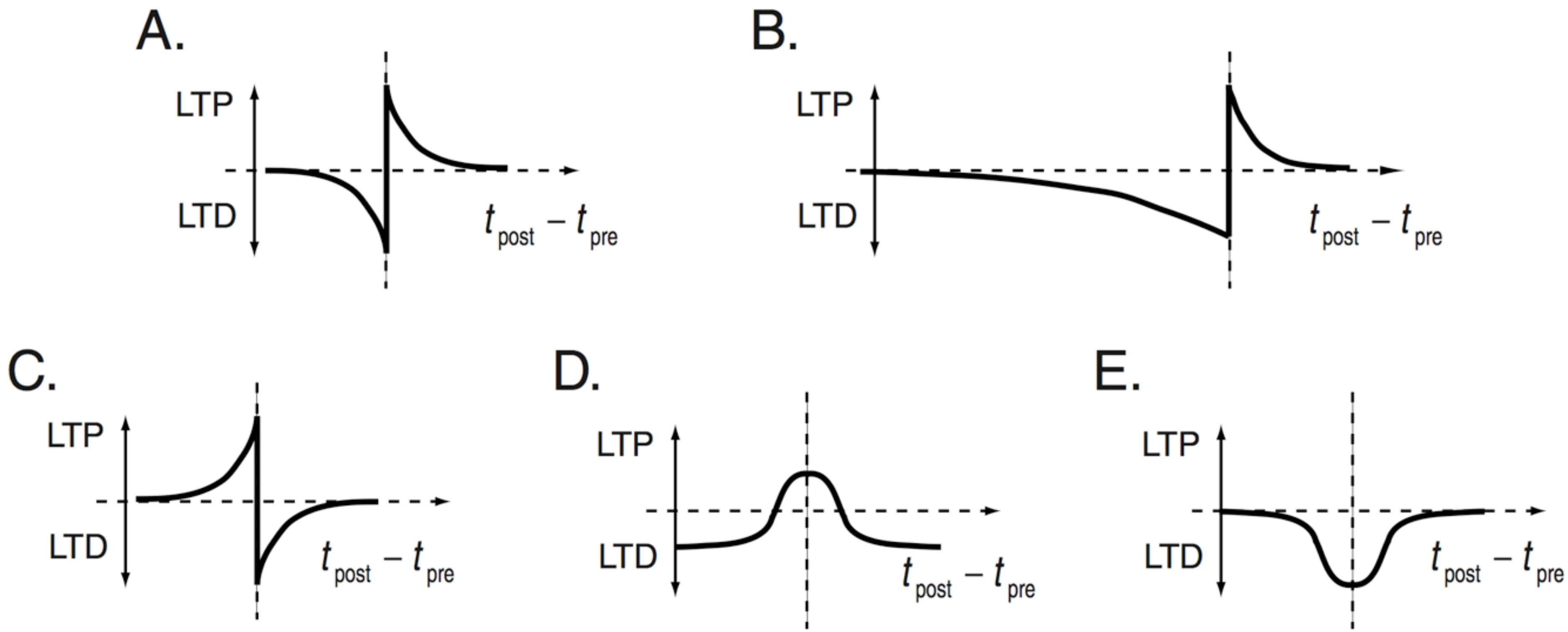
α-synapse



$$\Delta V_m^{\,non-NMDA} = wte^{-t/t^{peak}}$$

$$\Delta V_m^{\,NMDA} = c\left(V_m\right)e^{-t/\tau_1} - e^{-t/\tau_2}$$

Synaptic Plasticity

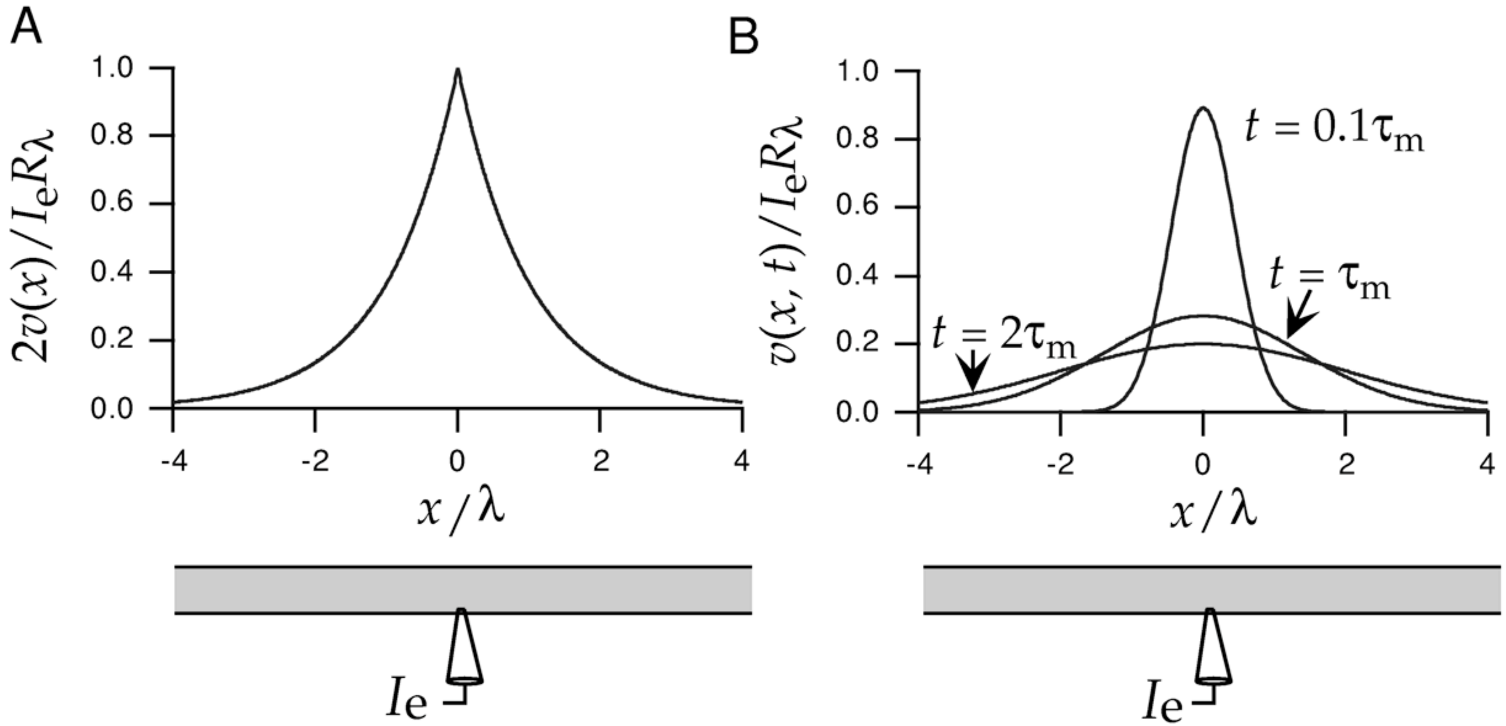


Abbott and Nelson, Nat Neurosci, 2000

Cable Equation

- How does the membrane voltage change over space?
- Why does myelin cause the speed of propagation to increase?

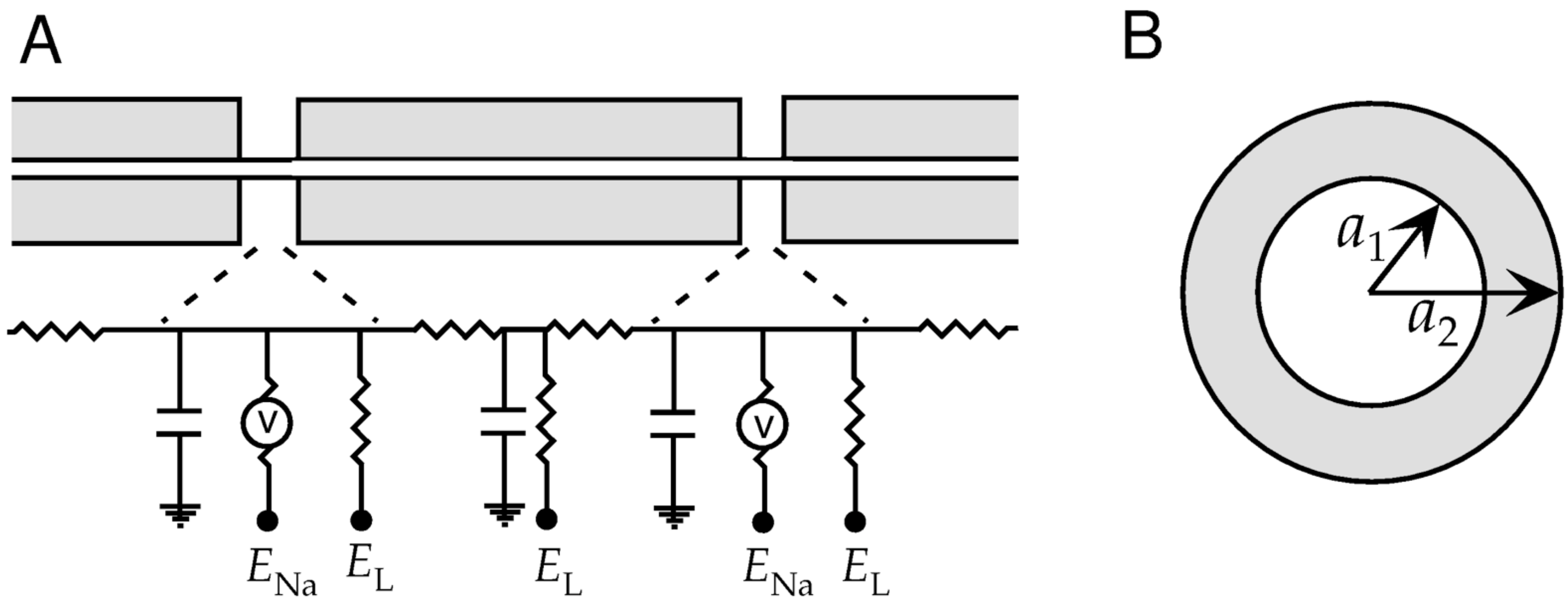
Cable Equation



$$\tau_m \frac{\partial v}{\partial t} = \lambda^2 \frac{\partial^2 v}{\partial x^2} - v + r_m i_e$$

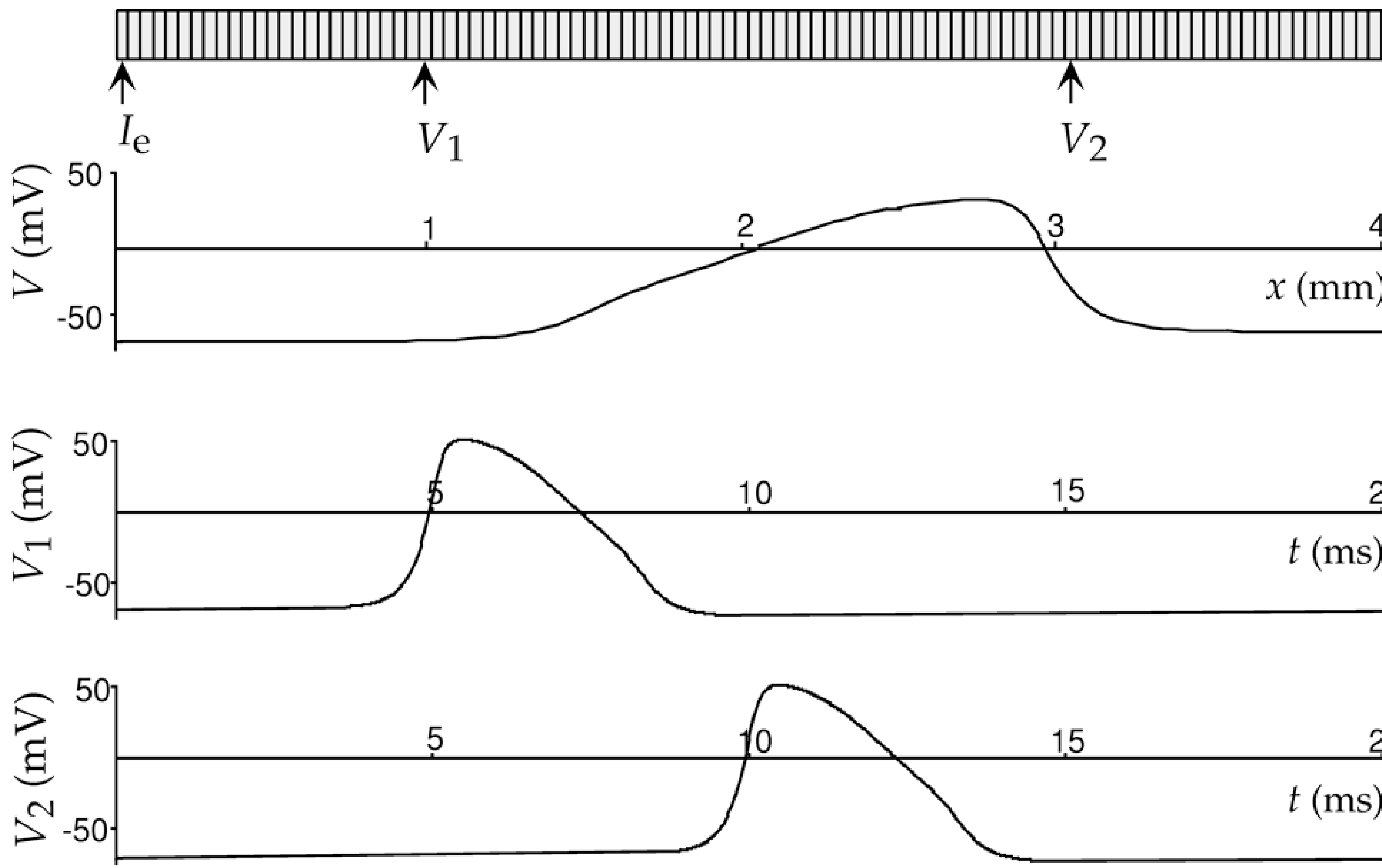
$$\lambda^2 = \frac{a r_m}{2 r_L}$$

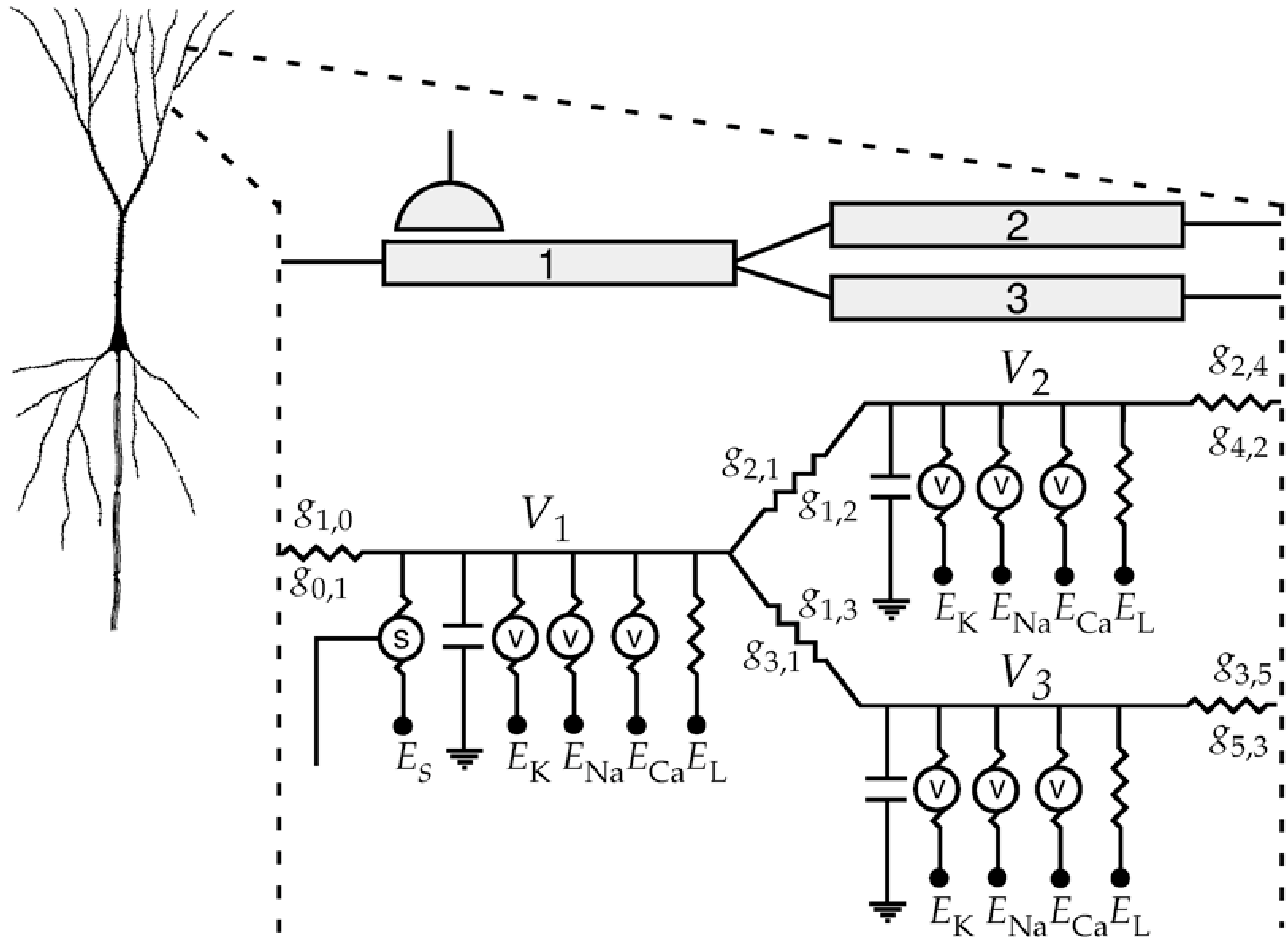
Myelinated Axon



$$\frac{C_m}{L}\frac{\partial v}{\partial t} = \frac{\pi a_1^2}{r_L}\frac{\partial^2 v}{\partial x^2}$$

Multiple Compartments



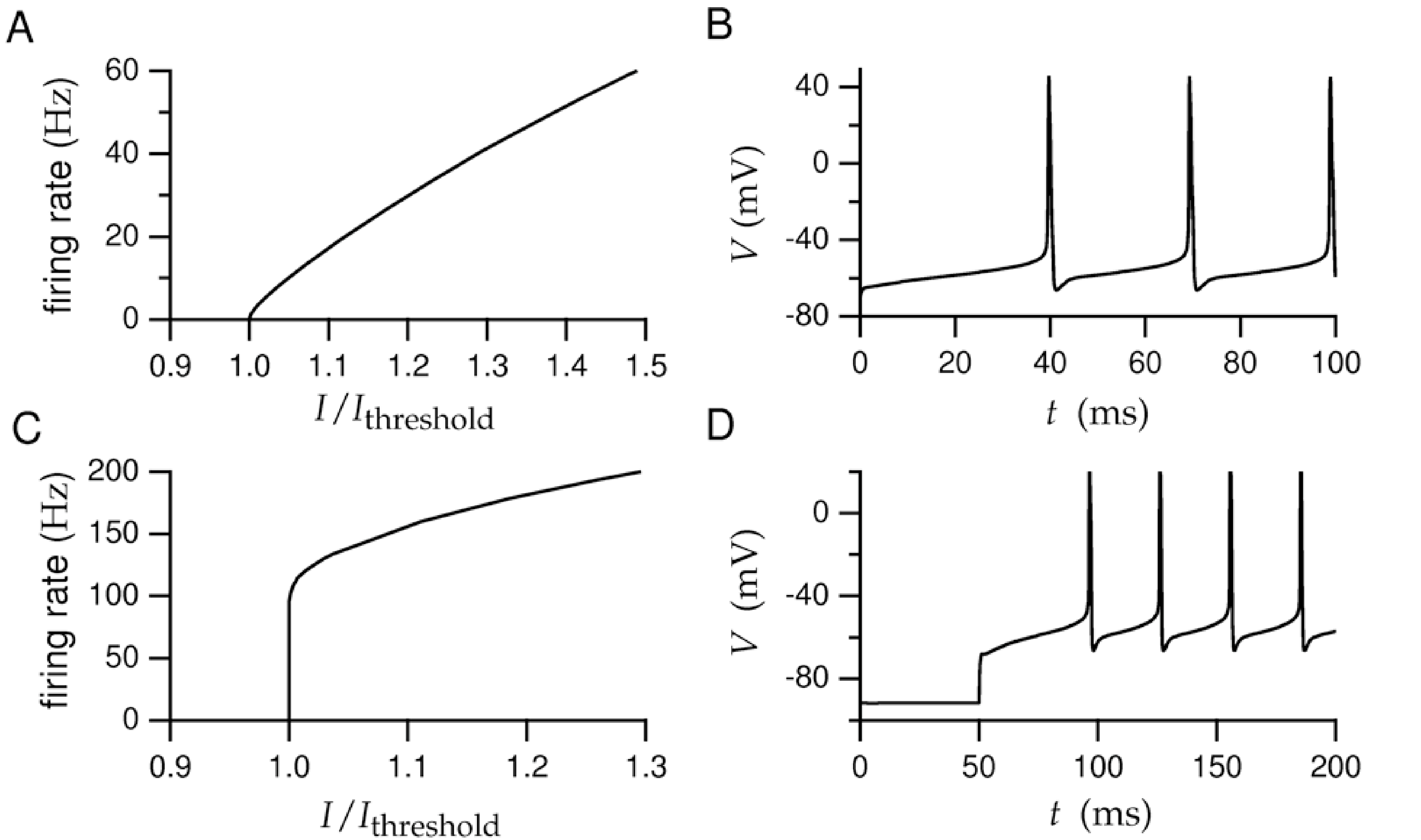


$$c_m \frac{dV_\mu}{dt} = -i_m^\mu + \frac{I_e^\mu}{A_\mu} + g_{\mu,\mu+1}\left(V_{\mu+1} - V_\mu\right) + g_{\mu,\mu-1}\left(V_{\mu-1} - V_\mu\right)$$

Beyond Hodgkin-Huxley

- Traub-Miles (~hippocampal/cortical)
  - Na, K, Leak, M or Ca-AHP
- Fleidervish (pyramidal cortical) or Miles (spinal motor neuron)
  - Slow Na inactivation
- Erisir (fast-spiking cortical)
  - Slow K
- Connor-Stevens (crab)
  - A Current (slowly inactivating K current)

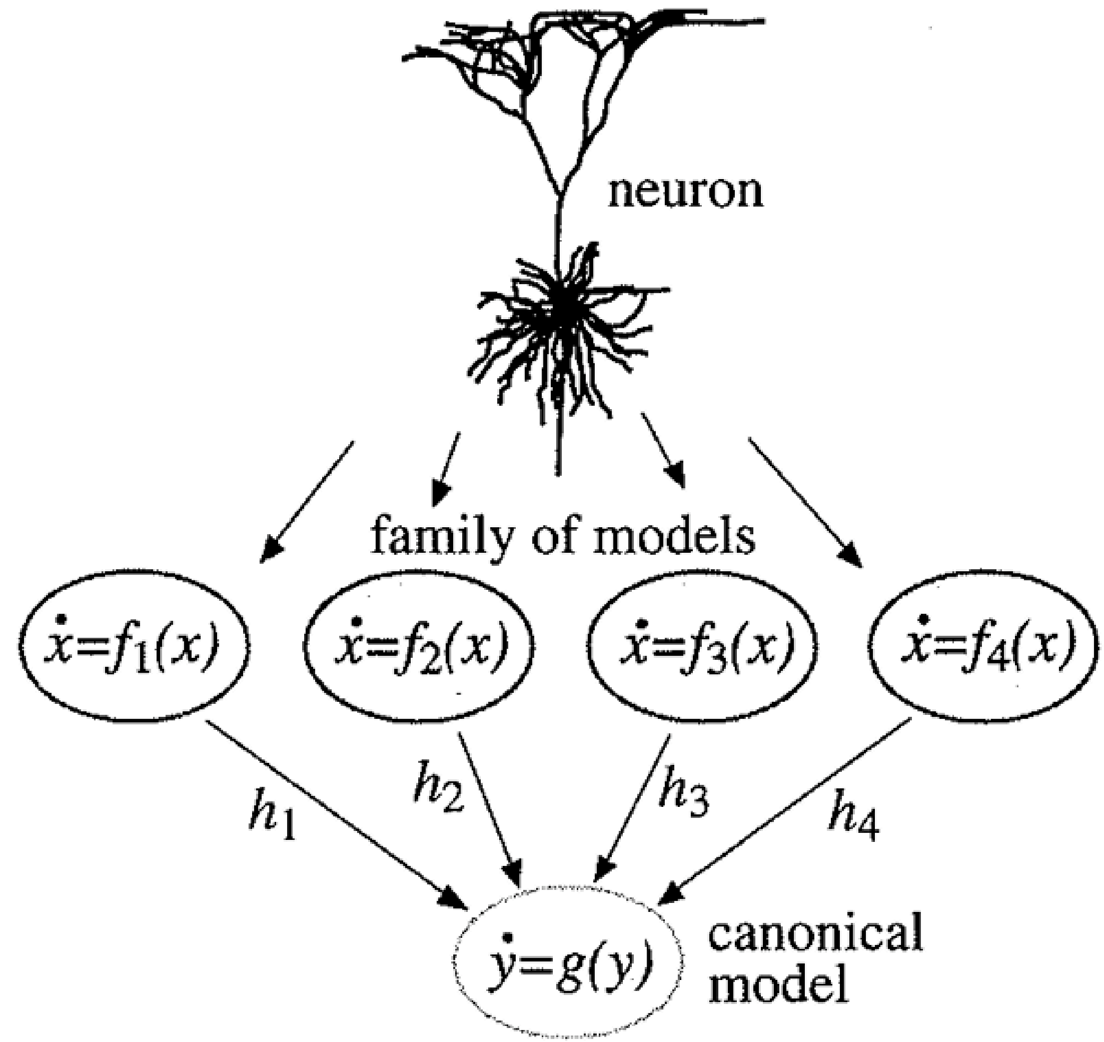
Connor-Stevens



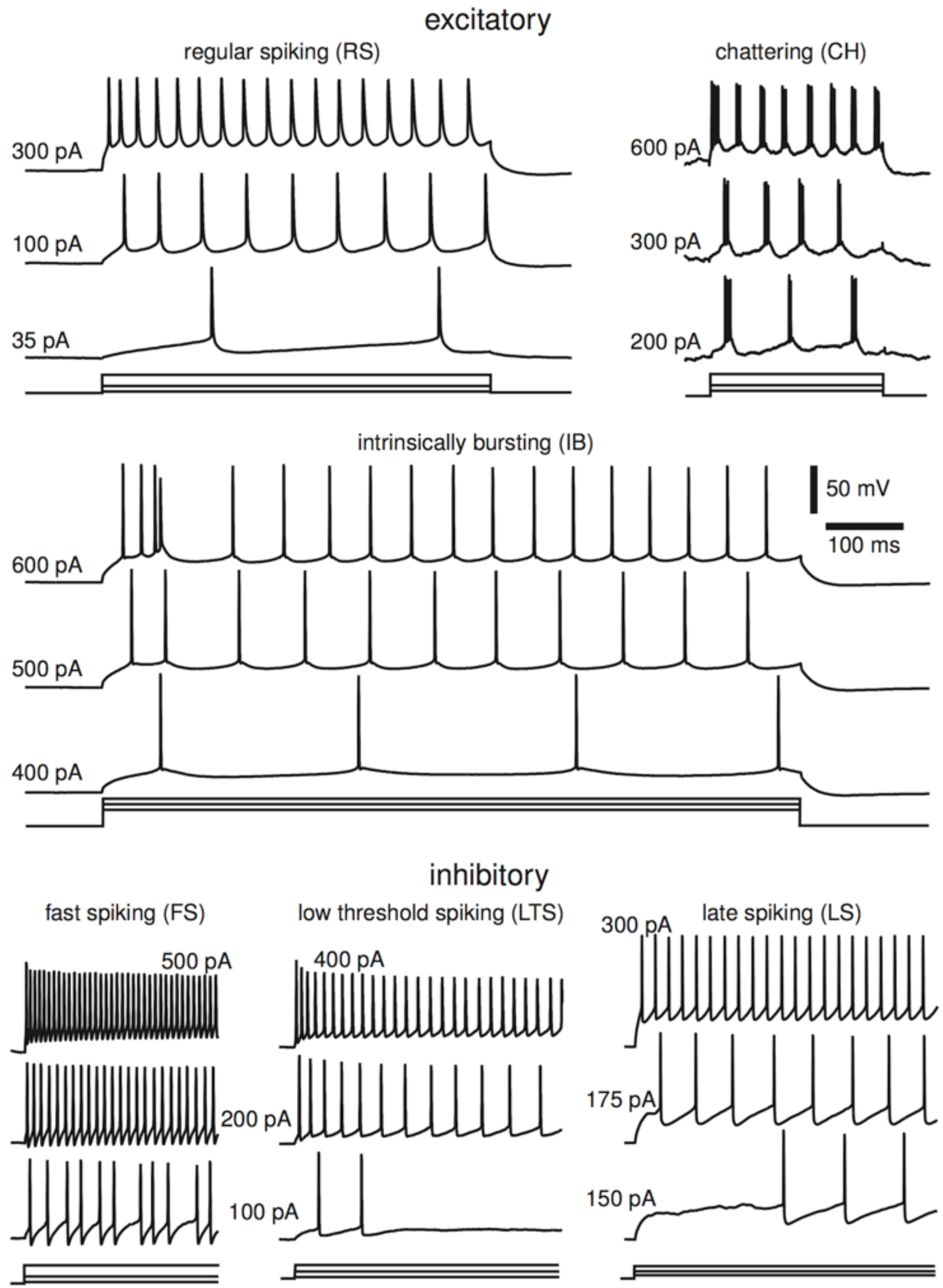
From Dynamical Systems...

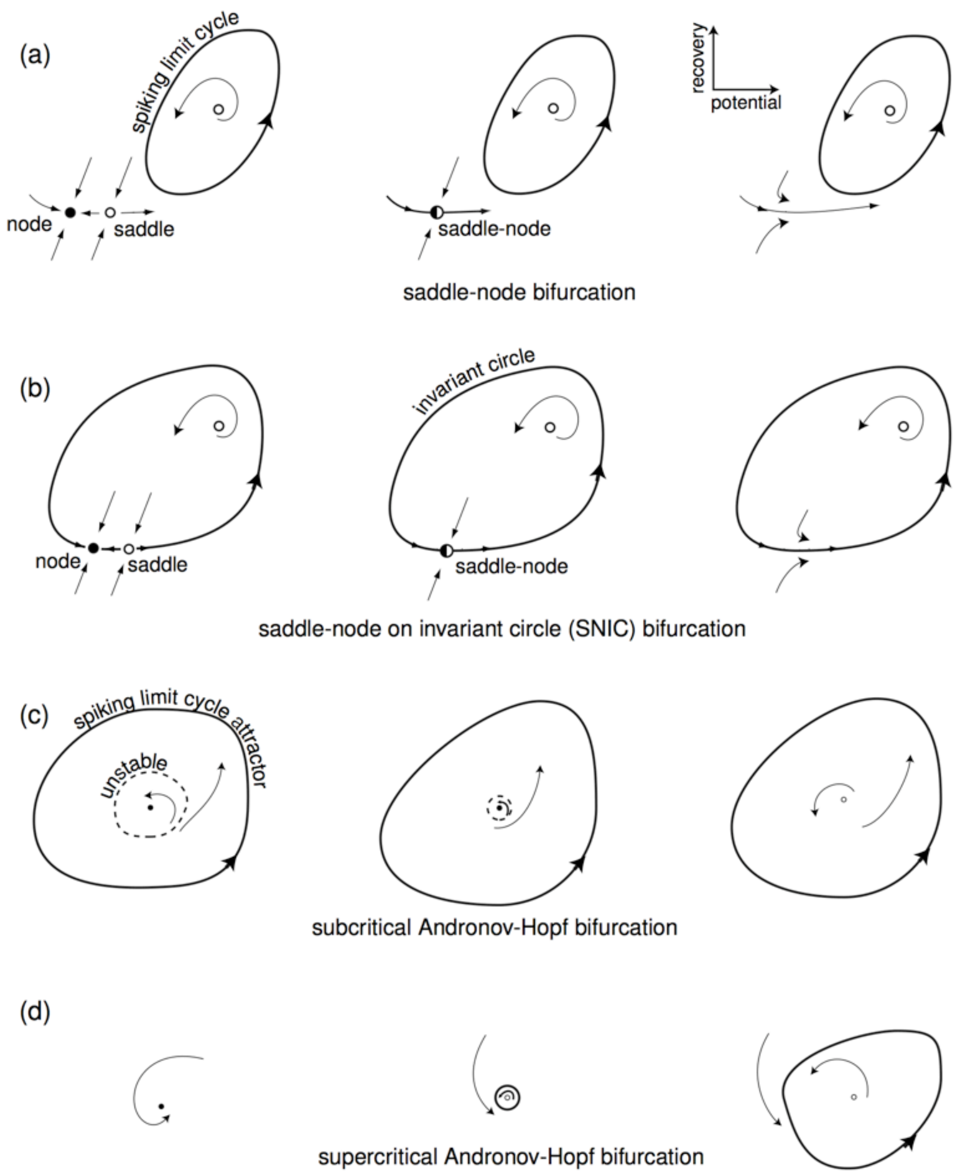
Canonical Models

- Quadratic IF
    - Type I (saddle-node on invariant circle)
- Phase model
    - nonlinear oscillators with exponentially stable limit cycle attractors
- Simple model
    - local canonical for HH-type models



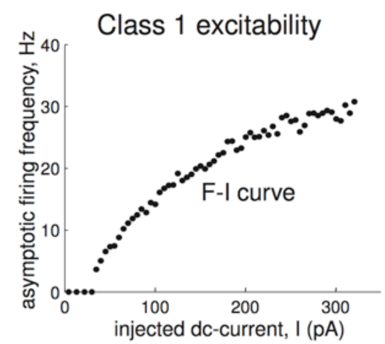
Izhikevich, *Dynamical Systems in Neuroscience*, 2007

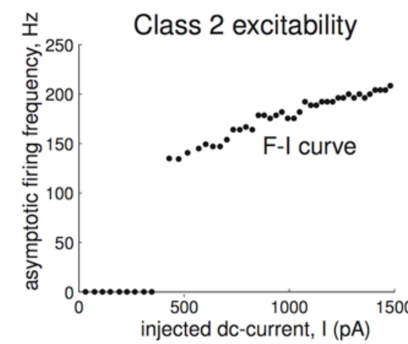

excitatory
regular spiking (RS)
chattering (CH)
300 pA
100 pA
35 pA
600 pA
300 pA
200 pA
intrinsically bursting (IB)
50 mV
100 ms
600 pA
500 pA
400 pA
inhibitory
fast spiking (FS)
low threshold spiking (LTS)
late spiking (LS)
500 pA
200 pA
100 pA
400 pA
300 pA
175 pA
150 pA



co-existance of resting and spiking states

subthreshold oscillations	YES (bistable)	NO (monostable)
NO (integrator)	saddle-node	saddle-node on invariant circle
YES (resonator)	subcritical Andronov-Hopf	supercritical Andronov-Hopf





Simple Model

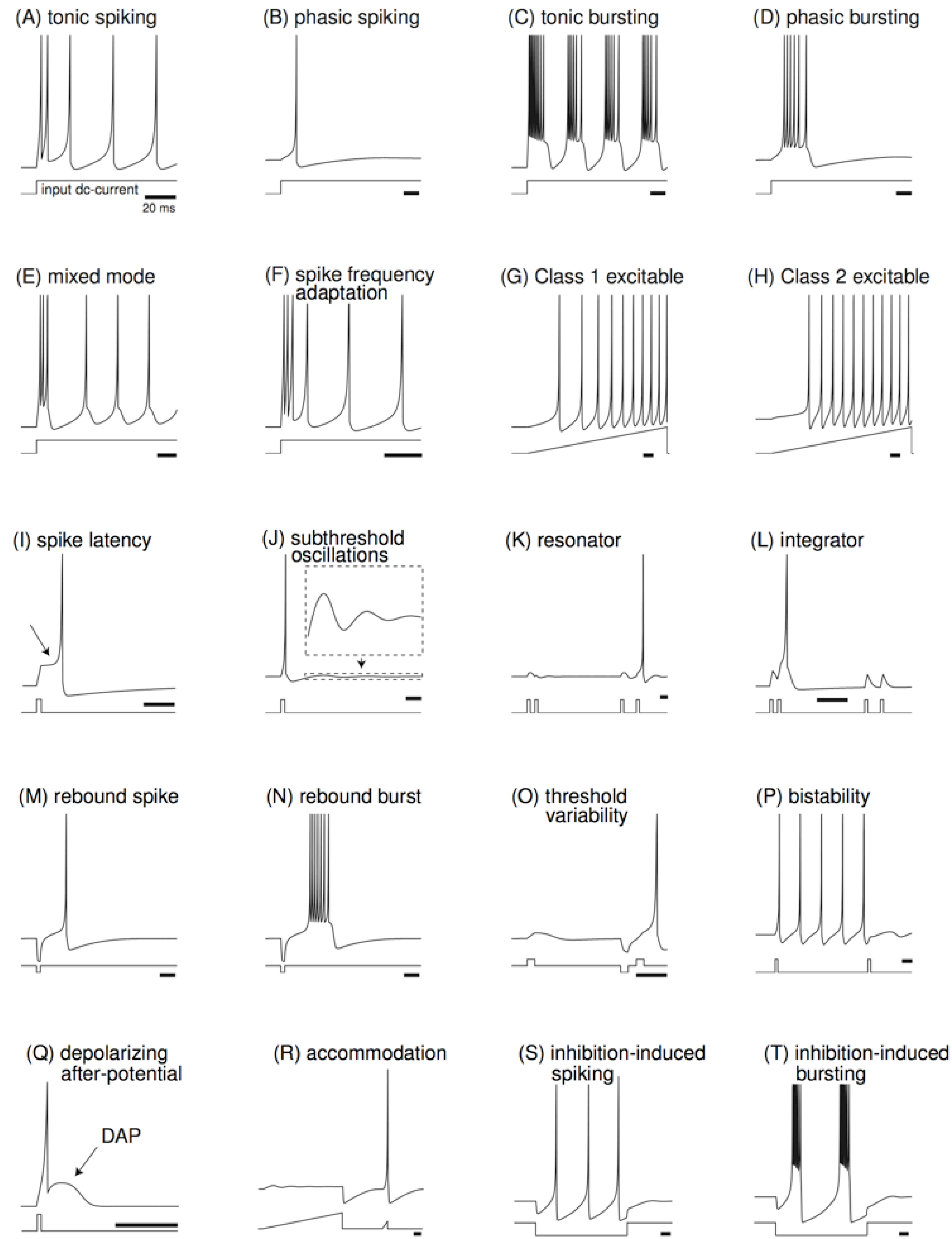
$$\frac{dv}{dt} = I + v^2 - u,$$

$$\frac{du}{dt} = a(bv - u)$$

if $v \geq 1$, then

$v \leftarrow c, u \leftarrow u + d$

- Advantages
  - 4 parameters
  - Simple (canonical)
- Disadvantages
  - No connection between model and biophysical parameters
  - Predictions for *in vitro/ vivo* neurons may be difficult



Hypotheses

- 2 “channels” are sufficient to describe neuronal computation
- 2D phase portraits can successfully depict even complicated neuronal function
- Biophysical parameters can be mapped into this 2D space