CSE/NEUBEH 528
Modeling Synapses and Networks
(Chapter 7)

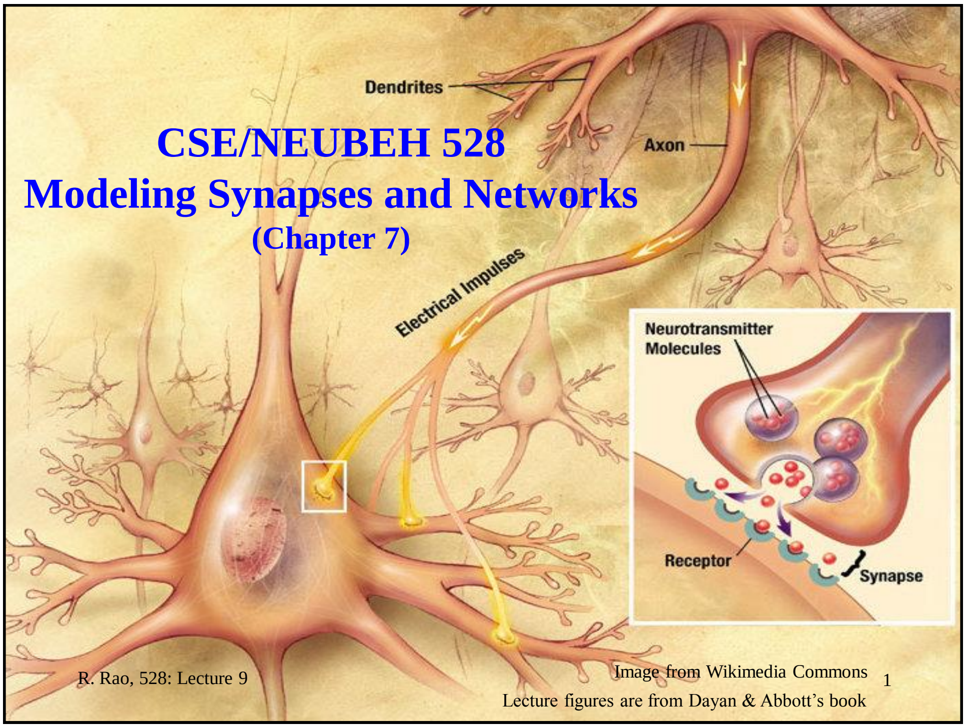


Image from Wikimedia Commons

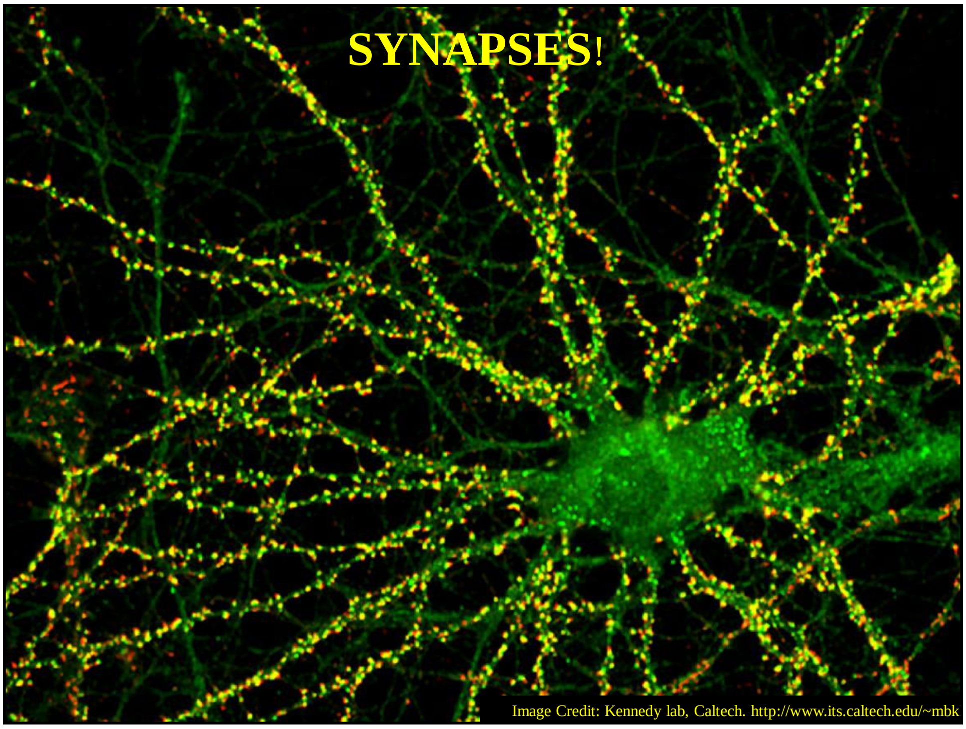
Lecture figures are from Dayan & Abbott's book

Course Summary (thus far)

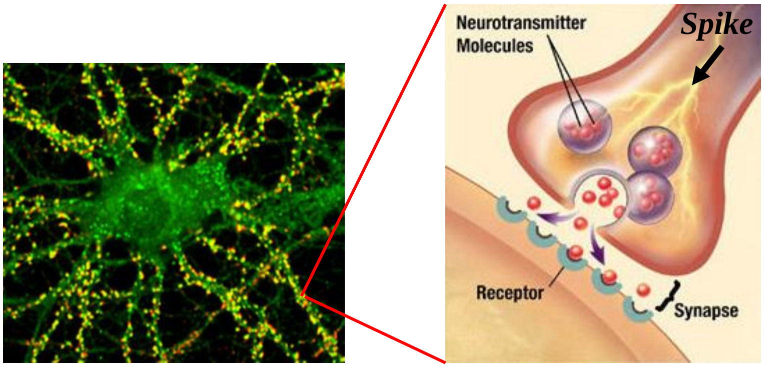
- Neural Encoding
  - What makes a neuron fire? (STA, covariance analysis)
  - Poisson model of spiking
- Neural Decoding
  - Spike-train based decoding of stimulus
  - Stimulus Discrimination based on firing rate
  - Population decoding (Bayesian estimation)
- Single Neuron Models
  - RC circuit model of membrane
  - Integrate-and-fire model
  - Conductance-based Models

Today's Agenda

- Computation in Networks of Neurons
  - Modeling synaptic inputs
  - From spiking to firing-rate based networks
  - Feedforward Networks
  - Linear Recurrent Networks



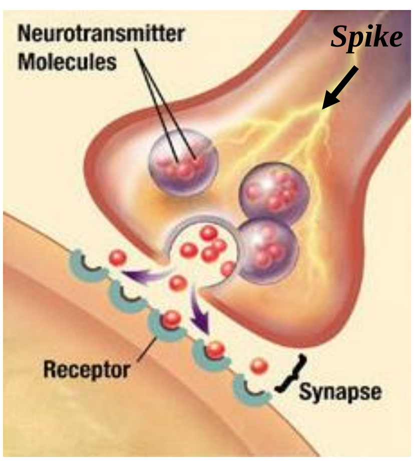
What do synapses do?



Increase or decrease postsynaptic membrane potential

Image Source: Wikimedia Commons

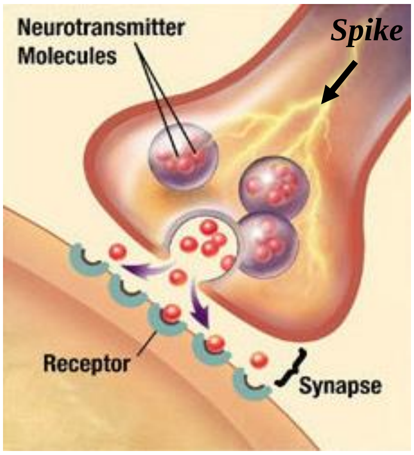
An Excitatory Synapse



Input spike →
Neurotransmitter release
(e.g., Glutamate) →
Binds to receptors →
Ion channels open →
positive ions (e.g. Na+)
enter cell →
Depolarization due to
EPSP (excitatory
postsynaptic potential)

Image Source: Wikimedia Commons

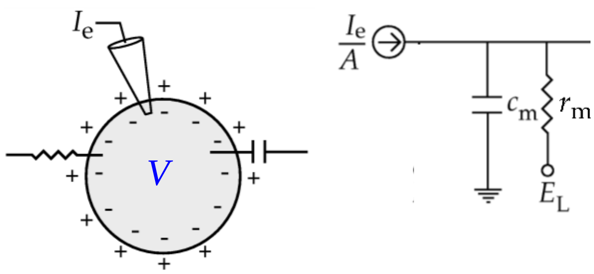
An Inhibitory Synapse



Input spike → Neurotransmitter release (e.g., GABA) → Binds to receptors → Ion channels open → positive ions (e.g., K+) leave cell → Hyperpolarization due to IPSP (inhibitory postsynaptic potential)

Image Source: Wikimedia Commons

Flashback Membrane Model

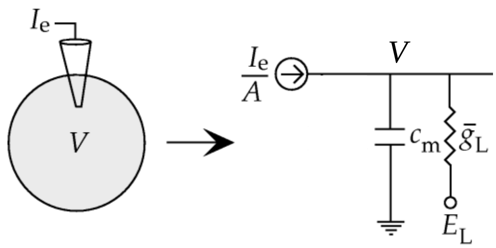


$$c_m \frac{dV}{dt} = -\frac{(V - E_L)}{r_m} + \frac{I_e}{A}, \text{ or equivalently}$$

$\tau_m = r_m c_m = R_m C_m$ membrane time constant

$$\tau_m \frac{dV}{dt} = -(V - E_L) + I_e R_m$$

Flashback! The Integrate-and-Fire Model



Models a passive leaky membrane

$$\tau_m \frac{dV}{dt} = -(V - E_L) + I_e R_m$$

If V > $V_{threshold}$ → Spike

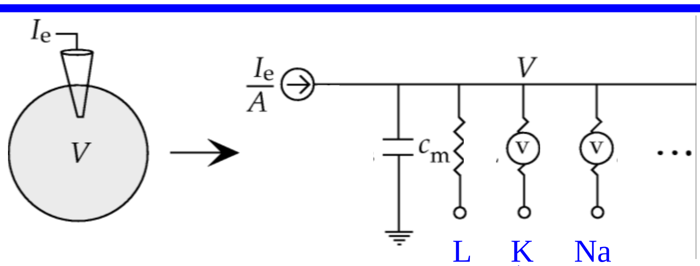
Then reset: V = V_{reset}

$E_L \approx -70$ mV (resting potential)

$V_{threshold} \approx -50$ mV

$V_{reset} \approx E_L$

Flashback! Hodgkin-Huxley Model

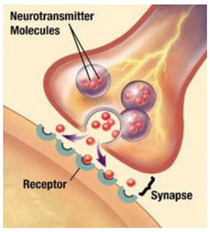


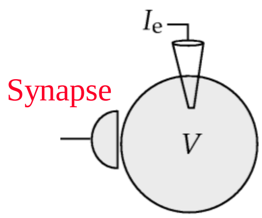
$$c_m \frac{dV}{dt} = -i_m + \frac{I_e}{A}$$

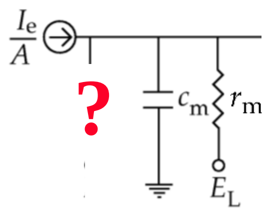
$$i_m = g_{L,\max}(V - E_L) + g_{K,\max} n^4 (V - E_K) + g_{Na,\max} m^3 h (V - E_{Na})$$

E_L = -54 mV, E_K = -77 mV, E_{Na} = +50 mV

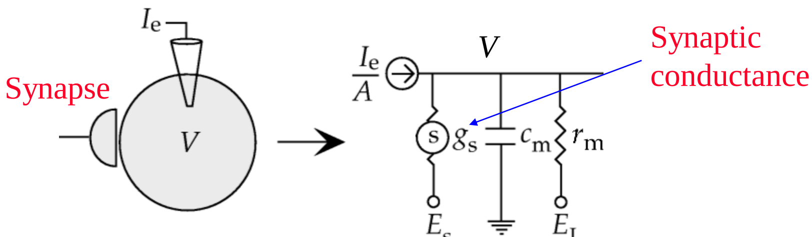
How do we model the effects of a synapse on the membrane potential V?



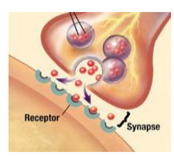




Modeling Synaptic Inputs



$$\tau_m \frac{dV}{dt} = -(V - E_L) - r_m g_s (V - E_s) + I_e R_m$$



$$g_s = g_{s,\max} P_{rel} P_s$$

P_s ← Probability of postsynaptic channel opening (= fraction of channels opened)

P_{rel} ← Probability of transmitter release given an input spike

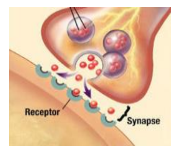
Basic Synapse Model

- Assume $P_{rel} = 1$
- Model the effect of a single spike input on P_s ← fraction of channels opened
- Kinetic Model of postsynaptic channels:

$$\text{Closed} \underset{\beta_s}{\overset{\alpha_s}{\rightleftharpoons}} \text{Open}$$

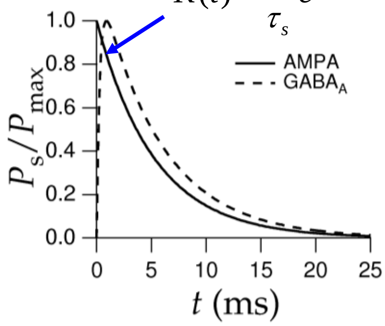
$$\frac{dP_s}{dt} = \alpha_s(1 - P_s) - \beta_s P_s$$

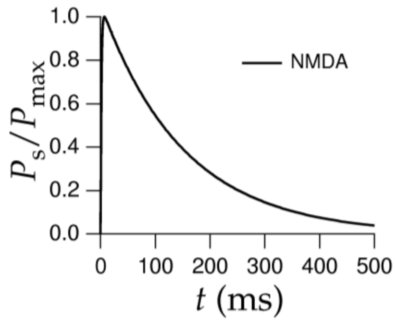
α_s: Opening rate; $(1 - P_s)$: Fraction of channels closed; β_s: Closing rate; P_s: Fraction of channels open



What does P_s look like over time?

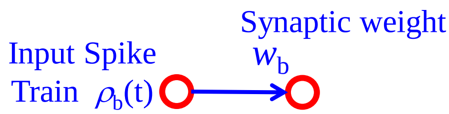
$$K(t) = \frac{1}{\tau_s} e^{-\frac{t}{\tau_s}}$$





Exponential function $K(t)$ gives reasonable fit to biological data (other options: difference of exponentials, “alpha” function)

Linear Filter Model of Synaptic Input to a Neuron



$\rho_b(t) = \Sigma_i \delta(t-t_i)$ (t_i are the input spike times)

Filter for synapse b: $$K(t) = \frac{1}{\tau_s} e^{-\frac{t}{\tau_s}}$$

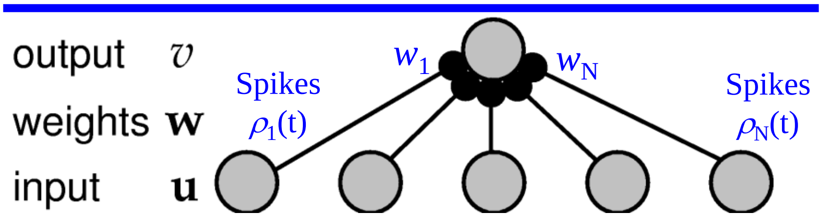
Synaptic current for b: $$I_b(t) = w_b \sum_{t_i < t} K(t - t_i)$$

$$= w_b \int_{-\infty}^{t} K(t-\tau)\rho_b(\tau)d\tau$$

Modeling Networks of Neurons

- Option 1: Use *spiking* neurons
  - *Advantages*: Model computation and learning based on:
    - Spike Timing
    - Spike Correlations/Synchrony between neurons
  - *Disadvantages*: Computationally expensive
- Option 2: Use neurons with *firing-rate outputs (real valued outputs)*
  - *Advantages*: Greater efficiency, scales well to large networks
  - *Disadvantages*: Ignores spike timing issues
- Question: How are these two approaches related?

From Spiking to Firing Rate Models



Total synaptic current $I_s(t) = \sum_b I_b(t)$

$$I_s(t) = \sum_b w_b \int_{-\infty}^{t} K(t-\tau)\rho_b(\tau)d\tau$$ **Spike train ρ_b(t)**

$$\approx \sum_b w_b \int_{-\infty}^{t} K(t-\tau)u_b(\tau)d\tau$$ **Firing rate u_b(t)**

Synaptic Current Dynamics in Firing Rate Model

- Suppose synaptic kernel K is exponential: $K(t) = \frac{1}{\tau_s} e^{-\frac{t}{\tau_s}}$

  Differentiating $I_s(t) = \sum_b w_b \int_{-\infty}^{t} K(t-\tau)u_b(\tau)d\tau$ w.r.t. time t,

  we get
$$\tau_s \frac{dI_s}{dt} = -I_s + \sum_b w_b u_b$$
$$= -I_s + \mathbf{w} \cdot \mathbf{u}$$

Output Firing-Rate Dynamics

- How is the output firing rate v related to synaptic inputs?

$$\tau_r \frac{dv}{dt} = -v + F(I_s(t)) \qquad \tau_s \frac{dI_s}{dt} = -I_s + \mathbf{w}\cdot\mathbf{u}$$

- Looks very much like membrane equation:

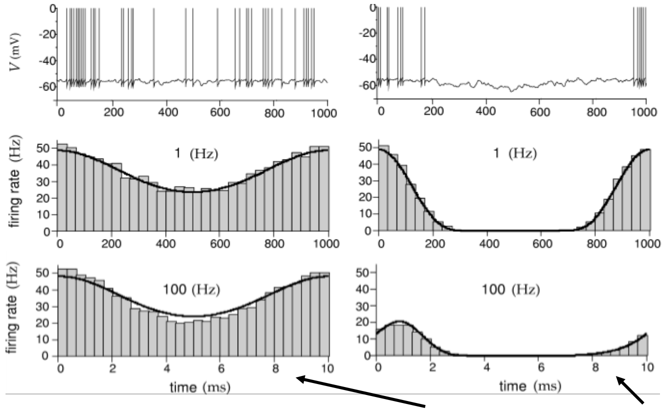
$$\tau_m \frac{dV}{dt} = -(V - E_L) + I_e R_m$$

- On-board derivations of special cases obtained from comparing the relative magnitudes of τ_r and τ_s …

  (see also pages 234-236 in the text)

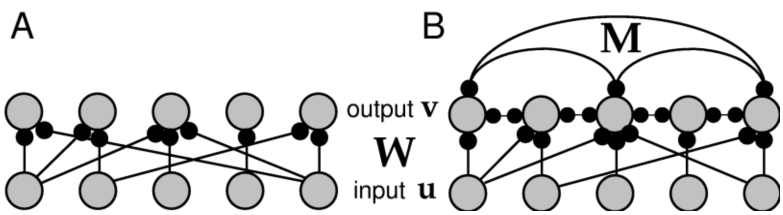
How good are Firing Rate Models?

Input $I(t) = I_0 + I_1\cos(\omega t)$



Firing rate model $v(t) = F(I(t))$ describes this well but not this case

Feedforward versus Recurrent Networks

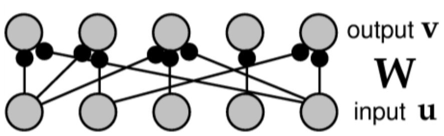


$$\tau \frac{d\mathbf{v}}{dt} = -\mathbf{v} + F(\mathrm{W}\mathbf{u} + \mathrm{M}\mathbf{v})$$

Output Decay Input Feedback

For feedforward networks, matrix M = 0

Example: Linear Feedforward Network



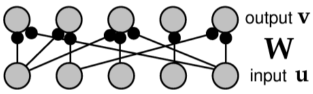
Dynamics: $\tau \frac{d\mathbf{v}}{dt} = -\mathbf{v} + \mathrm{W}\mathbf{u}$

Steady State (set $d\mathbf{v}/dt$ to 0): $\mathbf{v}_{ss} = \mathrm{W}\mathbf{u}$

$$\mathrm{W} = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & 0 & -1 \end{bmatrix} \qquad \mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 2 \\ 1 \end{bmatrix}$$

What is $\mathbf{v}_{ss}$?

Linear Feedforward Network

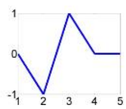


$$\mathbf{v}_{ss} = \mathrm{W}\mathbf{u} = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}$$

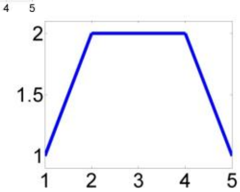
What is the network doing?

Linear Filtering for Edge Detection

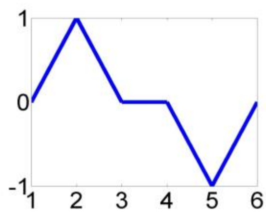
$\text{Filter} = [0 \quad -1 \quad 1 \quad 0 \quad 0]$
(and shifted versions in W)



$$\text{Input} = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 2 \\ 1 \end{bmatrix} \quad \text{Output} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}$$

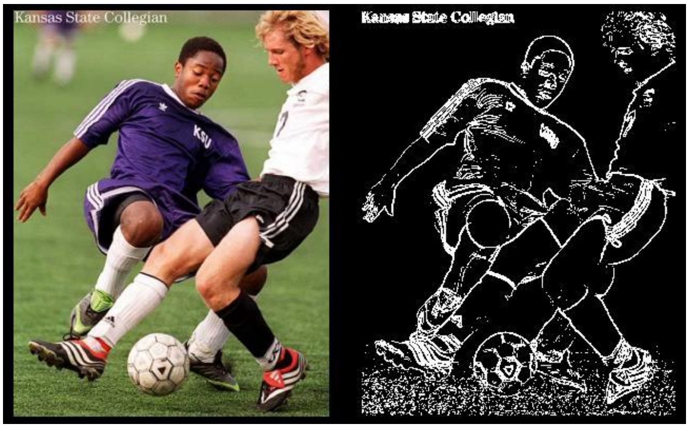


Input



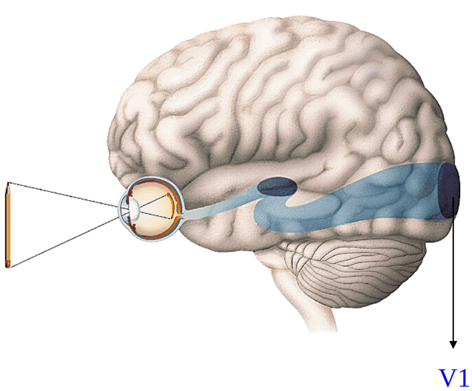
Output

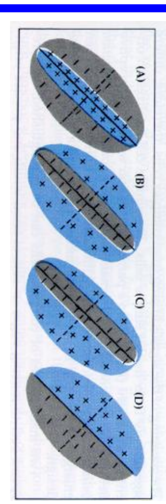
Example of Edge Detection in a 2D Image



http://www.alexandria.nu/ai/blog/entry.asp?E=51

Edge detectors in the visual system

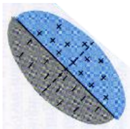




Examples of receptive fields in primary visual cortex (V1)

(From Nicholls et al., 1992)

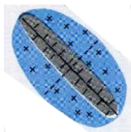
Filtering network is computing derivatives!



$$[0 \quad -1 \quad 1 \quad 0 \quad 0]$$

$$\frac{df}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Discrete approximation $\approx f(x+1) - f(x)$



$$[0 \quad 1 \quad -2 \quad 1 \quad 0]$$

$$\frac{d^2 f}{dx^2} = \lim_{h \to 0} \frac{f'(x+h) - f'(x)}{h}$$

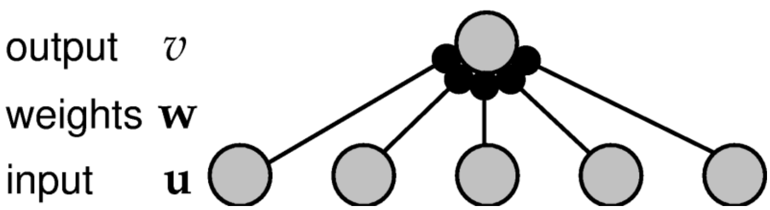
Disc. approx. $\approx \big(f(x+1) - f(x)\big) - \big(f(x) - f(x-1)\big)$

$= f(x+1) - 2f(x) + f(x-1)$

Feedforward Networks: Example 2

Coordinate Transformation

Output: Premotor Cortex Neuron with *Body-Based Tuning Curves*

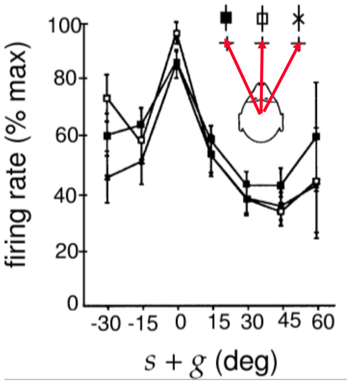


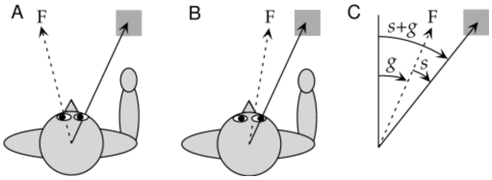
Input: Area 7a Neurons with *Gaze-Dependent Tuning Curves*

(From Section 7.3 in Dayan & Abbott)

Output of Coordinate Transformation Network

Head fixed;
gaze shifted to g_1 g_2 g_3



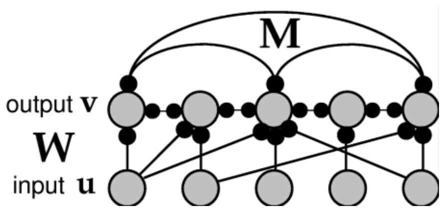


Same tuning curve regardless of gaze angle

Premotor cortex neuron responds to stimulus location *relative to body*, not retinal image location

(See section 7.3 in Dayan & Abbott for details)

Linear Recurrent Networks



$$\tau \frac{d\mathbf{v}}{dt} = -\mathbf{v} + \mathbf{W}\mathbf{u} + \mathbf{M}\mathbf{v}$$

Output Decay Input Feedback

Next Class: Recurrent Networks

- To Do:
  - Homework 2
  - Find a final project topic and partner(s)