CSE 531
Assignment 2
Due October 12, 2000

1. It can be instructive to convert an "implementation description" of a Turing machine to a "formal description". This exercise gives you confidence that implementation descriptions are adequate for describing Turing machine algorithms. In this problem you will convert a Turing machine with a two-way infinite tape to a Turing machine with a one-way infinite tape using a different construction than was given in class.

   Let $M = (Q, \Sigma, \Gamma, \delta, q_0, q_A, q_R)$ be a Turing machine that has a two-way infinite tape. Formally construct a Turing machine $M' = (Q', \Sigma, \Gamma', \delta', q'_0, q'_A, q'_R)$ such that M' has a one-way infinite tape and follows the implementation description below.

   Implementation description of M': M' begins by inserting a new symbol # at the left end of the tape, shifting the input string one cell to the right and resetting the head to the first symbol of the input. This new format prevents M' from ever running off the left end of the tape. M' now begins simulating M, but should M' ever read # then that indicates that M has moved left on its two-way tape to a cell that has never been visited before. In order to accommodate this M' must create a blank cell just to the right of #, shift the entire contents of the tape one cell to the right, then return the head to the blank cell just to the right of # to continue the simulation.

   Construct M' in full detail, specifying its states, tape alphabet, transition function, start state, accepting state, and rejecting state.

2. Let $s_1, s_2, \ldots$ be the strings in Σ^* in enumeration order. For example if $\Sigma = \{0, 1\}$ then enumeration order is $\{\varepsilon, 0, 1, 00, 01, 10, 11, 000, 001, \ldots\}$, that is, strings of length $n + 1$ are enumerated after strings of length n and strings of the same length are enumerated in lexicographical (alphabetical) order.

   (a) Let L be the language enumerated by a Turing enumerator E. Suppose E enumerates L in enumeration order, that is, if s_i is enumerated by E before s_j then $i < j$. Argue that L is Turing-decidable.

   (b) Suppose that L is an infinite Turing-recognizable language. Prove that L has an infinite Turing-decidable subset. (Hint: use part (a).)