CSE 531 MIDTERM EXAM

November 2, 1995

1. (30%) We have shown that $K = \{\langle M \rangle : \langle M \rangle \in L(M)\}$ is undecidable. Let $H = \{\langle M \rangle : M \text{ halts on all inputs}\}$.

   (a) Why can't H be shown to be undecidable using Rice's theorem?

   (b) Show that H is undecidable using a reduction from K.

   (c) Is H recursively enumerable? If so, why? If not, why not?

2. (30%) Define the *shuffle* of two languages by $L_1 \parallel L_2 = \{x_1 y_1 x_2 y_2 ... x_n y_n : x_i, y_i \in \Sigma^*$ for $1 \leq i \leq n$, $x_1 x_2 ... x_n \in L_1$, and $y_1 y_2 ... y_n \in L_2\}$.

   (a) If M_1 and M_2 are DFAs accepting L_1 and L_2 respectively, construct an NFA M which accepts $L_1 \parallel L_2$.

   (b) State carefully a behavioral lemma, which could be proved by induction, which expresses the relationship between the behavior of your NFA M and the behaviors of the two machines M_1 and M_2.

   (c) Use your behavioral lemma to prove $L(M) = L(M_1) \parallel L(M_2)$

3. (40%) Define a language $L \subseteq \Sigma^*$ to be *right locally testable over* Σ if $L = \Sigma^* F \cup G$ for finite sets $F, G \subseteq \Sigma^*$. Clearly, the right locally testable languages are closed under union.

   (a) Argue that the right locally testable languages are closed under intersection.

   (b) Argue that the complement of $\Sigma^* a$ is locally testable where $a \in \Sigma$.

   (c) Argue that the complement of $\Sigma^* a_1 a_2 ... a_n$ is locally testable where $a_i \in \Sigma$ for $1 \leq i \leq n$.

   (d) Using (a) and (c) above argue that the right locally testable languages are closed under complement.

   (e) From the above we know that the right locally testable languages are closed under the Boolean operations of union, intersection, and complement. How would you go about proving that the class of right locally testable languages over Σ is the smallest class of languages containing Σ^* which is closed under the Boolean operations and concatenation on the right by a single symbols from Σ.