

# CSE 544

# Principles of Database Management Systems

## Lecture 3 –Schema Normalization

# Announcements

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- Project groups due on Friday
- First review due on Tuesday (makeup lecture)
  - Run ‘git pull’ to get the ‘review’ subdirectory
  - Place your review there
  - Commit, push (see instructions in hw1.md); no need to tag
- Homework 1 due next Friday
  - Run `turnInHw.sh hw1`
  - Or manually: ‘git commit...’, ‘git tag...’, ‘git push...’

# Database Design

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- The relational model is great, but how do I design my database schema?

# Outline

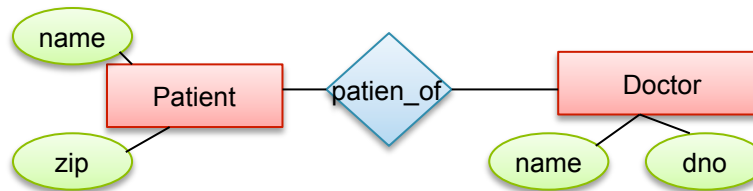
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- Conceptual db design: entity-relationship model
- Problematic database designs
- Functional dependencies
- Normal forms and schema normalization

# Conceptual Schema Design

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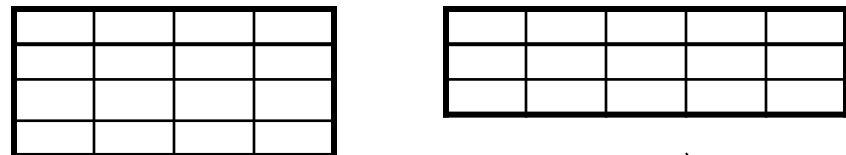
Conceptual Model:



Relational Model:

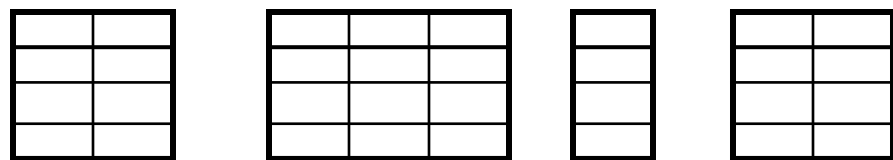
plus FD's

(FD = functional dependency)



Normalization:

Eliminates anomalies



# Entity-Relationship Diagram

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Attributes



Entity sets



Relationship sets



# Entity-Relationship Diagram

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Attributes



Entity sets

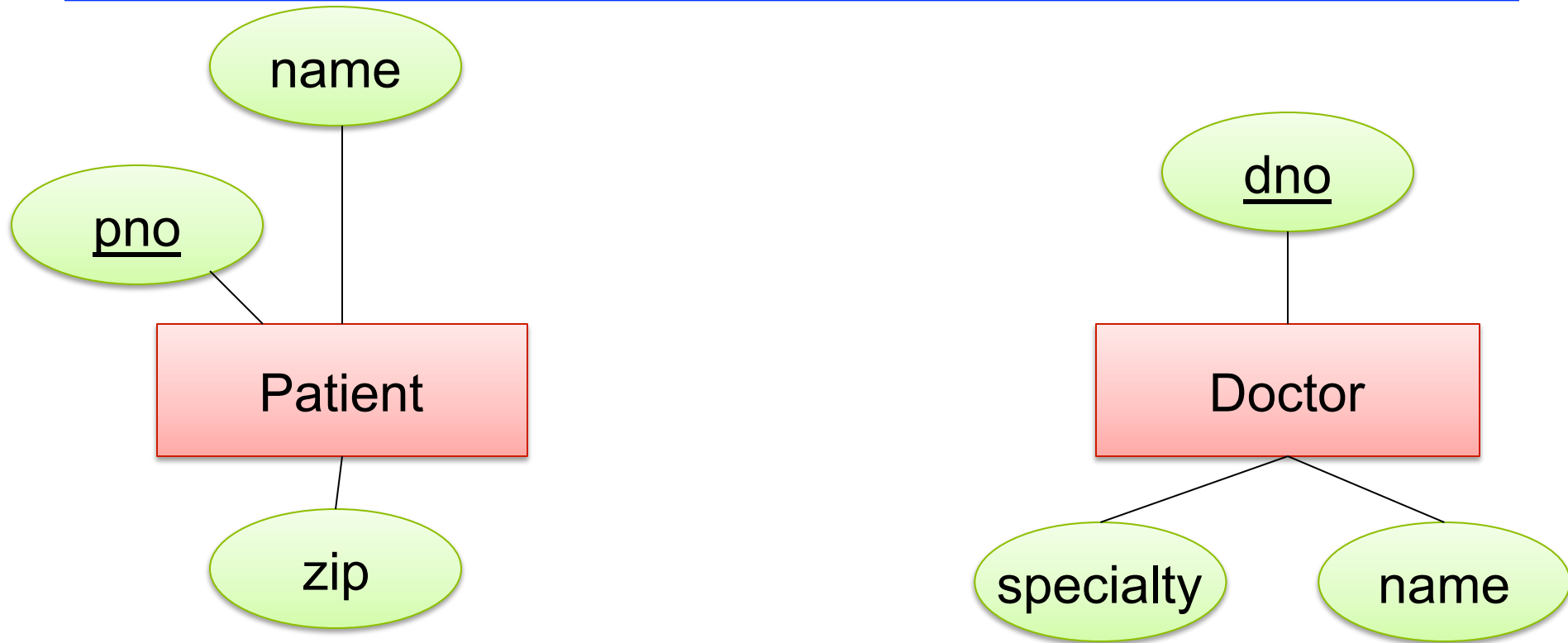


Relationship sets



# Entity-Relationship Diagram

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Attributes



Entity sets



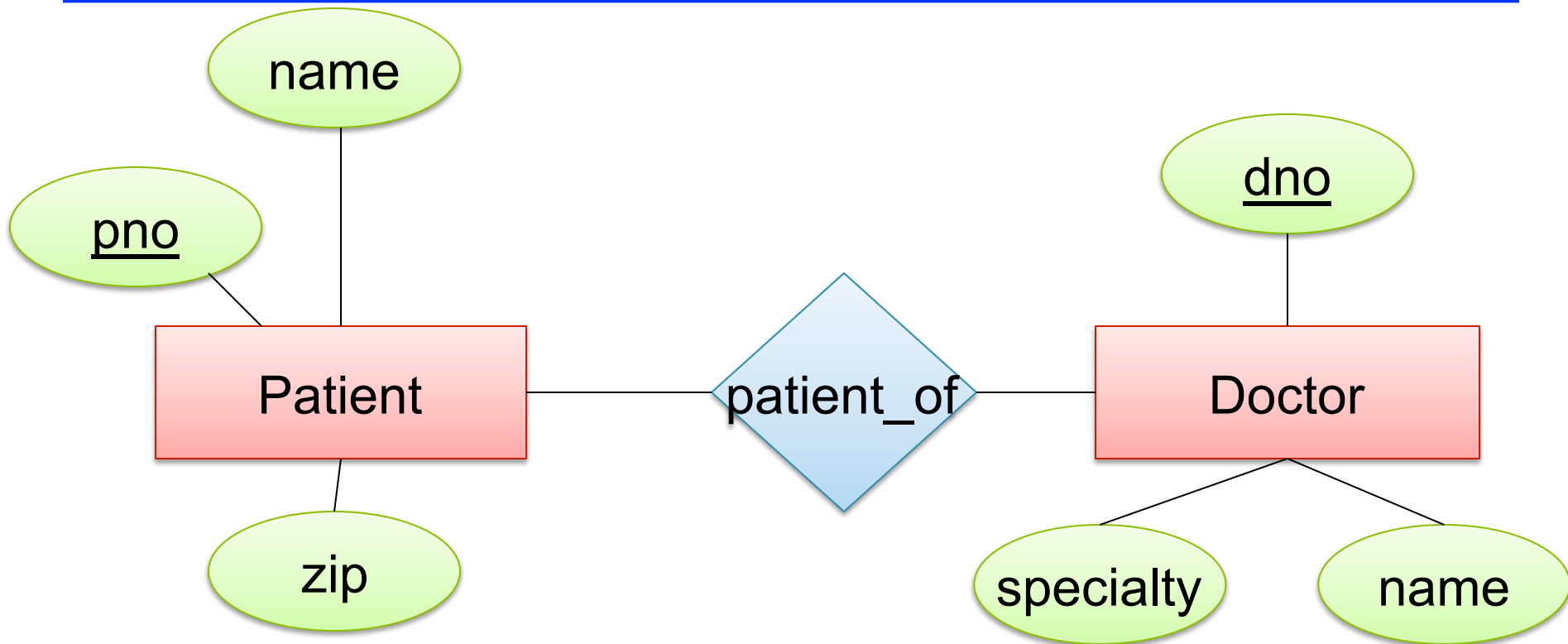
Relationship sets





# Entity-Relationship Diagram

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Attributes



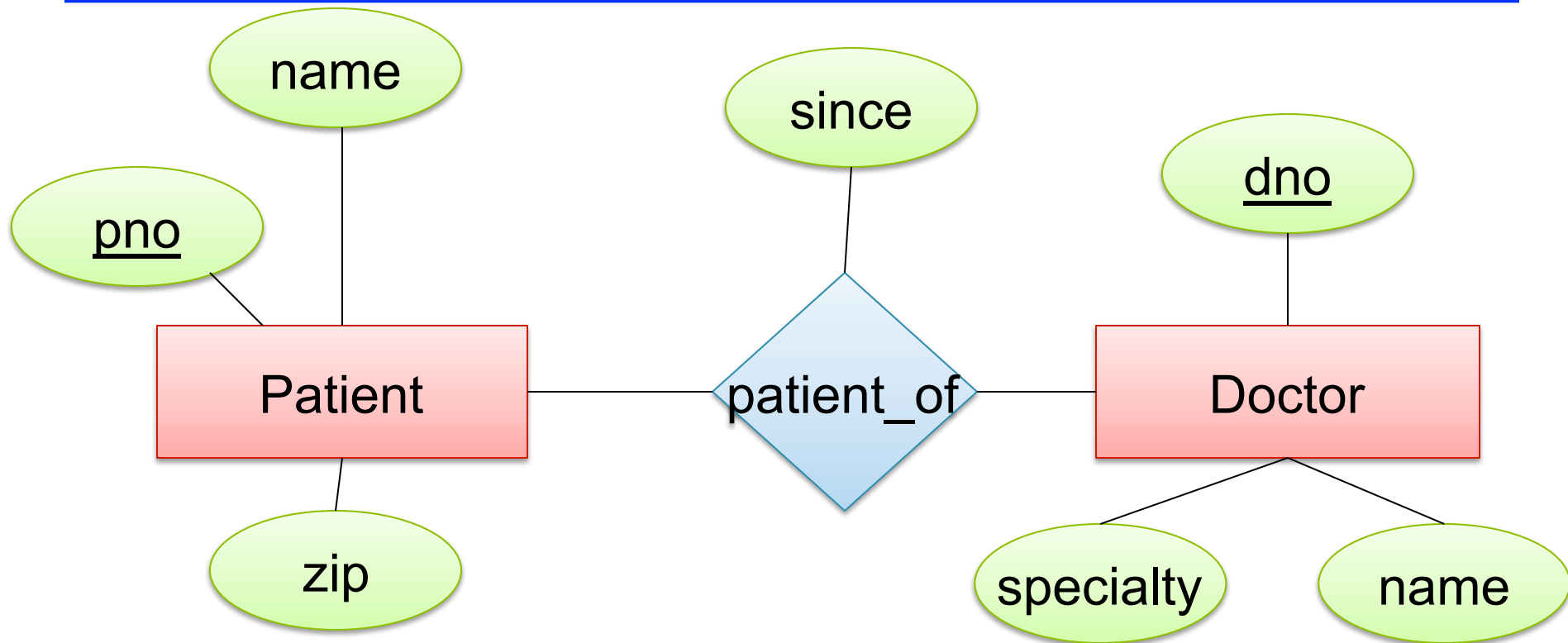
Entity sets



Relationship sets



# Entity-Relationship Diagram



Attributes



Entity sets



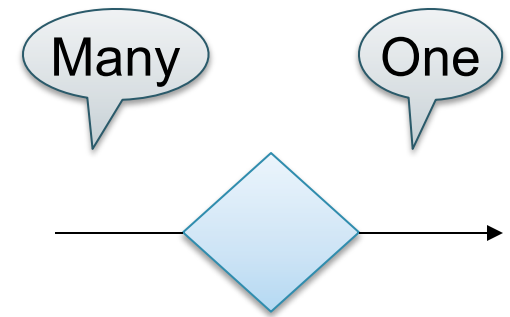
Relationship sets



# Entity-Relationship Model

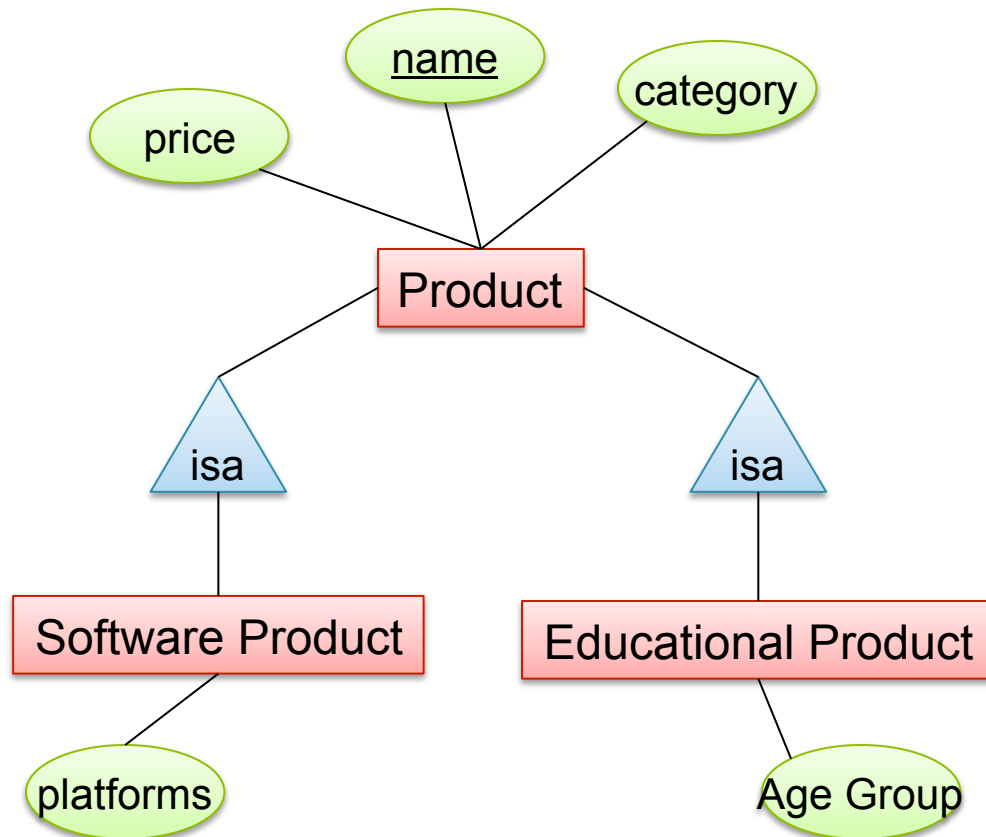
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- Typically, each entity has a key
- ER relationships can include multiplicity
  - One-to-one, one-to-many, etc.
  - Indicated with arrows
- Can model multi-way relationships
- Can model subclasses
- And more...

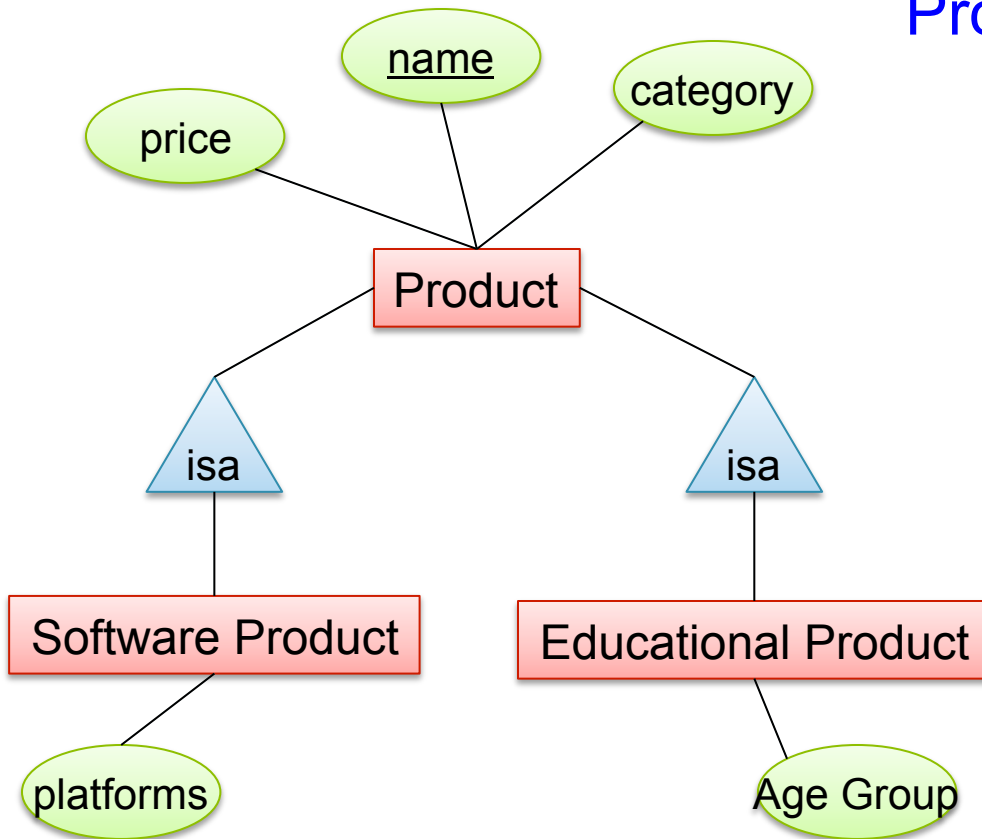


# Subclasses to Relations

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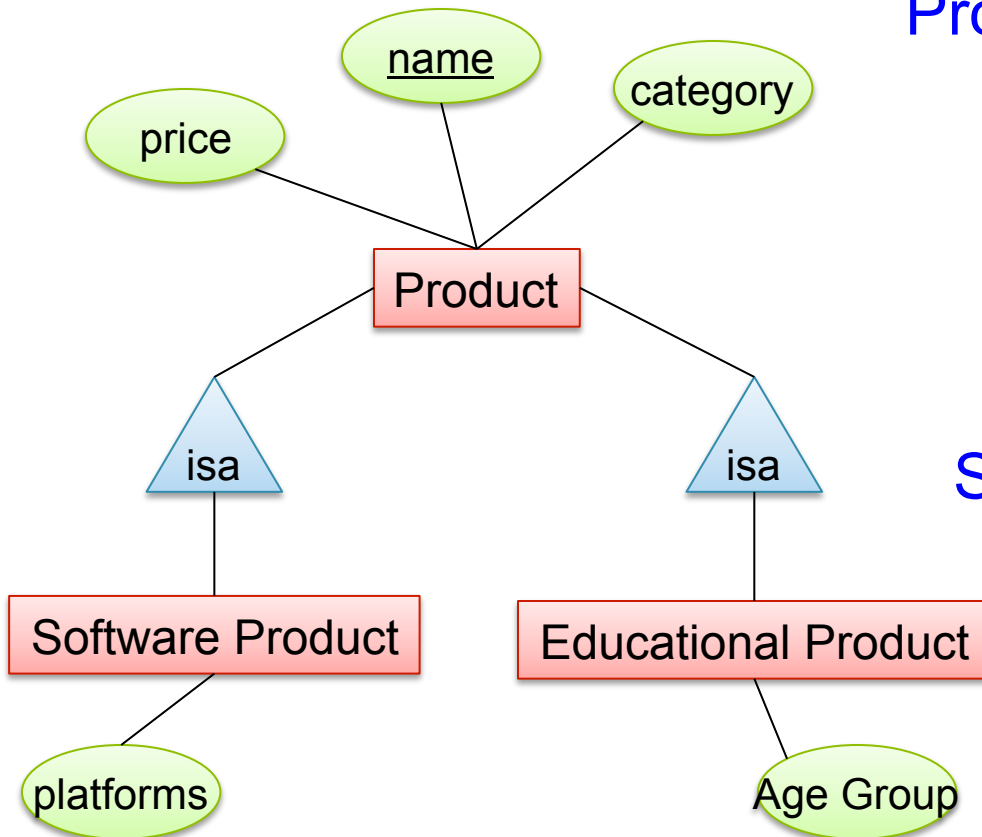
# Subclasses to Relations



Product

<u>Name</u>	Price	Category
Gizmo	99	gadget
Camera	49	photo
Toy	39	gadget

# Subclasses to Relations



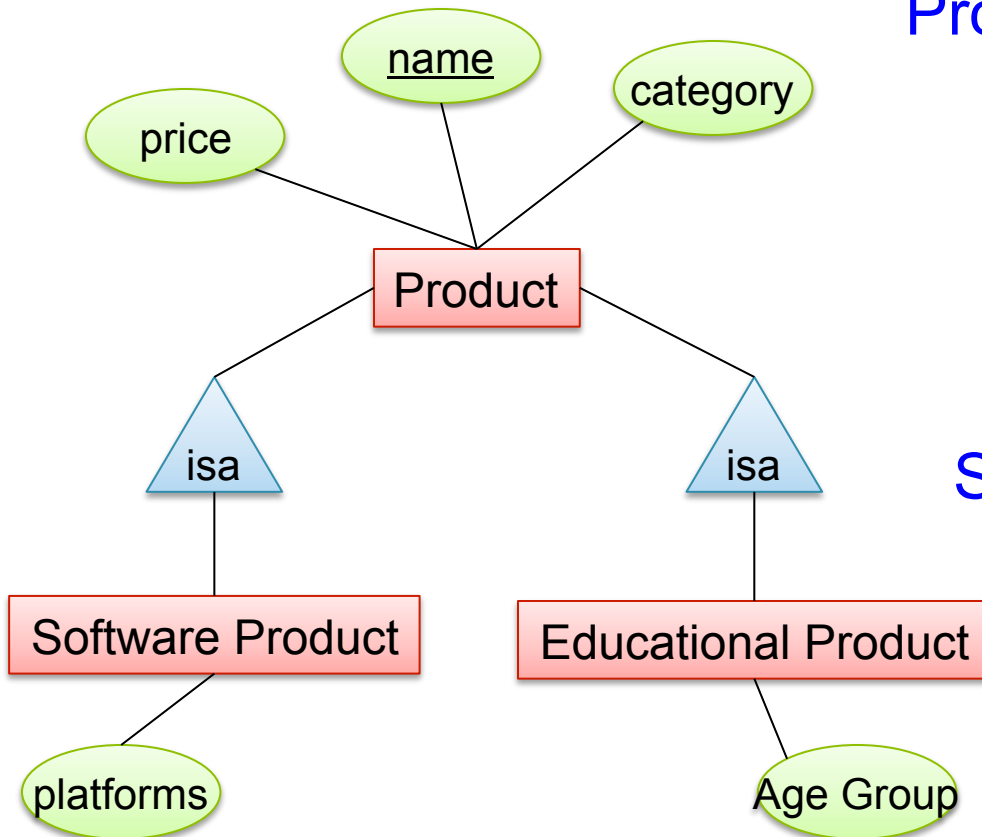
Product

<u>Name</u>	Price	Category
Gizmo	99	gadget
Camera	49	photo
Toy	39	gadget

Sw.Product

<u>Name</u>	platforms
Gizmo	unix

# Subclasses to Relations



Product

<u>Name</u>	Price	Category
Gizmo	99	gadget
Camera	49	photo
Toy	39	gadget

Sw.Product

<u>Name</u>	platforms
Gizmo	unix

Ed.Product

<u>Name</u>	Age Group
Gizmo	toddler
Toy	senior

# General approach to Translating Diagram into Relations

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- Each entity set becomes a relation with a key
- Each relationship set becomes a relation with foreign keys except many-one relationships: just add a fk
- Each isA relationship becomes another relation, with both a key and foreign key



# Outline

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- Conceptual db design: entity-relationship model
- Problematic database designs
- Functional dependencies
- Normal forms and schema normalization

# Relational Schema Design

---

<u>Name</u>	<u>SSN</u>	<u>PhoneNumber</u>	<u>City</u>
Fred	123-45-6789	206-555-1234	Seattle
Fred	123-45-6789	206-555-6543	Seattle
Joe	987-65-4321	908-555-2121	Westfield

One person may have multiple phones, but lives in only one city

Primary key is thus (SSN, PhoneNumber)

What is the problem with this schema?

# Relational Schema Design

---

Name	<u>SSN</u>	<u>PhoneNumber</u>	City
Fred	123-45-6789	206-555-1234	Seattle
Fred	123-45-6789	206-555-6543	Seattle
Joe	987-65-4321	908-555-2121	Westfield

## Anomalies:

- **Redundancy** = repeat data for Fred
- **Update anomalies** = what if Fred moves to “Bellevue”?
- **Deletion anomalies** = what if Joe deletes his phone number?

# Relation Decomposition

**Break the relation into two:**

Name	SSN	PhoneNumber	City
Fred	123-45-6789	206-555-1234	Seattle
Fred	123-45-6789	206-555-6543	Seattle
Joe	987-65-4321	908-555-2121	Westfield

Name	<u>SSN</u>	City
Fred	123-45-6789	Seattle
Joe	987-65-4321	Westfield

<u>SSN</u>	<u>PhoneNumber</u>
123-45-6789	206-555-1234
123-45-6789	206-555-6543
987-65-4321	908-555-2121

**Anomalies have gone:**

- No more repeated data
- Easy to move Fred to “Bellevue” (how ?)
- Easy to delete all Joe’s phone numbers (how ?)

# Relational Schema Design (or Logical Design)

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How do we do this systematically?

- Start with some relational schema
- Find out its **functional dependencies** (FDs)
- Use FDs to **normalize** the relational schema

# Functional Dependencies (FDs)

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## Definition

If two tuples agree on the attributes

$A_1, A_2, \dots, A_n$

then they must also agree on the attributes

$B_1, B_2, \dots, B_m$

Formally:

$A_1 \dots A_n$  determines  $B_1 \dots B_m$

$A_1, A_2, \dots, A_n \rightarrow B_1, B_2, \dots, B_m$

# Functional Dependencies (FDs)

**Definition**  $A_1, \dots, A_m \rightarrow B_1, \dots, B_n$  holds in R if:

$\forall t, t' \in R,$

$(t.A_1 = t'.A_1 \wedge \dots \wedge t.A_m = t'.A_m \rightarrow t.B_1 = t'.B_1 \wedge \dots \wedge t.B_n = t'.B_n)$

R		$A_1$	...	$A_m$		$B_1$	...	$B_n$		
t										
t'										

if t, t' agree here then t, t' agree here

# Example

---

An FD holds, or does not hold on an instance:

<b>EmpID</b>	<b>Name</b>	<b>Phone</b>	<b>Position</b>
E0045	Smith	1234	Clerk
E3542	Mike	9876	Salesrep
E1111	Smith	9876	Salesrep
E9999	Mary	1234	Lawyer

EmpID → Name, Phone, Position

Position → Phone

but not Phone → Position



# Example

---

<b>EmpID</b>	<b>Name</b>	<b>Phone</b>	<b>Position</b>
E0045	Smith	1234	Clerk
E3542	Mike	9876 ←	Salesrep
E1111	Smith	9876 ←	Salesrep
E9999	Mary	1234	Lawyer

**Position → Phone**

# Example

---

<b>EmpID</b>	<b>Name</b>	<b>Phone</b>	<b>Position</b>
E0045	Smith	1234 →	Clerk
E3542	Mike	9876	Salesrep
E1111	Smith	9876	Salesrep
E9999	Mary	1234 →	Lawyer

But not Phone → Position

# Example

name → color  
category → department  
color, category → price

name	category	color	department	price
Gizmo	Gadget	Green	Toys	49
Tweaker	Gadget	Green	Toys	99

Which FD's hold?

# Buzzwords

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- FD **holds** or **does not hold** on an instance
- If we can be sure that *every instance of R* will be one in which a given FD is true, then we say that **R satisfies the FD**
- If we say that R satisfies an FD, we are **stating a constraint on R**

# An Interesting Observation

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If all these FDs are true:

$\text{name} \rightarrow \text{color}$   
 $\text{category} \rightarrow \text{department}$   
 $\text{color, category} \rightarrow \text{price}$

Then this FD also holds:

$\text{name, category} \rightarrow \text{price}$

Find out from application domain some FDs,  
Compute all FD's implied by them

# Closure of a set of Attributes

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**Given** a set of attributes  $A_1, \dots, A_n$

The **closure** is the set of attributes  $B$ , denoted  $\{A_1, \dots, A_n\}^+$ ,  
s.t.  $A_1, \dots, A_n \rightarrow B$

# Closure of a set of Attributes

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s.t.  $A_1, \dots, A_n \rightarrow B$

Example:

1. name  $\rightarrow$  color
2. category  $\rightarrow$  department
3. color, category  $\rightarrow$  price

# Closure of a set of Attributes

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**Given** a set of attributes  $A_1, \dots, A_n$

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s.t.  $A_1, \dots, A_n \rightarrow B$

Example:

1. name  $\rightarrow$  color
2. category  $\rightarrow$  department
3. color, category  $\rightarrow$  price

Closures:

name<sup>+</sup> = {name, color}



# Closure of a set of Attributes

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**Given** a set of attributes  $A_1, \dots, A_n$

The **closure** is the set of attributes  $B$ , denoted  $\{A_1, \dots, A_n\}^+$ ,  
s.t.  $A_1, \dots, A_n \rightarrow B$

Example:

1. name  $\rightarrow$  color
2. category  $\rightarrow$  department
3. color, category  $\rightarrow$  price

Closures:

$\text{name}^+ = \{\text{name}, \text{color}\}$

$\{\text{name}, \text{category}\}^+ = \{\text{name}, \text{category}, \text{color}, \text{department}, \text{price}\}$

# Closure of a set of Attributes

---

**Given** a set of attributes  $A_1, \dots, A_n$

The **closure** is the set of attributes  $B$ , denoted  $\{A_1, \dots, A_n\}^+$ ,  
s.t.  $A_1, \dots, A_n \rightarrow B$

Example:

1. name  $\rightarrow$  color
2. category  $\rightarrow$  department
3. color, category  $\rightarrow$  price

Closures:

$\text{name}^+ = \{\text{name}, \text{color}\}$

$\{\text{name}, \text{category}\}^+ = \{\text{name}, \text{category}, \text{color}, \text{department}, \text{price}\}$

$\text{color}^+ = \{\text{color}\}$

# Keys

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- A **superkey** is a set of attributes  $A_1, \dots, A_n$  s.t. for any other attribute  $B$ , we have  $A_1, \dots, A_n \rightarrow B$
- A **key** is a minimal superkey (no subset is a superkey)

# Computing (Super)Keys

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- For all sets  $X$ , compute  $X^+$
- If  $X^+ = [\text{all attributes}]$ , then  $X$  is a superkey
- Try reducing to the minimal  $X$ 's to get the key

# Example

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Product(name, price, category, color)

name, category → price  
category → color

What is the key ?

# Example

---

Product(name, price, category, color)

name, category  $\rightarrow$  price  
category  $\rightarrow$  color

What is the key ?

(name, category) + = { name, category, price, color }

# Example

---

Product(name, price, category, color)

name, category  $\rightarrow$  price  
category  $\rightarrow$  color

What is the key ?

$(\text{name, category})^+ = \{ \text{name, category, price, color} \}$

Hence (name, category) is a key

# Key or Keys ?

---

Can we have more than one key ?



# Key or Keys ?

---

Can we have more than one key ?

A	→	B
B	→	C
C	→	A

what are the keys here ?

# Key or Keys ?

---

Can we have more than one key ?

$A \rightarrow B$   
 $B \rightarrow C$   
 $C \rightarrow A$

$AB \rightarrow C$   
 $BC \rightarrow A$

$A \rightarrow BC$   
 $B \rightarrow AC$

what are the keys here ?

# Eliminating Anomalies

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Main idea:

- $X \rightarrow A$  is OK if  $X$  is a (super)key
- $X \rightarrow A$  is not OK otherwise
  - Need to decompose the table, but how?

# Boyce-Codd Normal Form

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There are no  
“bad” FDs:

**Definition.** A relation R is in BCNF if:

Whenever  $X \rightarrow B$  is a non-trivial dependency, then X is a superkey.

Equivalently:

**Definition.** A relation R is in BCNF if:

$\forall X$ , either  $X^+ = X$  or  $X^+ = [\text{all attributes}]$

# BCNF Decomposition Algorithm

Normalize(R)

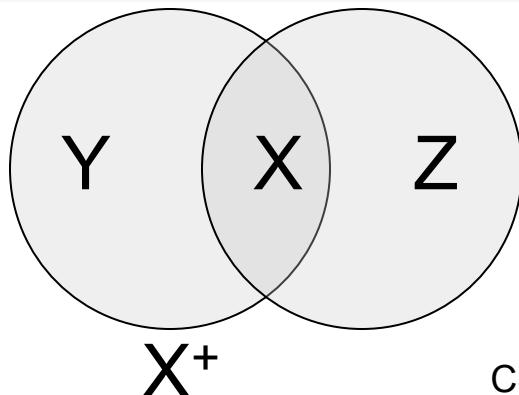
find  $X$  s.t.:  $X \neq X^+$  and  $X^+ \neq [\text{all attributes}]$

**if** (not found) **then** “R is in BCNF”

**let**  $Y = X^+ - X$ ;  $Z = [\text{all attributes}] - X^+$

decompose R into  $R_1(X \cup Y)$  and  $R_2(X \cup Z)$

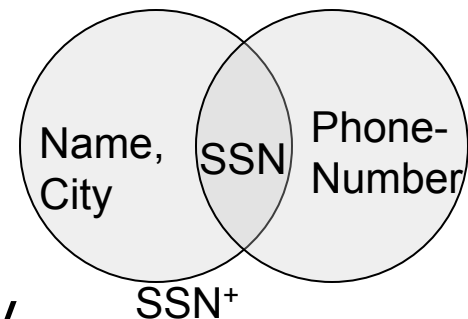
Normalize( $R_1$ ); Normalize( $R_2$ );



# Example

Name	SSN	PhoneNumber	City
Fred	123-45-6789	206-555-1234	Seattle
Fred	123-45-6789	206-555-6543	Seattle
Joe	987-65-4321	908-555-2121	Westfield
Joe	987-65-4321	908-555-1234	Westfield

$SSN \rightarrow Name, City$



The only key is:  $\{SSN, PhoneNumber\}$

Hence  $SSN \rightarrow Name, City$  is a “bad” dependency

In other words:

$SSN^+ = SSN, Name, City$  and is neither  $SSN$  nor  $All\ Attributes$

Find  $X$  s.t.:  $X \neq X^+$  and  $X^+ \neq$  [all attributes]

## Example BCNF Decomposition

Person(name, SSN, age, hairColor, phoneNumber)

SSN  $\rightarrow$  name, age

age  $\rightarrow$  hairColor



Find  $X$  s.t.:  $X \neq X^+$  and  $X^+ \neq$  [all attributes]

## Example BCNF Decomposition

Person(name, SSN, age, hairColor, phoneNumber)

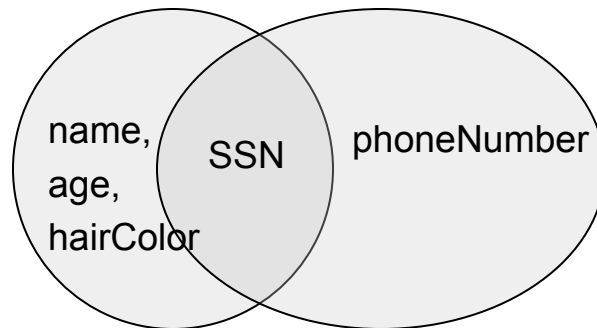
SSN  $\rightarrow$  name, age

age  $\rightarrow$  hairColor

Iteration 1: **Person**: SSN<sup>+</sup> = SSN, name, age, hairColor

Decompose into: **P**(SSN, name, age, hairColor)

**Phone**(SSN, phoneNumber)





Find  $X$  s.t.:  $X \neq X^+$  and  $X^+ \neq$  [all attributes]

## Example BCNF Decomposition

Person(name, SSN, age, hairColor, phoneNumber)

SSN  $\rightarrow$  name, age

age  $\rightarrow$  hairColor

What are  
the keys ?

Iteration 1: **Person**: SSN<sup>+</sup> = SSN, name, age, hairColor

Decompose into: **P**(SSN, name, age, hairColor)

**Phone**(SSN, phoneNumber)

Iteration 2: **P**: age<sup>+</sup> = age, hairColor

Decompose: **People**(SSN, name, age)

**Hair**(age, hairColor)

**Phone**(SSN, phoneNumber)

Find  $X$  s.t.:  $X \neq X^+$  and  $X^+ \neq$  [all attributes]

## Example BCNF Decomposition

Person(name, SSN, age, hairColor, phoneNumber)

SSN  $\rightarrow$  name, age

age  $\rightarrow$  hairColor

Note the keys!

Iteration 1: **Person**: SSN<sup>+</sup> = SSN, name, age, hairColor

Decompose into: **P**(SSN, name, age, hairColor)

**Phone**(SSN, phoneNumber)

Iteration 2: **P**: age<sup>+</sup> = age, hairColor

Decompose: **People**(SSN, name, age)

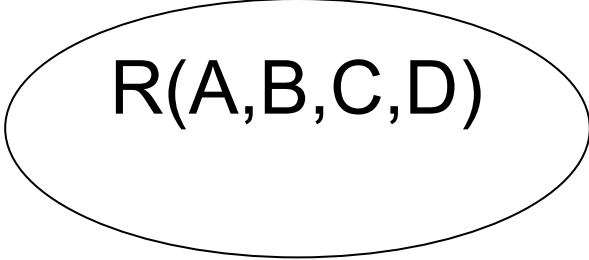
**Hair**(age, hairColor)

**Phone**(SSN, phoneNumber)

R(A,B,C,D)

# Example: BCNF

A	→	B
B	→	C



R(A,B,C,D)

$R(A,B,C,D)$

# Example: BCNF

$A \rightarrow B$   
 $B \rightarrow C$

Recall: find  $X$  s.t.  
 $X \subsetneq X^+ \subsetneq [\text{all-attrs}]$

$R(A,B,C,D)$

R(A,B,C,D)

# Example: BCNF

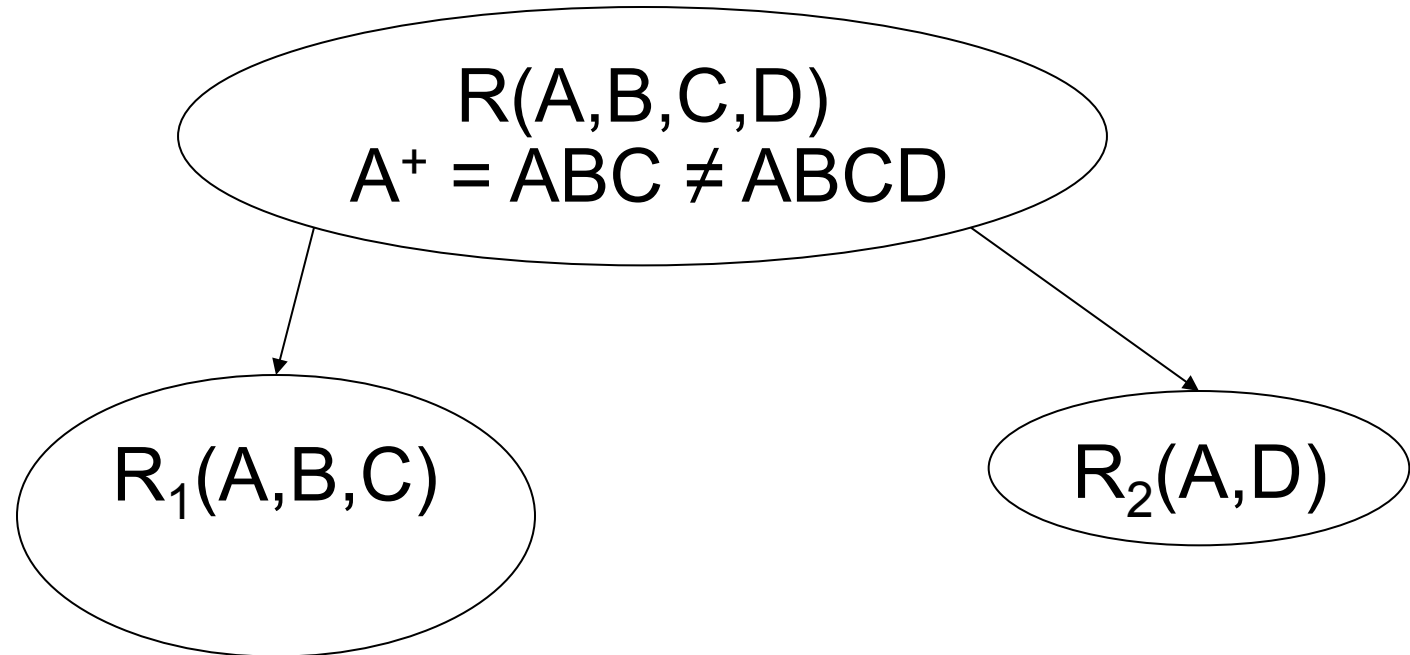
A	→	B
B	→	C

R(A,B,C,D)  
 $A^+ = ABC \neq ABCD$

R(A,B,C,D)

# Example: BCNF

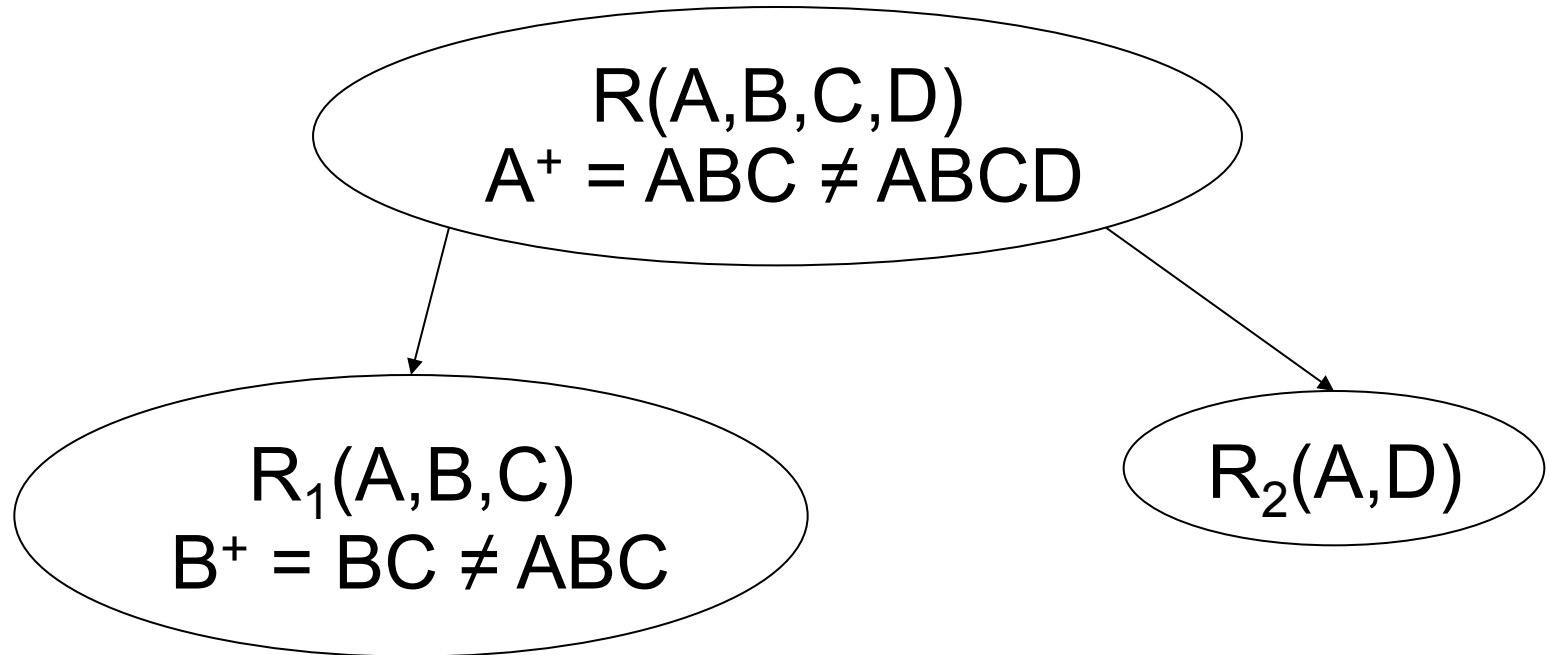
A → B  
B → C



R(A,B,C,D)

# Example: BCNF

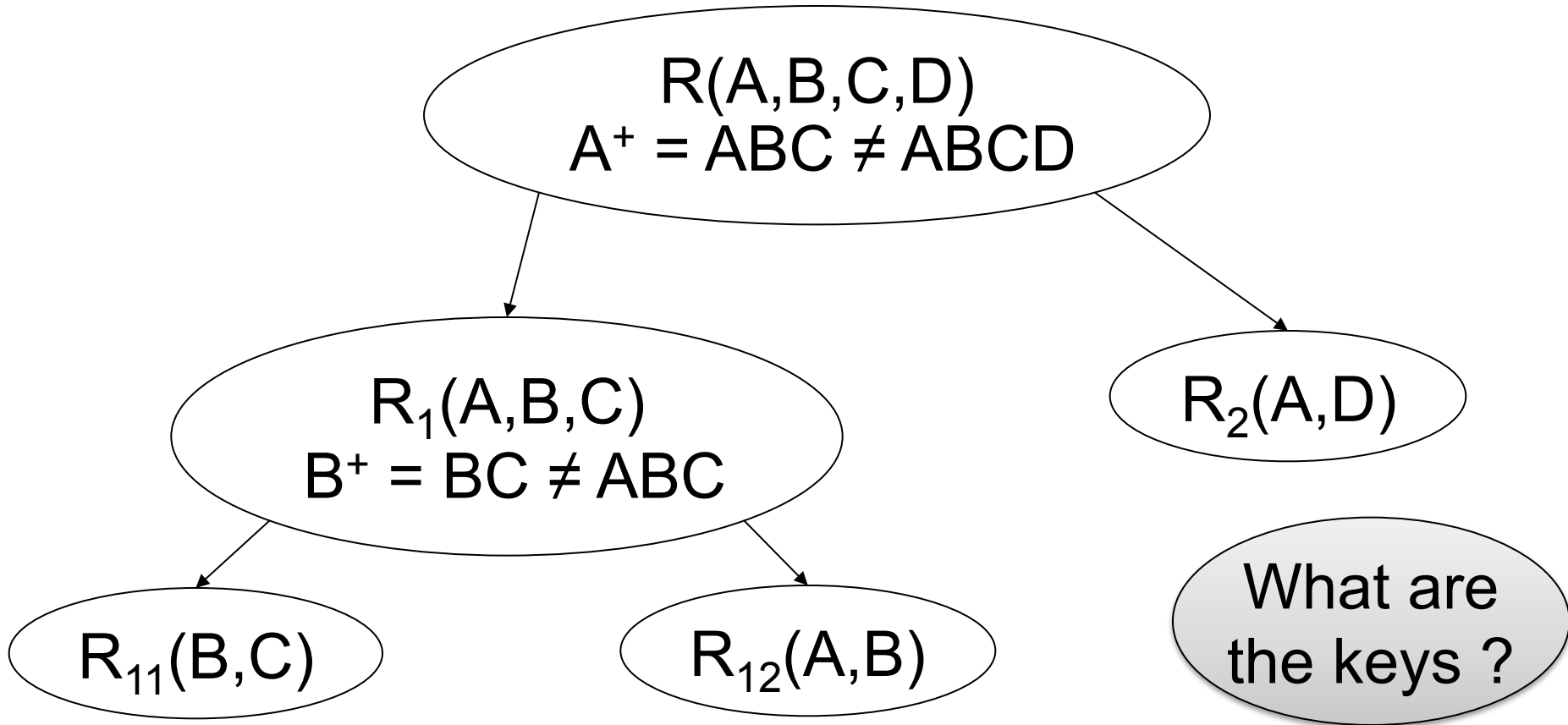
A → B  
B → C



R(A,B,C,D)

# Example: BCNF

A → B  
B → C

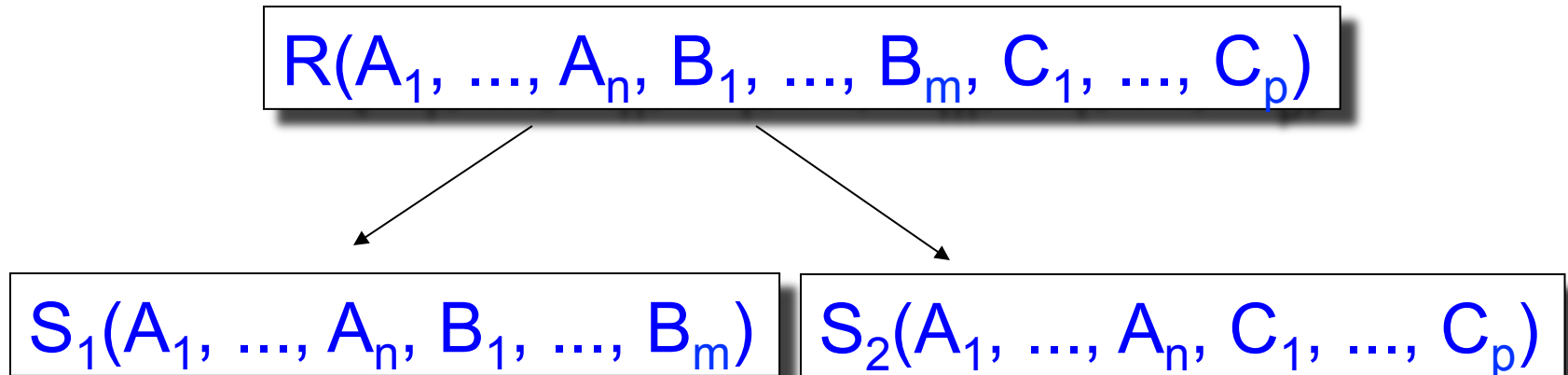


What happens if in R we first pick  $B^+$  ? Or  $AB^+$  ? 56



# Decompositions in General

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$S_1$  = projection of  $R$  on  $A_1, \dots, A_n, B_1, \dots, B_m$

$S_2$  = projection of  $R$  on  $A_1, \dots, A_n, C_1, \dots, C_p$


# Lossless Decomposition

---

Name	Price	Category
Gizmo	19.99	Gadget
OneClick	24.99	Camera
Gizmo	19.99	Camera



Name	Price
Gizmo	19.99
OneClick	24.99
<del>Gizmo</del>	<del>19.99</del>



Name	Category
Gizmo	Gadget
OneClick	Camera
Gizmo	Camera

# Lossy Decomposition

---

What is lossy here?

Name	Price	Category
Gizmo	19.99	Gadget
OneClick	24.99	Camera
Gizmo	19.99	Camera

Name	Category
Gizmo	Gadget
OneClick	Camera
Gizmo	Camera

Price	Category
19.99	Gadget
24.99	Camera
19.99	Camera

# Lossy Decomposition

---

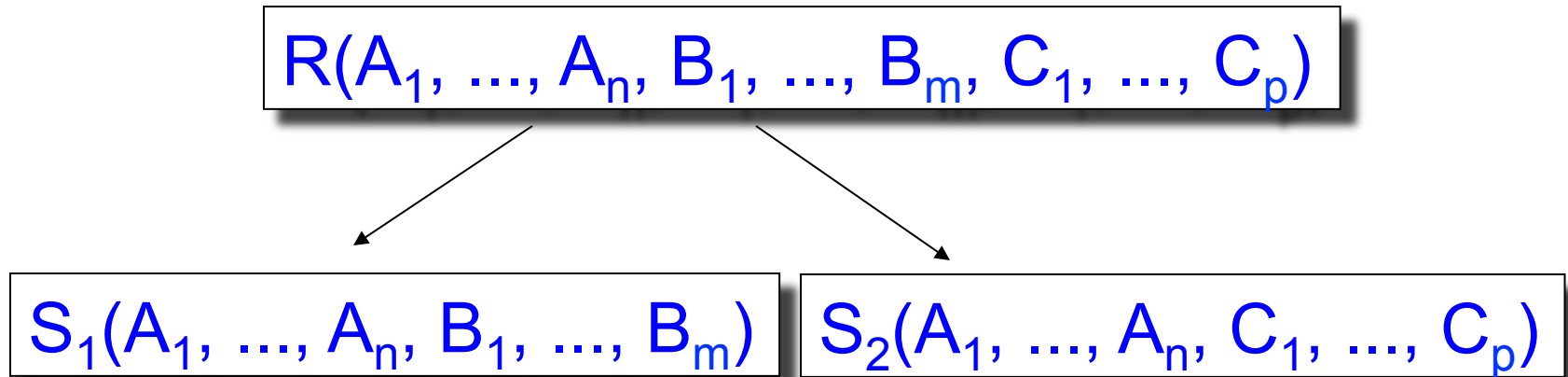
Name	Price	Category
Gizmo	19.99	Gadget
OneClick	24.99	Camera
Gizmo	19.99	Camera

Name	Category
Gizmo	Gadget
OneClick	Camera
Gizmo	Camera

Price	Category
19.99	Gadget
24.99	Camera
19.99	Camera

# Decomposition in General

---



Let:  $S_1$  = projection of  $R$  on  $A_1, \dots, A_n, B_1, \dots, B_m$   
 $S_2$  = projection of  $R$  on  $A_1, \dots, A_n, C_1, \dots, C_p$

The decomposition is called lossless if  $R = S_1 \bowtie S_2$

Fact: If  $A_1, \dots, A_n \rightarrow B_1, \dots, B_m$  then the decomposition is lossless

It follows that every BCNF decomposition is lossless

# Testing for Lossless Join

---

If we decompose  $R$  into  $\Pi_{S_1}(R)$ ,  $\Pi_{S_2}(R)$ ,  $\Pi_{S_3}(R)$ , ...  
Is it true that  $S_1 \bowtie S_2 \bowtie S_3 \bowtie \dots = R$  ?

That is true if we can show that:

$R \subseteq S_1 \bowtie S_2 \bowtie S_3 \bowtie \dots$  always holds (why?)

$R \supseteq S_1 \bowtie S_2 \bowtie S_3 \bowtie \dots$  **need to check**

Example from textbook Ch. 3.4.2

# The Chase Test for Lossless Join

---

$R(A,B,C,D) = S1(A,D) \bowtie S2(A,C) \bowtie S3(B,C,D)$

R satisfies:  $A \rightarrow B$ ,  $B \rightarrow C$ ,  $CD \rightarrow A$

$S1 = \Pi_{AD}(R)$ ,  $S2 = \Pi_{AC}(R)$ ,  $S3 = \Pi_{BCD}(R)$ ,

hence  $R \subseteq S1 \bowtie S2 \bowtie S3$

Need to check:  $R \supseteq S1 \bowtie S2 \bowtie S3$

Example from textbook Ch. 3.4.2

# The Chase Test for Lossless Join

---

$R(A,B,C,D) = S1(A,D) \bowtie S2(A,C) \bowtie S3(B,C,D)$   
R satisfies:  $A \rightarrow B$ ,  $B \rightarrow C$ ,  $CD \rightarrow A$

$S1 = \Pi_{AD}(R)$ ,  $S2 = \Pi_{AC}(R)$ ,  $S3 = \Pi_{BCD}(R)$ ,

hence  $R \subseteq S1 \bowtie S2 \bowtie S3$

Need to check:  $R \supseteq S1 \bowtie S2 \bowtie S3$

Suppose  $(a,b,c,d) \in S1 \bowtie S2 \bowtie S3$  Is it also in R?



Example from textbook Ch. 3.4.2

# The Chase Test for Lossless Join

$R(A,B,C,D) = S1(A,D) \bowtie S2(A,C) \bowtie S3(B,C,D)$   
R satisfies:  $A \rightarrow B$ ,  $B \rightarrow C$ ,  $CD \rightarrow A$

$S1 = \Pi_{AD}(R)$ ,  $S2 = \Pi_{AC}(R)$ ,  $S3 = \Pi_{BCD}(R)$ ,  
hence  $R \subseteq S1 \bowtie S2 \bowtie S3$

Need to check:  $R \supseteq S1 \bowtie S2 \bowtie S3$

Suppose  $(a,b,c,d) \in S1 \bowtie S2 \bowtie S3$  Is it also in R?

R must contain the following tuples:

A	B	C	D
a	b1	c1	d

Why ?

$(a,d) \in S1 = \Pi_{AD}(R)$

Example from textbook Ch. 3.4.2

# The Chase Test for Lossless Join

$R(A,B,C,D) = S1(A,D) \bowtie S2(A,C) \bowtie S3(B,C,D)$   
R satisfies:  $A \rightarrow B$ ,  $B \rightarrow C$ ,  $CD \rightarrow A$

$S1 = \Pi_{AD}(R)$ ,  $S2 = \Pi_{AC}(R)$ ,  $S3 = \Pi_{BCD}(R)$ ,  
hence  $R \subseteq S1 \bowtie S2 \bowtie S3$

Need to check:  $R \supseteq S1 \bowtie S2 \bowtie S3$

Suppose  $(a,b,c,d) \in S1 \bowtie S2 \bowtie S3$  Is it also in R?

R must contain the following tuples:

A	B	C	D
a	b1	c1	d
a	b2	c	d2

Why ?

$(a,d) \in S1 = \Pi_{AD}(R)$

$(a,c) \in S2 = \Pi_{AC}(R)$

Example from textbook Ch. 3.4.2

# The Chase Test for Lossless Join

$R(A,B,C,D) = S1(A,D) \bowtie S2(A,C) \bowtie S3(B,C,D)$   
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a	b1	c1	d
a	b2	c	d2
a3	b	c	d

Why ?

$(a,d) \in S1 = \Pi_{AD}(R)$

$(a,c) \in S2 = \Pi_{AC}(R)$

$(b,c,d) \in S3 = \Pi_{BCD}(R)$

## Example from textbook Ch. 3.4.2

# The Chase Test for Lossless Join

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Why ?


$(a,d) \in S1 = \Pi_{AD}(R)$

$(a,c) \in S2 = \Pi_{AC}(R)$

$(b,c,d) \in S3 = \Pi_{BCD}(R)$

“Chase” them (apply FDs):

$A \rightarrow B$



A	B	C	D
a	b1	c1	d
a	b1	c	d2
a3	b	c	d

## Example from textbook Ch. 3.4.2

# The Chase Test for Lossless Join

$R(A,B,C,D) = S1(A,D) \bowtie S2(A,C) \bowtie S3(B,C,D)$   
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R must contain the following tuples:

A	B	C	D
a	b1	c1	d
a	b2	c	d2
a3	b	c	d

Why ?

$(a,d) \in S1 = \Pi_{AD}(R)$

$(a,c) \in S2 = \Pi_{AC}(R)$

$(b,c,d) \in S3 = \Pi_{BCD}(R)$

“Chase” them (apply FDs):

$A \rightarrow B$

A	B	C	D
a	b1	c1	d
a	b1	c	d2
a3	b	c	d

$B \rightarrow C$

A	B	C	D
a	b1	c	d
a	b1	c	d2
a3	b	c	d

Example from textbook Ch. 3.4.2

# The Chase Test for Lossless Join

$R(A,B,C,D) = S1(A,D) \bowtie S2(A,C) \bowtie S3(B,C,D)$   
R satisfies:  $A \rightarrow B$ ,  $B \rightarrow C$ ,  $CD \rightarrow A$

$S1 = \Pi_{AD}(R)$ ,  $S2 = \Pi_{AC}(R)$ ,  $S3 = \Pi_{BCD}(R)$ ,  
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a	b1	c1	d
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Why ?

$(a,d) \in S1 = \Pi_{AD}(R)$

$(a,c) \in S2 = \Pi_{AC}(R)$

$(b,c,d) \in S3 = \Pi_{BCD}(R)$

“Chase” them (apply FDs):

$A \rightarrow B$

A	B	C	D
a	b1	c1	d
a	b1	c	d2
a3	b	c	d

$B \rightarrow C$

A	B	C	D
a	b1	c	d
a	b1	c	d2
a3	b	c	d

$CD \rightarrow A$

A	B	C	D
a	b1	c	d
a	b1	c	d2
a	b	c	d

Hence R  
contains  $(a,b,c,d)$

# Schema Refinements = Normal Forms

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- 1st Normal Form = all tables are flat
- 2nd Normal Form = obsolete
- Boyce Codd Normal Form = no bad FDs
- 3rd Normal Form = see book
  - BCNF is lossless but can cause loss of ability to check some FDs
  - 3NF fixes that (is lossless and dependency-preserving), but some tables might not be in BCNF – i.e., they may have redundancy anomalies