Computer Graphics — Zoran Popovi}
CSE 557 — Winter 2001

Homework 1

Received: Fri, Feb. 2
Due: Wed, Feb. 16

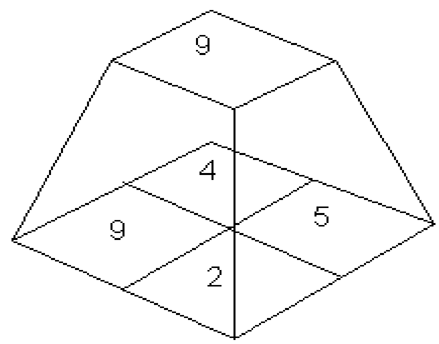
DIRECTIONS

Please provide short written answers to the questions in the space provided. If you require extra space, you may staple additional pages to the back of your assignment. Please answer the questions on your own.

NAME: ________________________________

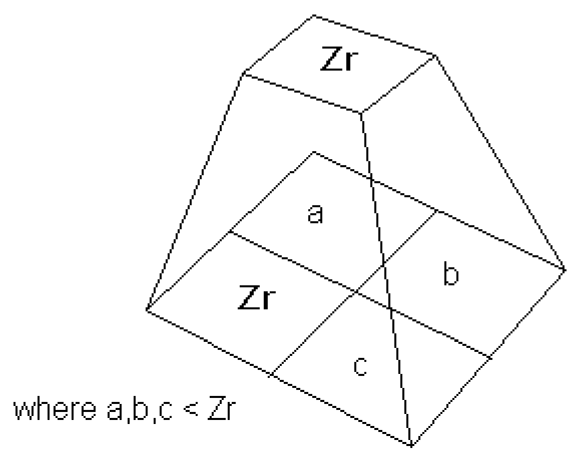
Problem 1. (20 Points)

The Z-buffer algorithm can be improved by using an image space “Z-pyramid.” The basic idea of the Z-pyramid is to use the original Z-buffer as the finest level in the pyramid, and then combine four Z-values at each level into one Z-value at the next coarser level by choosing the farthest (largest) Z from the observer. Every entry in the pyramid therefore represents the farthest (largest) Z for a square area of the Z-buffer. A Z-pyramid for a single 2x2 image is shown below:



a) At the coarsest level of the pyramid there is just a single Z value. What does that Z value represent?

Suppose we wish to test the visibility of a polygon **P**. Let **Zp** be the nearest (smallest) Z value of polygon **P**. Let **R** be the smallest region in the Z-pyramid that completely covers polygon **P**, and let **Zr** be the Z value that is associated with region **R** in the Z-pyramid.



b) What can we conclude if **Zr** < **Zp**?

c) What can we conclude if **Zp** < **Zr**?

If the visibility test is inconclusive, then the algorithm applies the same test recursively: it goes to the next finer level of the pyramid, where the region **R** is divided into four quadrants, and attempts to prove that polygon **P** is hidden in each of the quadrants **R** of that **P** intersects. Since it is expensive to compute the closest Z value of **P** within each quadrant, the algorithm just uses the same **Zp** (the nearest Z of the *entire* polygon) in making the comparison in every quadrant. If at the bottom of the pyramid the test is still inconclusive, the algorithm resorts to ordinary Z-buffered scan conversion to resolve visibility.

d) Suppose that, instead of using the above algorithm, we decided to go to the expense of computing the closest Z value of **P** within each quadrant. Would it then be possible to always make a definitive conclusion about the visibility **P** of within each pixel, without resorting to scan conversion? Why or why not?

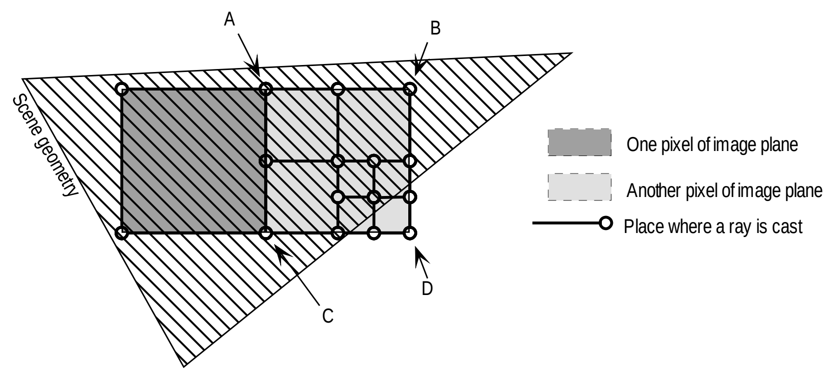
Problem 2. (10 Points)

The company you work for has just bought rights to a raytracing engine. Unfortunately, you don't have the source code, just a compiled library. You have been asked to determine how rays are terminated. So, you call the authors you find out even they don't remember for sure. All they can tell you is this: *The termination criteria for tracing rays is either (a) rays are traced to a maximum recursion depth of 5, or (b) rays are adaptively terminated based on their contribution to a pixel color.*

a) Describe a scene that can be used to determine which method is used. Be specific about all relevant aspects of the scene and what you would look for in the resulting image to determine which termination method is used.

One of the features included in the raytracing engine your company bought is a brand new algorithm for <u>antialiasing by adaptive supersampling</u>.

The normal implementation is to sample rays at the corner of every pixel, compare the colors of each sample, and if the difference between neighboring sample colors is too great, subdivide that region recursively and sample more times. (See the diagram below, or Foley, et al., 15.10.4)



However, in this new algorithm, we subdivide and supersample if neighboring rays *intersect different objects*. In other words, note the light-grey pixel above. Three of the four corner samples (a, b, and c) intersect the scene geometry. The fourth corner (d), misses the geometry completely. So we choose to supersample this pixel without ever comparing colors.

Problem 2 - continued.

b) In what ways is this better than the traditional way? In what ways is it worse?

Problem 3. (10 Points)

The Phong shading model can be summarized by the following equation:

$$I_{phong} = k_e + k_a I_a + \sum_i \left[I_{l_i} \left[k_d (\mathbf{N} \cdot \mathbf{L}_i)_+ + k_s (\mathbf{V} \cdot \mathbf{R}_i)_+^{n_s} \right] \min\left\{ 1, \frac{1}{a_0 + a_1 d_i + a_2 d_i^2} \right\} \right]$$

where the summation i is taken over all light sources.

e) Describe the relationships between **N**, $\mathbf{L}_i$, and $\mathbf{R}_i$ that would result in a point shaded with the Phong model appearing maximally bright.

Blinn and Newell have suggested that, when **V** and **L** are assumed to be constants, the computation of $\mathbf{V} \cdot \mathbf{R}$ can be simplified by associating with each light source a fictitious light source that will generate specular reflections. This second light source is located in a direction **H** halfway between **L** and **V**. The specular component is then computed from $(\mathbf{N} \cdot \mathbf{H})_+^{n_s}$ instead of from $(\mathbf{V} \cdot \mathbf{R})_+^{n_s}$.

b) Under what circumstances might **L** and **V** be assumed to be constant?

c) How does the new equation using **H** simplify shading equations?

Problem 4. (20 Points)

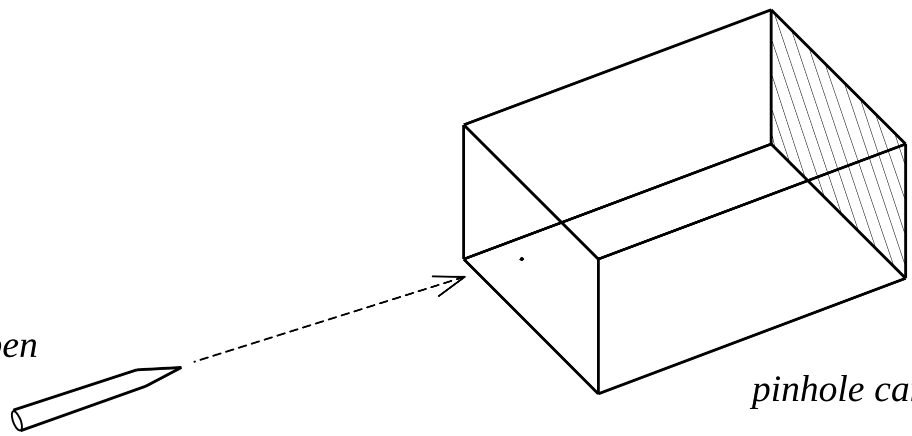
An oblique projection for which the foreshortening factor for edges perpendicular to the plane of projection is one-half is called a cabinet projection. Show that for a cabinet projection the angle between the projection lines and the plane of projection is 63.43º.

Problem 5. (20 Points)

In 2D, a rotation transformation by angle θ can be specified as a series of shear transformation matrices. Give these matrices, or if it can't be done, prove it.

Problem 6 – 10 Pts.

Your pinhole camera is not collecting enough light, so you decide to create a "penhole" camera by poking the barrier with your ballpoint pen. This creates a round aperture with a 1cm diameter. You proceed in viewing a scene of various objects through your camera.



(a) You notice after the "penhole" modification that the objects on the file plane became blurry. Which became blurrier: far away objects or close objects?

(b) Assume that a given object, when struck by light, reflects it equally in all directions. Will the film receive more total energy from a point on the object if the object is close or if it is far away?

(c) Suppose that the film plane is 12cm from the aperture, and you have an object 1m away from the aperture (on the other side, of course). How big a spot will a point on the object project to on the film?

Problem 7 – 10 Pts.

Convolution filtering can modify images in a variety of ways. Describe the expected effect of filtering an image using the following convolution kernel. Justify your answer.

$$\begin{matrix} \mathbf{1} & \mathbf{3} & \mathbf{1} \\ \mathbf{3} & \mathbf{-15} & \mathbf{3} \\ \mathbf{1} & \mathbf{3} & \mathbf{1} \end{matrix}$$