Computer Graphics	**Prof. Brian Curless**
CSE 557	**Winter 2002**

Homework #1

Sampling theory, image processing, affine transformations

Assigned: Tuesday, January 22, 2002
Due: Tuesday, February 5, 2002

Directions: Please provide short written answers to the following questions. For problem #1, use and attach additional sheets of paper. You can use the space provided for the other problems. Feel free to talk over the problems in general terms with classmates, but please *answer the questions on your own*.

Name:____

1. Fourier transforms and signal reconstruction
57 Points

In this problem, you will take a closer look at Fourier transforms and signal reconstruction. We begin with a couple of preliminaries. Recall that the box function II(x) is defined as:

$$\mathrm{II}(x)=\begin{cases}1 & |x|<1/2\\ 1/2 & |x|=1/2\\ 0 & |x|>1/2\end{cases}$$

and its Fourier transform is sinc(s) = sin(πs)/πs. Likewise, the Fourier transform of sinc(x) is II(s). sinc(0)=1.

The hat function, $\Lambda(x)$, is defined as:

$$\Lambda(x)=\begin{cases}1-|x| & |x|<1\\ 0 & |x|>1\end{cases}$$

We sample a function by multiplying with the comb or shah function:

$$\hat{f}(x)=f(x)\mathrm{III}(x)=\sum_{i=-\infty}^{i=\infty} f(i)\delta(x-i)$$

We reconstruct by convolving with a reconstruction filter $r(x)$:

$$\tilde{f}(x)=r(x)*\hat{f}(x)=\sum_{i=-\infty}^{i=\infty} f(i)r(x-i)$$

a) The "ideal" reconstruction filter is r(x) = sinc(x). Is this filter an interpolating filter, i.e., will the reconstructed function $\tilde{f}(x)$ pass through the original sample points $f(i)$? In other words, do we expect to find that $\tilde{f}(i)=f(i)$? Justify your answer.

b) Let's say we reconstruct with the sinc(x) function, and we want to evaluate $\tilde{f}(1/2)$. Assume that the samples $f(i)$ are positive 8-bit numbers (0-255), that we perform the reconstruction using floating point math (including negative numbers), and that we want an accurate reconstruction such that the smallest contribution from *any* original sample to the reconstruction summation can be at most ½ bit, i.e, +/-0.5 on a scale of 0-255. Given no prior knowledge about $f(x)$, how many samples $f(i)$ must, in the worst case, be included to compute $\tilde{f}(1/2)$?

c) It is easy to show that $\Lambda(x)$ = II(x)*II(x). (Try integrating it yourself, or use the convolution applet.) What is the Fourier transform of $\Lambda(x)$? Sketch the function.

d) If we use $\Lambda(x)$ as a reconstruction filter, what is the equation for computing $\tilde{f}(a)$ where a is a floating point number, $a = i + \Delta$, and i = floor(a)?

e) Is $\Lambda(x)$ an interpolating filter? How many samples are evaluated to compute $\tilde{f}(1/2)$ to the same precision as in (b)?

f) One of the most common resampling methods used in graphics is *bilinear interpolation*, defined mathematically as using a reconstruction filter $r(x, y) = \Lambda(x)\Lambda(y)$. Recall that for separable filters such as this one, we can compute the reconstruction by first convolving in the x direction then in the y direction. What is the equation for $f(a,b)$ where $a = i + \Delta x$, $b = j + \Delta y$, i = floor(a), and j = floor(b)?

g) Sketch the shape of $\Lambda(x)\Lambda(y)$. Note: there is a partial sketch of this in the notes, but it really only shows the "footprint" and the x- and y-axis cross-sections. Your sketch should give a better idea of the rest of its shape.

h) (Extra credit) What is the Fourier transform of the filter $r(x, y) = \Lambda(x)\Lambda(y)$? You will need to manipulate the equation for the 2D Fourier transform to justify your answer. You do not need to sketch the function.

2. Image processing
10 points

Describe the effect of each of the following filters. In addition, indicate which filter will cause the most blurring and which, when convolved with a solid intensity image, will produce the brightest image. Justify your answers.

0.1	0.1	0.1
0.1	0.1	0.1
0.1	0.1	0.1

0	-1	0
-1	5	-1
0	-1	0

0	0.2	0
0.2	0.4	0.2
0	0.2	0

0	0	0
-1	3	-1
0	0	0

3. Affine Transformations
33 points

$$A=\begin{bmatrix}1 & 0 & 0 & 6\\ 0 & 1 & 0 & 3\\ 0 & 0 & 1 & 2\\ 0 & 0 & 0 & 1\end{bmatrix} \quad B=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \quad C=\begin{bmatrix}0 & -1 & 0 & 0\\ 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$D=\begin{bmatrix}2 & 0 & 0 & 0\\ 0 & 2 & 0 & 0\\ 0 & 0 & 2 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \quad E=\begin{bmatrix}2 & 0 & 0 & 0\\ 0 & 3 & 0 & 0\\ 0 & 0 & 2 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \quad F=\begin{bmatrix}1 & 0 & 0 & 0\\ 1 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$G=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \quad H=\begin{bmatrix}1 & 0 & -1 & 0\\ 0 & 1 & 0 & 0\\ 1 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \quad I=\begin{bmatrix}1 & 0 & 0 & 1\\ 0 & .6 & .8 & 0\\ 0 & -.8 & .6 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

a) As discussed in class, any three-dimensional affine transformation can be represented with a 4x4 matrix. Match the matrices above to the following transformations (not all blanks will be filled):

____ Differential (Non-Uniform) Scaling

____ Reflection

____ Rotation about the z-axis with non-uniform scaling

____ Rotation about the y-axis with non-uniform scaling

____ Translation

____ Rotation about the x-axis

____ Rotation about the y-axis

____ Rotation about the z-axis

____ Shearing with respect to the x-y plane

____ Shearing with respect to the y-z plane

____ Rotation about the x-axis and translation

____ Uniform scaling

____ Reflection with uniform scaling

Problem 3 (cont'd.)

b) Consider a line that passes through a point $\mathbf{p} = (p_x, p_y, p_z)$ in the direction $\mathbf{v} = (\cos\alpha, 0, \sin\alpha)$. Write out the product of matrices that would perform a rotation by θ about this line. You should **not** multiply these matrices out, but you do need to write out all of the elements in these matrices.