18. Color

Reading

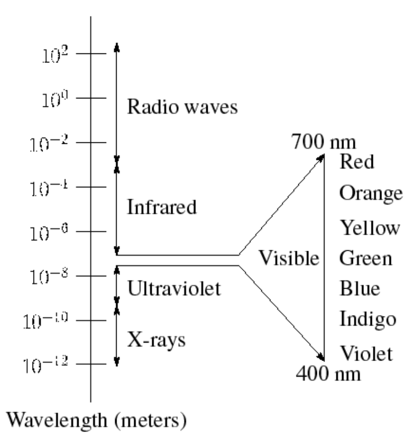
Watt , Chapter 15.

Brian Wandell. *Foundations of Vision. Chapter 4.* Sinauer Associates, Sunderland, MA, pp. 69-97, 1995.

The radiant energy spectrum

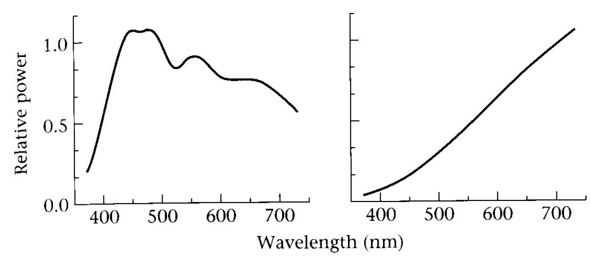
We can think of light as waves, instead of rays.

Wave theory allows a nice arrangement of electromagnetic radiation (EMR) according to wavelength:



Emission spectra

A light source can be characterized by an emission spectrum:



Emission spectra for daylight and a tungsten lightbulb (Wandell, 4.4)

The spectrum describes the energy at each wavelength.

What is color?

The eyes and brain turn an incoming emission spectrum into a discrete set of values.

The signal sent to our brain is somehow interpreted as *color*.

Color science asks some basic questions:

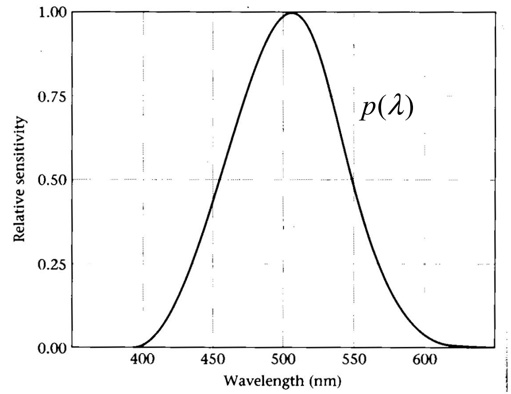
- When are two colors alike?
- How many pigments or primaries does it take to match another color?

One more question: why should we care?

Photopigments

Photopigments are the chemicals in the rods and cones that react to light. Can respond to a single photon!

Rods contain **rhodopsin**, which has peak sensitivity at about 500nm.

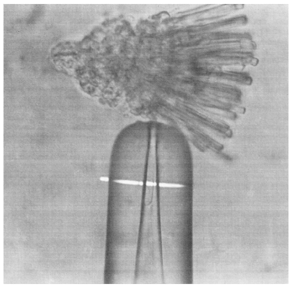


Rod sensitivity (Wandell ,4.6)

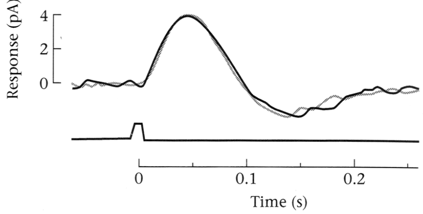
Rods are active under low light levels, i.e., they are responsible for **scotopic** vision.

Univariance

Principle of univariance: For any single photoreceptor, no information is transmitted describing the wavelength of the photon.



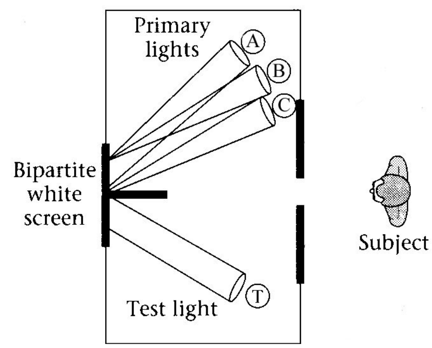
Measuring photoreceptor photocurrent (Wandell, 4.15)



Photocurrents measured for two light stimuli: 550nm (solid) and 659 nm (gray). The brightnesses of the stimuli are different, but the shape of the response is the same. (Wandell 4.17)

The color matching experiment

We can construct an experiment to see how to match a given test light using a set of lights called **primaries** with power control knobs.



The color matching experiment (Wandell, 4.10)

The primary spectra are $a(\lambda)$, $b(\lambda)$, $c(\lambda)$, …

The power knob settings are A, B, C, …

Rods and "color matching"

A rod responds to a spectrum through its spectral sensitivity function, *p(λ)*. The response to a test light, *t(λ)*, is simply:

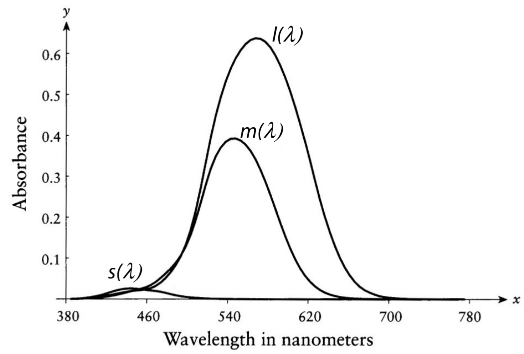
$$P_t = \int t(\lambda)p(\lambda)d\lambda$$

How many primaries are needed to match the test light?

What does this tell us about rod color discrimination?

Cone photopigments

Cones come in three varieties: L, M, and S.



Cone photopigment absorption (Glassner, 1.1)

Cones are active under high light levels, i.e., they are responsible for **photopic** vision.

Cones and color matching

Color is perceived through the responses of the cones to light.

The response of each cone can be written simply as:

$$L_t = \int t(\lambda)l(\lambda)d\lambda$$

$$M_t = \int t(\lambda)m(\lambda)d\lambda$$

$$S_t = \int t(\lambda)s(\lambda)d\lambda$$

These are the only three numbers used to determine color.

Any pair of stimuli that result in the same three numbers will be indistinguishable.

How many primaries do you think we'll need to match *t*?

Color matching

Let's assume that we need 3 primaries to perform the color matching experiment.

Consider three primaries, *a(λ), b (λ), c (λ)*, with three emissive power knobs, *A, B, C*.

The three knobs create spectra of the form:

$$e(\lambda) = Aa(\lambda) + Bb(\lambda) + Cc(\lambda)$$

What is the response of the l-cone?

$$\begin{aligned} L_{abc} &= \int e(\lambda)l(\lambda)d\lambda \\ &= \int \left[Aa(\lambda) + Bb(\lambda) + Cc(\lambda) \right] l(\lambda)d\lambda \\ &= \int Aa(\lambda)l(\lambda)d\lambda + \int Bb(\lambda)l(\lambda)d\lambda + \int Cc(\lambda)l(\lambda)d\lambda \\ &= A\int a(\lambda)l(\lambda)d\lambda + B\int b(\lambda)l(\lambda)d\lambda + C\int c(\lambda)l(\lambda)d\lambda \\ &= AL_a + BL_b + CL_c \end{aligned}$$

How about the m- and s-cones?

Color matching, cont'd

We end up with similar relations for all the cones:

$$L_{abc} = AL_a + BL_b + CL_c$$
$$M_{abc} = AM_a + BM_b + CM_c$$
$$S_{abc} = AS_a + BS_b + CS_c$$

We can re-write this as a matrix:

$$\begin{bmatrix} L_{abc} \\ M_{abc} \\ S_{abc} \end{bmatrix} = \begin{bmatrix} L_a & L_b & L_c \\ M_a & M_b & M_c \\ S_a & S_b & S_c \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

and then solve for the knob settings:

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} L_a & L_b & L_c \\ M_a & M_b & M_c \\ S_a & S_b & S_c \end{bmatrix}^{-1} \begin{bmatrix} L_{abc} \\ M_{abc} \\ S_{abc} \end{bmatrix}$$

In other words, we can choose the knob settings to cause the cones to react as we please!

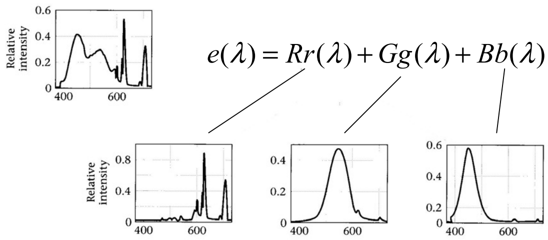
Well, one little "gotcha" – we may need to set the knob values to be negative.

Choosing Primaries

The primaries could be three color (monochromatic) lasers.

But, they can also be non-monochromatic, e.g., monitor phosphors:

$$e(\lambda) = Rr(\lambda) + Gg(\lambda) + Bb(\lambda)$$



Emission spectra for RGB monitor phosphors (Wandell B.3)

Color as linear projection

We can think of spectral functions in sampled form as *n*-dimensional vectors, where *n* is the number of samples.

$$\begin{bmatrix} \vdots \\ t(\lambda) \\ \vdots \end{bmatrix} = \begin{bmatrix} t(\lambda_o) \\ t(\lambda_o + \Delta_\lambda) \\ t(\lambda_o + 2\Delta_\lambda) \\ \vdots \\ t(\lambda_o + (n-1)\Delta_\lambda) \end{bmatrix}$$

In that case, computing the rod response is a projection from n dimensions to 1 dimension:

$$P_t = \begin{bmatrix} \cdots & p(\lambda) & \cdots \end{bmatrix} \begin{bmatrix} \vdots \\ t(\lambda) \\ \vdots \end{bmatrix}$$

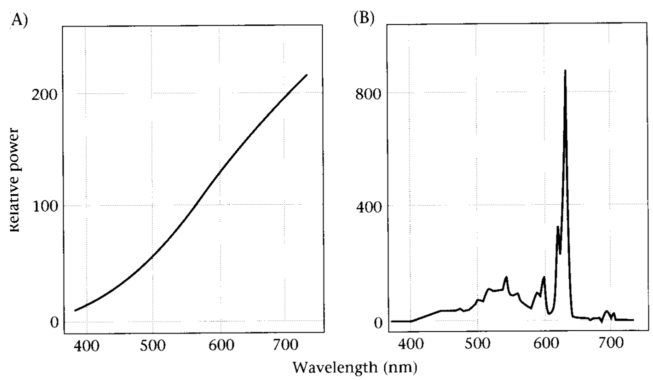
Likewise, computing cone responses is a projection down to 3 dimensions:

$$\begin{bmatrix} L_t \\ M_t \\ S_t \end{bmatrix} = \begin{bmatrix} \cdots & l(\lambda) & \cdots \\ \cdots & m(\lambda) & \cdots \\ \cdots & s(\lambda) & \cdots \end{bmatrix} \begin{bmatrix} \vdots \\ t(\lambda) \\ \vdots \end{bmatrix}$$

Emission Spectrum is not Color

Clearly, information is lost in this projection step…

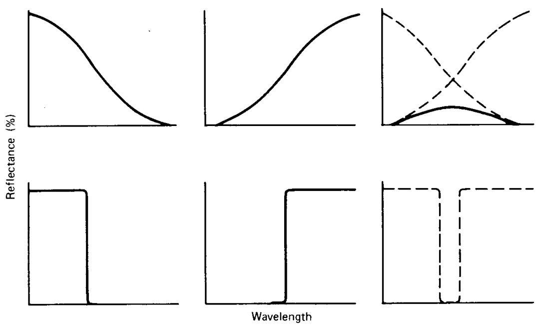
Different light sources can evoke exactly the same colors. Such lights are called **metamers**.



A dim tungsten bulb and an RGB monitor set up to emit a metameric spectrum (Wandell 4.11)

Colored Surfaces

So far, we've discussed the colors of lights. How do *surfaces* acquire color?



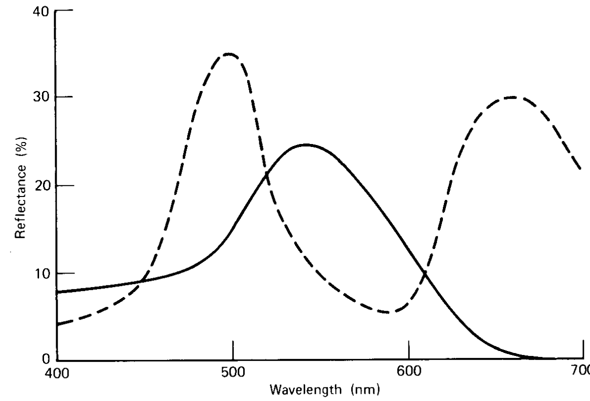
Subtractive colour mixing (Wasserman 2.2)

A surface's **reflectance**, $\rho(\lambda)$, is its tendency to reflect incoming light across the spectrum.

Reflectance is combined "**subtractively"** with incoming light. Actually, the process is *multiplicative*:

$$I(\lambda) = \rho(\lambda)t(\lambda)$$

Subtractive Metamers

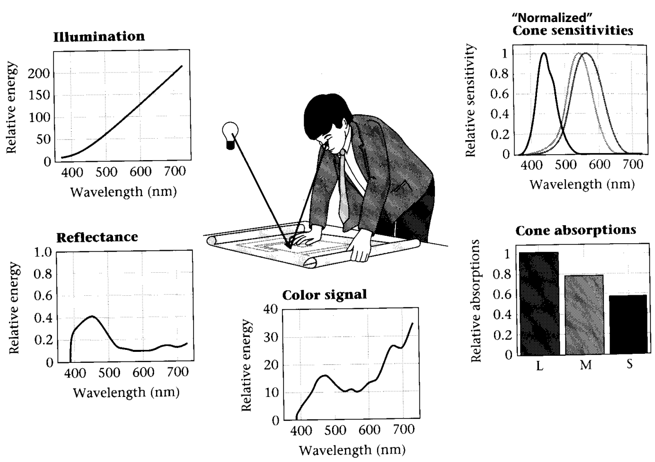


Surfaces that are metamers under only some lighting conditions (Wasserman 3.9)

Reflectance adds a whole new dimension of complexity to color perception.

The solid curve appears green indoors and out. The dashed curve looks green outdoors, but brown under incandescent light.

Illustration of Color Appearance



How light and reflectance become cone responses (Wandell, 9.2)

Lighting design

When deciding the kind of "feel" for an architectural space, the spectra of the light sources is critical.

Lighting design centers have displays with similar scenes under various lighting conditions.

For example:

We have one such center on Capitol Hill: The Northwest Lighting Design Lab.

http://www.northwestlighting.com/

Go visit in person sometime – it's really cool!!

The shape of color space

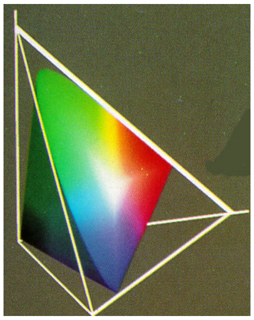
What is the "shape" of color space? How can we visualize it?

To answer this, we begin by thinking of the curves $l(\lambda)$, $m(\lambda)$, and $s(\lambda)$ as a single parametric curve in LMS space.

Convex cones

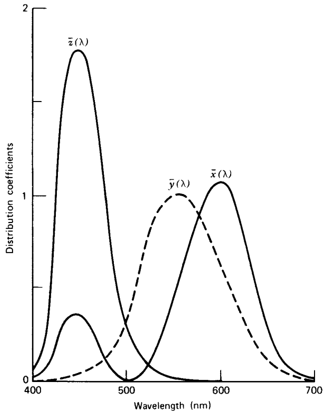
A spectrum can then be considered a positive linear combination of points on the curve.

All such linear combinations must lie within a **convex cone**.



The CIE *XYZ* System

A standard created in 1931 by CIE, defined in terms of three color matching functions.



The XYZ color matching functions (Wasserman 3.8)

These functions are related to the cone responses roughly as:

$$\bar{x}(\lambda) \approx k_1 s(\lambda) + k_2 l(\lambda)$$
$$\bar{y}(\lambda) \approx k_3 m(\lambda)$$
$$\bar{z}(\lambda) \approx k_4 s(\lambda)$$

CIE Coordinates

Given an emission spectrum, we can use the CIE matching functions to obtain the *X, Y* and *Z* coordinates.

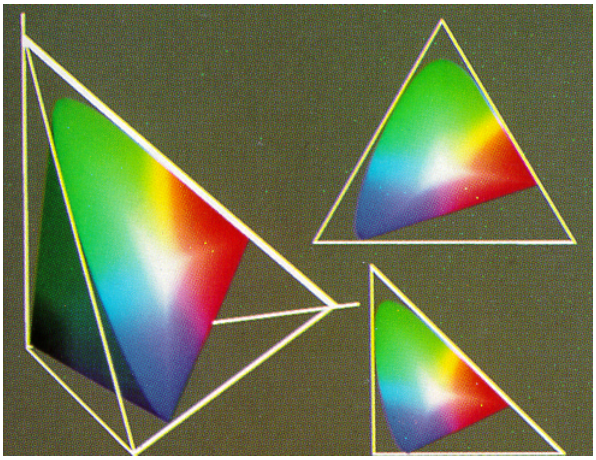
$$X = \int \bar{x}(\lambda) t(\lambda) d\lambda$$

$$Y = \int \bar{y}(\lambda) t(\lambda) d\lambda$$

$$Z = \int \bar{z}(\lambda) t(\lambda) d\lambda$$

Using the equations from the previous page, we can see that XYZ coordinates are closely related to LMS responses.

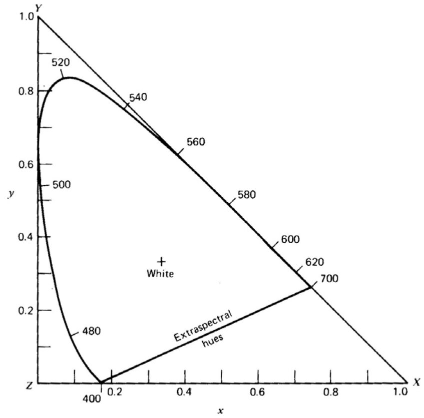
The CIE Colour Blob



Different views of the CIE color space (Foley II.1)

The CIE Chromaticity Diagram

The CIE Chromaticity Diagram is a projection of the plane $X+Y+Z=1$.

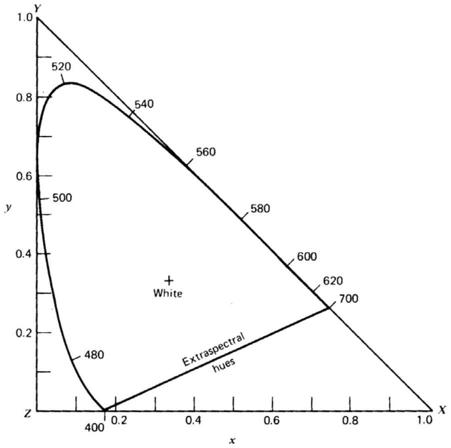


The chromaticity diagram (a kind of slice through CIE space, Wasserman 3.7)

Dominant wavelengths or **hues** go around the perimeter of the chromaticity diagram.

- A color's dominant wavelength is where a line from white through that color intersects the perimeter.
- Some colors, called *non-spectral* color's, don't have a dominant wavelength.

More About Chromaticity

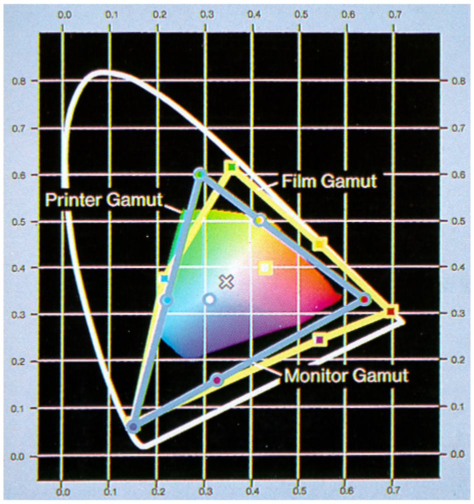


Excitation purity or **saturation** is measured in terms of a color's position on the line to its dominant wavelength.

Complementary colors lie on opposite sides of white, and can be mixed to get white.

Gamuts

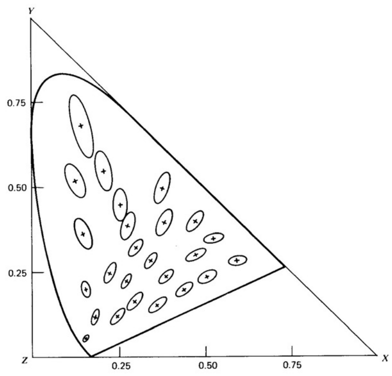
Not every output device can reproduce every color. A device's range of reproducible colors is called its **gamut**.



Gamuts of a few common output devices in CIE space (Foley, II.2)

Perceptually uniform spaces

The XYZ space is not perceptually uniform.



Some perceptually uniform spaces attempt to transform the color space so that Euclidean distance is meaningful:

L*u*v*
L*a*b*
"Farnsworth's non-linear transformation"

Color Spaces for Computer Graphics

In practice, there's a set of more commonly-used color spaces in computer graphics:

- RGB for display
- CMY (or CMYK) for hardcopy
- HSV for user selection

RGB

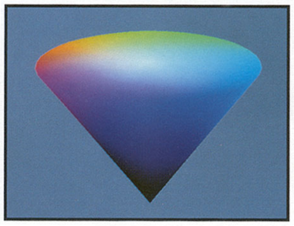
Perhaps the most familiar color space, and the most convenient for display on a CRT.

What does the RGB color space look like?

HSV

More natural for user interaction, corresponds to the artistic concepts of tint, shade and tone.

The HSV space looks like a cone:



CMY

A subtractive color space used for printing.

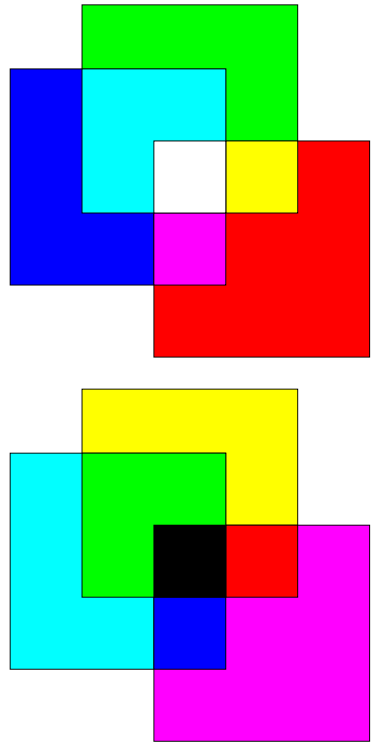
Involves three subtractive primaries:

- Cyan - subtracts red
- Magenta - subtracts green
- Yellow - subtracts blue

Mixing two pigments subtracts their opposites from white.

CMYK adds blacK ink rather than using equal amounts of all three.

RGB vs. CMY



YIQ

Used in TV broadcasting, YIQ exploits useful properties of the visual system.

- Y - luminance (taken from CIE)
- I - major axis of remaining color space
- Q - remaining axis

Key insight: we are most sensitive to changes in luminance.

YIQ is broadcast with relative bandwidth ratios 8:3:1