Computer Graphics	**Prof. Brian Curless**
CSE 557	**Autumn 2009**

Homework #2

Textures, shading, ray tracing, parametric curves, final project

Assigned: Tuesday, November 24

Project proposal due in email: Thursday, December 3

Project meetings with instructor and TA: Friday, December 4

Written homework due: Friday, December 4 at project meeting

Directions: Please provide short written answers to the following questions, using this page as a cover sheet. Feel free to discuss the problems with classmates, but please ***answer the questions on your own.***

Name:____

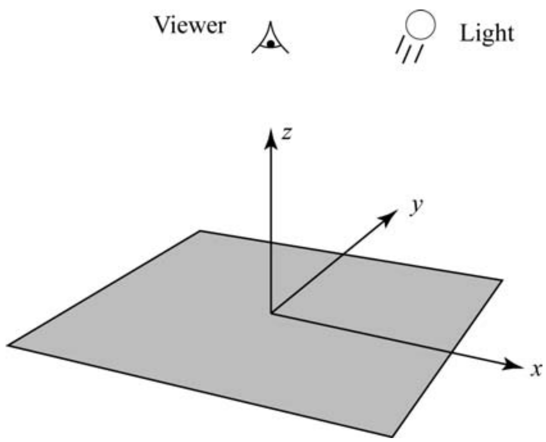
Problem 1. Shading, displacement mapping, and normal mapping

In this problem, an opaque surface will be illuminated by one directional light source and will reflect light according to the following Blinn-Phong shading equation:

$$I = A_{shadow} L\left(k_d\left(\mathbf{N}\cdot\mathbf{L}\right)_+ + k_s\left(\mathbf{N}\cdot\mathbf{H}\right)^{n_s}\right)$$

Note the inclusion of a shadowing term, which takes on a value of 0 or 1. For simplicity, we will assume a monochrome world where I, L, k_d, and k_s are scalar values.

Suppose a viewer is looking down at an infinite plane (the x-y plane) as illustrated below. The scene is illuminated by a directional light source, also pointing straight down on the scene.



Answer the following questions below, giving brief justifications of each answer. Note that lighting and viewing directions are from the point of view of the light and viewer, respectively, and need to be negated when considering the surface-centric shading equation above. We will consider perspective and orthographic viewing in this problem. For perspective viewing, all rays pass through a single (finite) point; for orthographic viewing, all viewing rays are parallel to each other, equivalent to having a center of projection at infinity, looking in a specific direction

In general, you don't need to solve equations and precisely plot functions. It is enough to describe the variables involved, how they relate to each other, and how this relationship will determine, e.g., the appearance of the surface. If you're more comfortable making the answers analytical with equations and plots, however, you are welcome to do so.

a) Assume: Perspective viewer at (0,0,1) looking in the (0,0,-1) direction, angular field of view of 90 degrees, lighting direction of (0,0,-1), $k_d = 1$, $k_s = 0$. Describe the brightness variation over the image seen by the viewer. Justify your answer.

b) Assume: Perspective viewer at (0,0,1) looking in the (0,0,-1) direction, angular field of view of 90 degrees, lighting direction of (0,0,-1), $k_d = 1$, $k_s = 1$, $n_s = 10$. Describe the brightness variation over the image seen by the viewer. Justify your answer.

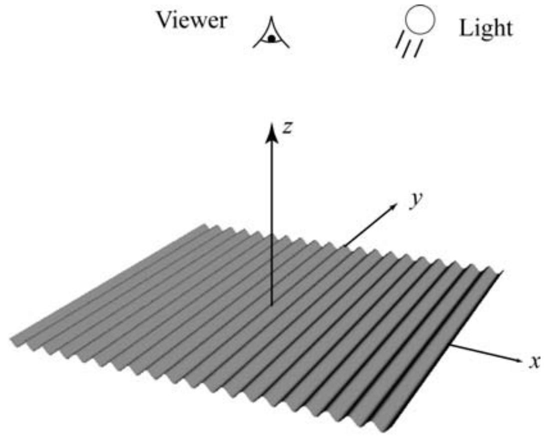
c) Assume: Orthographic viewer looking in the (0,0,-1) direction, lighting direction of (0,0,-1), $k_d = 1$, $k_s = 1$, $n_s = 10$. Describe the brightness variation over the image seen by the viewer. Justify your answer.

d) Assume: Orthographic viewer looking in the (0,0,-1) direction, $k_d = 1$, $k_s = 0$. The lighting direction starts at (-sqrt(2)/2 ,0, -sqrt(2)/2) and then rotates around the z-axis. Describe the brightness variation over time, as seen by the viewer. Justify your answer.

e) Assume: Orthographic viewer looking in the (0,0,-1) direction, $k_d = 1$, $k_s = 1$, $n_s = 10$. The lighting direction starts at (-sqrt(2)/2 ,0, -sqrt(2)/2) and then rotates around the z-axis. Describe the brightness variation over time, as seen by the viewer. Justify your answer.

Problem 1. (cont'd)

Suppose now the infinite plane is replaced with a surface $z = \cos(x)$:



We can think of this as simply adding a displacement $d=\cos(x)$ in the normal direction to the x-y plane.

f) What is the normal to the surface as a function of x? Show your work.

g) Assume: Orthographic viewer looking in the (0,0,-1) direction, lighting direction of (0,0,-1), $k_d = 1$, $k_s = 0$. At what values of x is the surface brightest? At what values is it dimmest? Describe the appearance of the surface. Justify your answers.

h) Assume: Orthographic viewer looking in the (0,0,-1) direction, lighting direction of (0,0,-1), $k_d =0$, $k_s =1$, $n_s =10$. At what values of x is the surface brightest? Describe the appearance of the surface. How does the appearance change as n_s increases to 100? Justify your answers.

Suppose now that we simply keep the normals computed in (f) and map them over the plane from the first part of the problem. The geometry will be flat, but the shading will be based on the varying normals.

i) Assume: Orthographic viewer looking in the (0,0,-1) direction, $k_d =1$, $k_s =0$. If we define the lighting to have direction ($-\sin\theta$, 0, $-\cos\theta$), will the normal mapped rendering look the same as the displacement mapped rendering for each of $\theta = 0$, 10, and 80 degrees? Justify your answer.

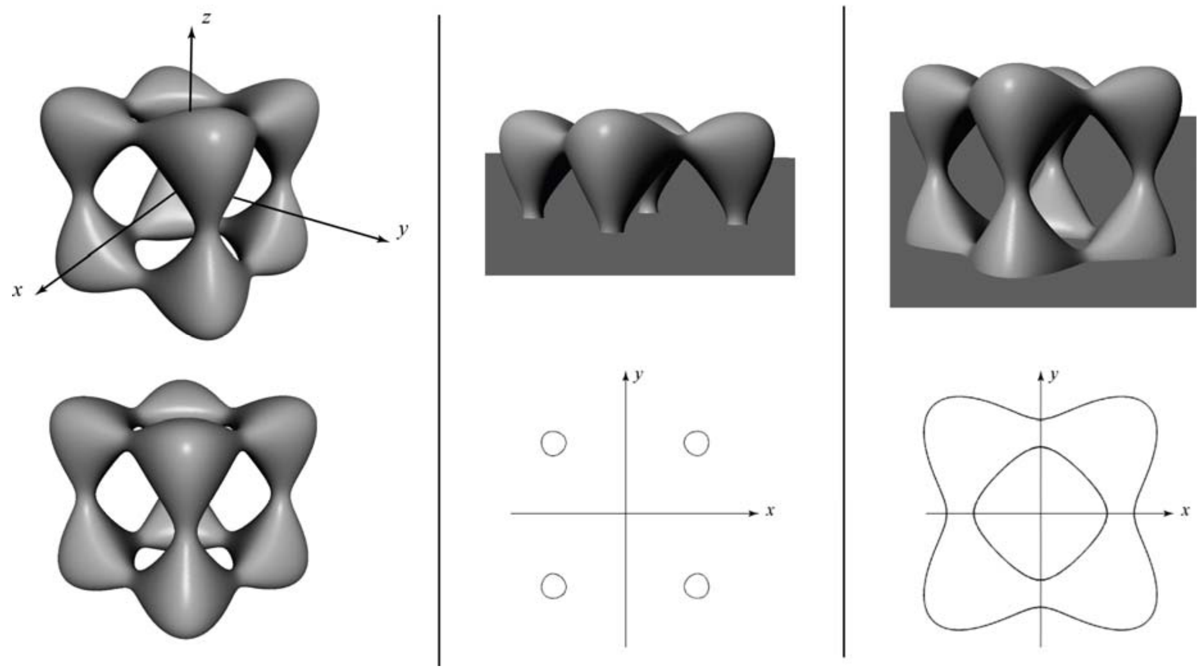
j) Assume: Orthographic viewer, lighting direction of (0,0,-1), $k_d =1$, $k_s =0$. As we move the viewer around, will the normal mapped rendering look the same as the displacement mapped rendering? Justify your answer.

Problem 2. Ray intersection with implicit surfaces

There are many ways to represent a surface. One way is to define a function of the form $f(x,y,z)=0$. Such a function is called an *implicit surface* representation. For example, the equation $f(x,y,z)=x^2+y^2+z^2-r^2=0$ defines a sphere of radius *r*. Suppose we wanted to ray trace a so-called "tangle cube," described by the equation:

$$x^4+y^4+z^4-5x^2-5y^2-5z^2+12=0$$

In the left column are two renderings of the tangle cube, the middle column illustrates taking a slice through the *x*-*y* plane (at *z* = 0), and the middle column shows a slice parallel to the *x*-*y* plane taken toward the bottom of the tangle cube (plane at z ≈ -1.5):



In the next problem steps, you will be asked to solve for and/or discuss ray intersections with this primitive. Performing the ray intersections will amount to solving for the roots of a polynomial, much as it did for sphere intersection. For your answers, you need to keep a few things in mind:

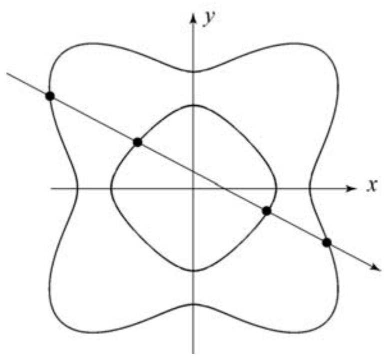
- You will find as many roots as the order (largest exponent) of the polynomial.
- You may find a mixture of real and complex roots. When we say complex here, we mean a number that has a non-zero imaginary component.
- All complex roots occur in complex conjugate pairs. If A + iB is a root, then so is A – iB.
- Sometimes a real root will appear more than once, i.e., has multiplicity > 1. Consider the case of sphere intersection, which we solve by computing the roots of a quadratic equation. A ray that intersects the sphere will usually have two distinct roots (each has multiplicity = 1) where the ray enters and leaves the sphere. If we were to take such a ray and translate it away from the center of the sphere, those roots get closer and closer together, until they merge into one root. They merge when the ray is tangent to the sphere. The result is one distinct real root with multiplicity = 2.

(a) Consider the ray $P+t\mathbf{d}$, where $P=\begin{pmatrix}0 & 0 & 0\end{pmatrix}$ and $\mathbf{d}=\begin{pmatrix}1 & 1 & 0\end{pmatrix}$. Typically, we normalize **d,** but for simplicity (and without loss of generality) you can work with the un-normalized **d** as given here.

- Solve for all values of *t* where the ray intersects the tangle cube (**including** any negative values of *t*). Show your work.
- Which value of *t* represents the intersection we care about for ray tracing?
- In the process of solving for t, you will be computing the roots of a polynomial. How many distinct real roots do you find? How many of them have multiplicity > 1? How many complex roots do you find?

Problem 2 (cont'd)

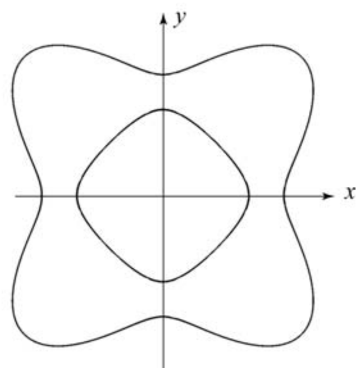
b) What are all the possible combinations of roots, not counting the one in part (a)? For each combination, describe the 4 roots as in part (a), draw a ray in the *x*-*y* plane that gives rise to that combination, and place a dot at each intersection point. Assume the origin of the ray is outside of the bounding box of the object. There are five diagrams below that have not been filled in. You may not need all five; on the other hand, if you can actually think of more distinct cases than spaces provided, then we might just give extra credit. The first one has already been filled in. (Note: not all conceivable combinations can be achieved on this particular implicit surface. For example, there is no ray that will give a root with multiplicity 4.) ***Please write on this page and include it with your homework solution. You do not need to justify your answers.***



# of distinct real roots: **4**

# of real roots w/ multiplicity > 1: **0**

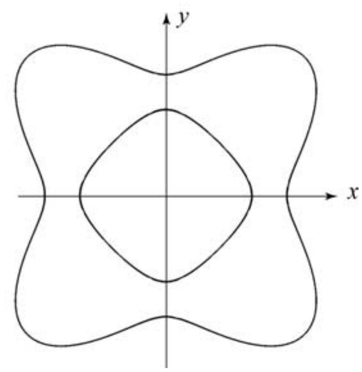
# of complex roots: **0**



# of distinct real roots:

# of real roots w/ multiplicity > 1:

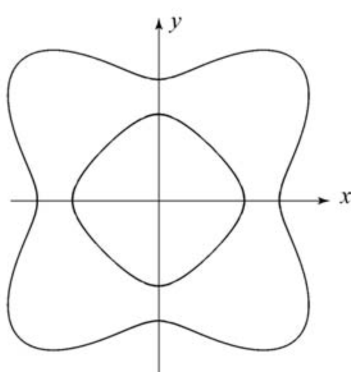
# of complex roots:



# of distinct real roots:

# of roots w/ multiplicity > 1:

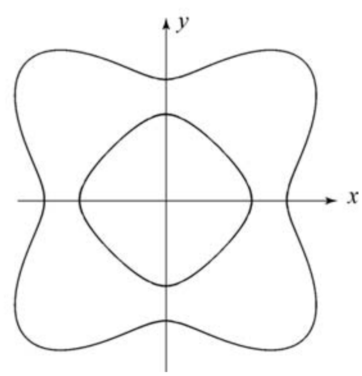
# of complex roots:



# of distinct real roots:

# of real roots w/ multiplicity > 1:

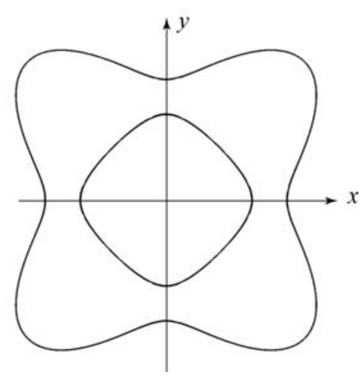
# of complex roots:



# of distinct real roots:

# of real roots w/ multiplicity > 1:

# of complex roots:



# of distinct real roots:

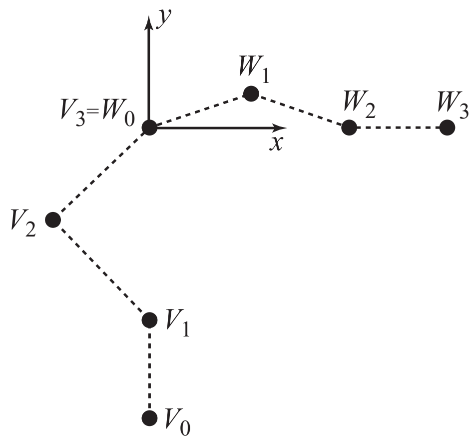
# of real roots w/ multiplicity > 1:

# of complex roots:

Problem 3. **Parametric vs. geometric continuity**

In this problem, we examine the differences between parametric and geometric continuity of curves. Recall that C^1 continuity implies that the curve has no sudden jumps in it for neighboring parametric values (i.e., is C^0 continuous) and is also continuous in the 1st parametric derivative. G^1 continuity means that all points on the curve have immediate neighbors (i.e., the curve is G^0), and the direction of the tangent to the curve is continuous. Recall also that the tangent to a curve points in the same direction as the first parametric derivative at that point.

Now consider the following control polygon defined by consecutive sets of Bezier control points (V_0, V_1, V_2, V_3) and (W_0, W_1, W_2, W_3):



where:

$$V_0 = \begin{bmatrix} 0 \\ -3 \end{bmatrix} \quad V_1 = \begin{bmatrix} 0 \\ -2 \end{bmatrix} \quad V_2 = V_3 - a\begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad V_3 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$W_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad W_1 = W_0 + b\begin{bmatrix} 1 \\ c \end{bmatrix} \quad W_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \quad W_3 = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

a. Sketch the control points and curve you get when setting $a = b = c = 1$. Note, your sketch does not need to be precise, but it should show which control points are interpolated, suggest the directions of the tangents at those points, and fit within the regions to which the curve should be restricted. (This note applies to the remainder of the problem.)

b. Sketch the control points and curve you get when setting $a = 1$, $b = 1/2$, $c = 1$.

c. Sketch the control points and curve you get when setting $a = b = 1$, $c = 0$.

d. For $a, b > 0$, what constraints on a, b, and/or c will ensure that the curve is C^1 continuous?

e. For $a, b > 0$, what constraints on a, b, and/or c will ensure that the curve is G^1 continuous?

f. Now consider the case where $a = b = 0$. Sketch the component curves $x(u)$ vs. u and $y(u)$ vs. u as two separate plots where u varies from –1 to 0 for the first set of Bezier control points, (V_0, V_1, V_2, V_3), and then 0 to 1 for the second set, (W_0, W_1, W_2, W_3).

g. Sketch the control points and curve you get when setting $a = b = 0$ (as in (f)).

h. Is the curve in part (f) C^1 continuous, G^1 continuous, both, or neither? Justify your answer.

Problem 4. Final project

Send email to both Alex and myself describing who you plan to work with and what you plan to develop for your final project. Explain your division of labor, and describe the artifact you hope to produce. This email is due by midnight of December 3. We will set up a meeting with each team to discuss your project the next day. The project itself will be demoed on Friday, December 18, with a concise web page write-up (with artifacts) due by midnight of that day.