Surfaces of Revolution

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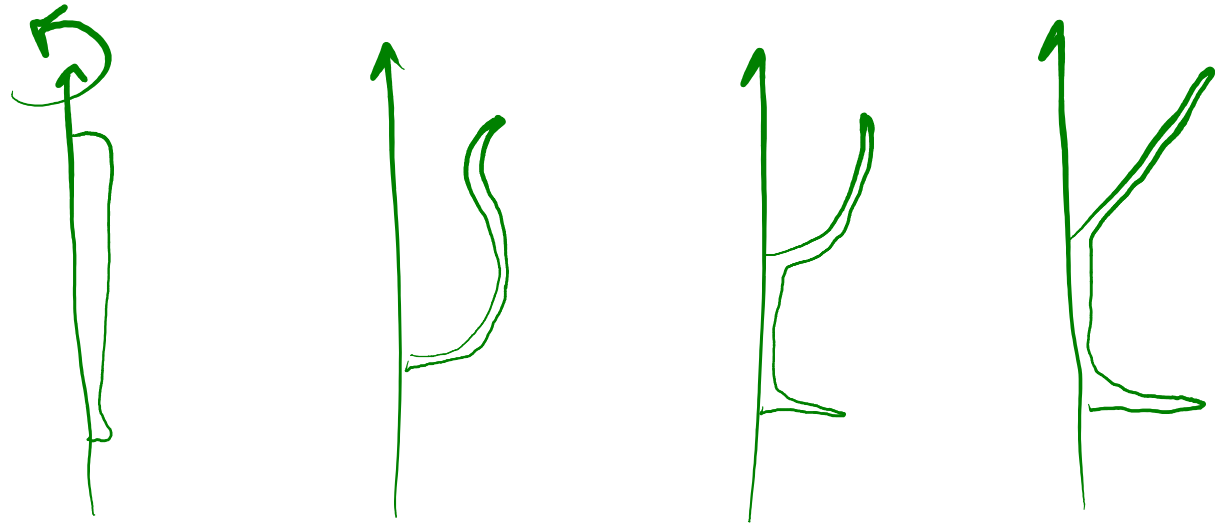
CSE 557
Fall 2015

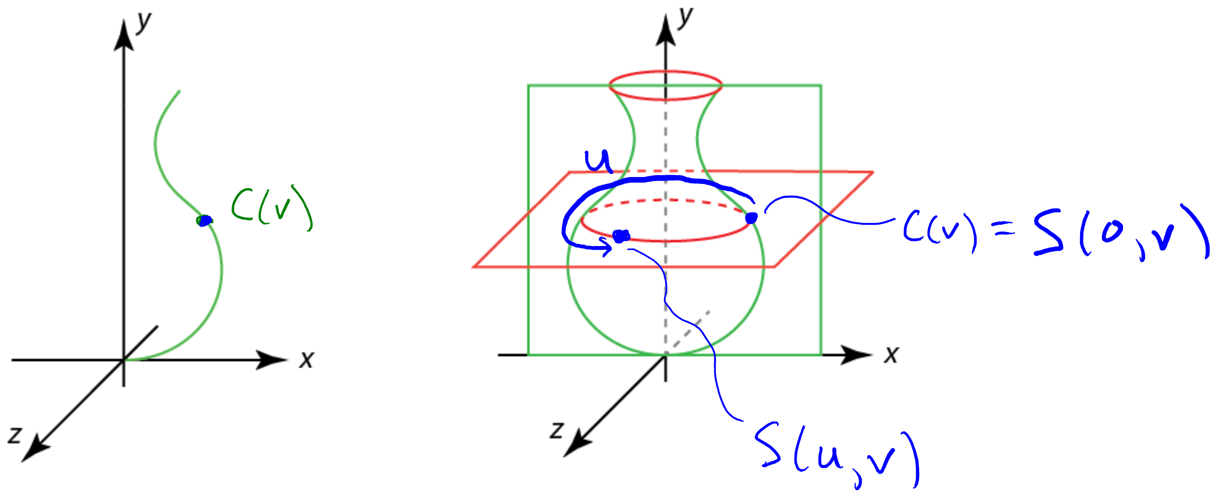
Surfaces of revolution



Idea: rotate a 2D **profile curve** around an axis.

What kinds of shapes can you model this way?

Constructing surfaces of revolution



Given: A curve $C(v)$ in the xy-plane:

$$C(v) = \begin{bmatrix} C_x(v) \\ C_y(v) \\ 0 \\ 1 \end{bmatrix}$$

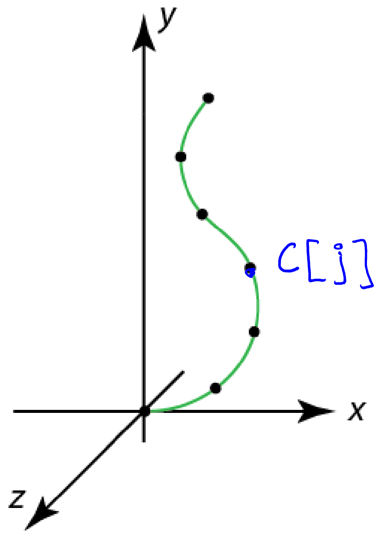
Let $R_y(\theta)$ be a rotation about the y-axis.

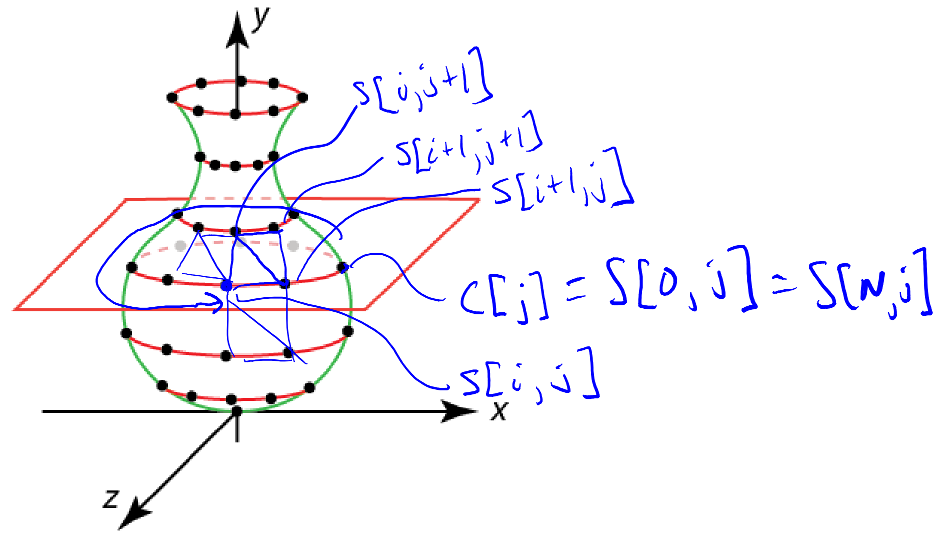
Find: A surface $S(u,v)$ which is $C(v)$ rotated about the y-axis, where $u, v \in [0, 1]$.

Solution: $S(u,v) = R_y(2\pi u)\, C(v)$

Constructing surfaces of revolution

We can sample in u and v to get a grid of points over the surface.





Suppose we sample:

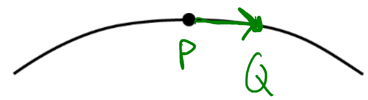
- in v, to give $C[j]$ where $j \in [0..M-1]$
- in u, to give rotation angle $\theta[i] = 2\pi i / N$ where $i \in [0..N]$

We can now write the surface as:

$$S[i,j] = R_y\left(\frac{2\pi i}{N}\right) C[j]$$

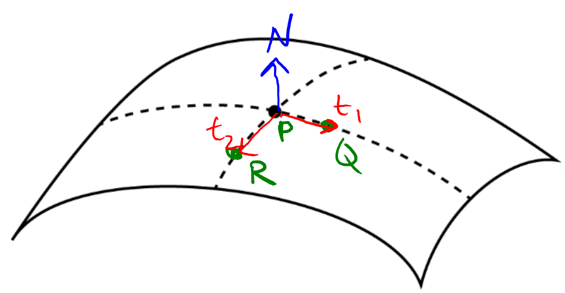
How would we turn this into a mesh of triangles?
How do we assign per-vertex normals?

Tangent vectors, tangent planes, and normals



$$t \approx Q - P$$

$$t = \lim_{Q \to P} \frac{Q - P}{\|Q - P\|}$$

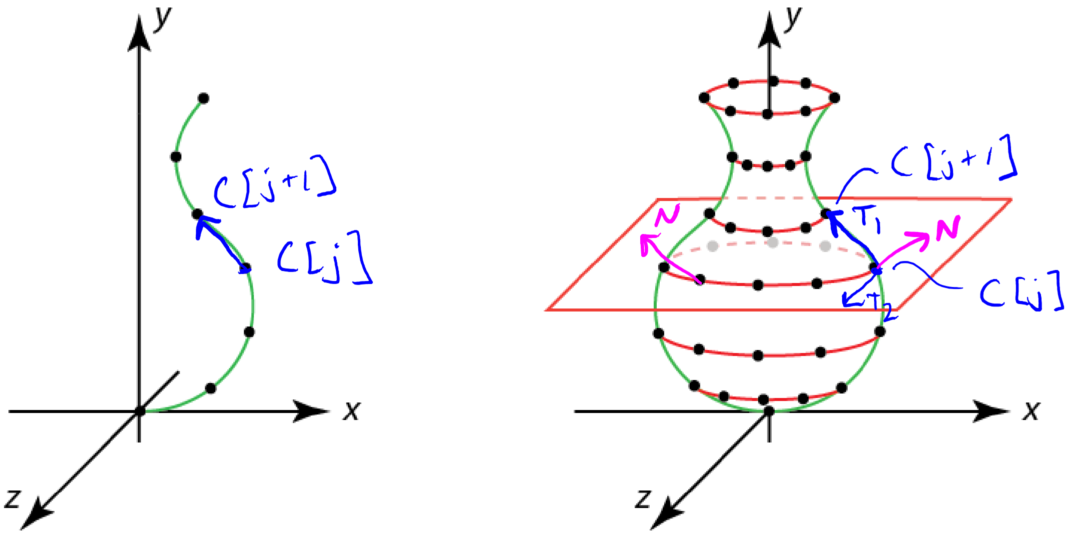


$$t_1 \approx Q - P$$

$$t_2 \approx R - P$$

$$N \approx \frac{t_2 \times t_1}{\|t_2 \times t_1\|}$$

Normals on a surface of revolution



We can compute tangents in the *x*-*y* plane:

$$\mathbf{T}_1[0, j] \approx C[j+1] - C[j]$$

$$\mathbf{T}_2[0, j] = [0 \quad 0 \quad 1]^T$$

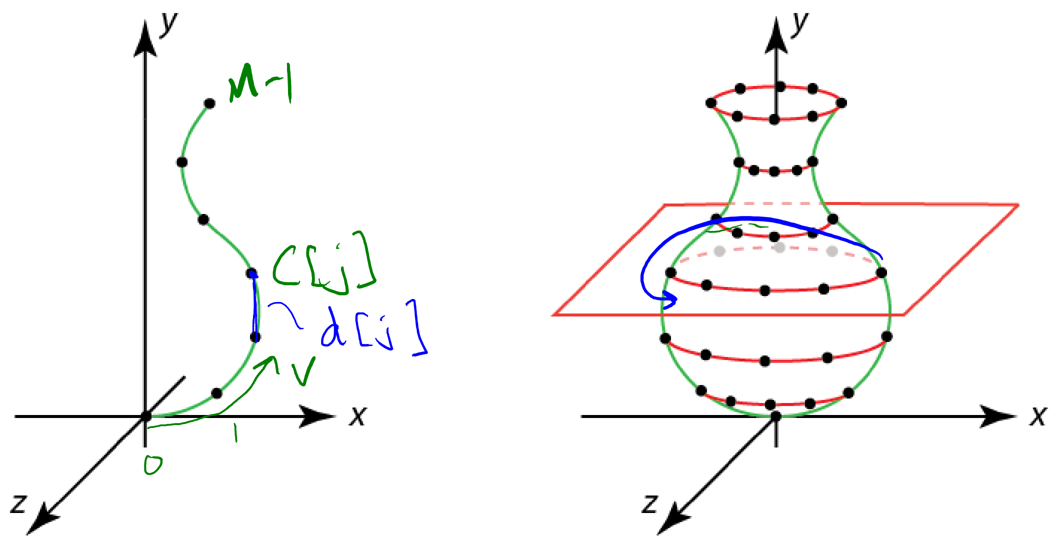
to get the normal in that plane:

$$\mathbf{N}[0, j] = T_1[0,j] \times T_2[0,j] \quad \text{.... then normalize!}$$

and then rotate it around:

$$N[i,j] = R_y\left(\frac{2\pi i}{N}\right)\mathbf{N}[0,j]$$

Texture coordinates on a surface of revolution



The simplest assignment of texture coordinates would be:

$$\cancel{v = \frac{j}{M-1}} \qquad u = \frac{i}{N}$$

We can do better for v to reduce distortion. Define:

$$d[j] = \begin{cases} \left\| C[j] - C[j-1] \right\|, & \text{if } j \neq 0 \\ 0, & \text{if } j = 0 \end{cases}$$

and set v to fractional distance along the curve:

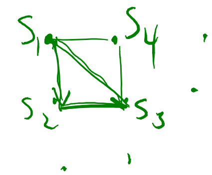
$$v[j] = \frac{\sum_{k=0}^{j} d[k]}{\sum_{k=0}^{M-1} d[k]}$$

"arc length parameterization"

You must do this for v for the assignment!

Triangle meshes

How should we generally represent triangle meshes?



S_1, N_1, u_1, v_1

S_2, N_2, u_2, v_2

S_3, N_3, u_2, v_2

S_1, N_1, u_1, v_1

S_3, N_3, u_3, v_3

S_4, N_4, u_4, v_4

Vertices

S_1, N_1, u_1, v_1

S_2, N_2, u_2, v_2

Δ's

(1, 2, 3)

(1, 3, 4)

Do this