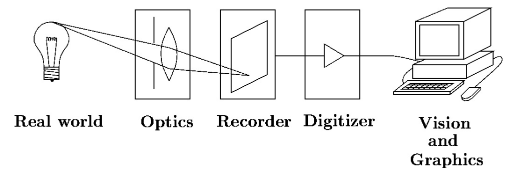
Lenses

Overview

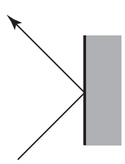


Pinhole camera

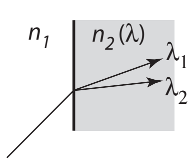
Lenses

- **Principles of operation**
- **Limitations**

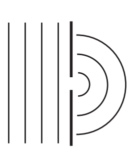
Terminology



Reflection



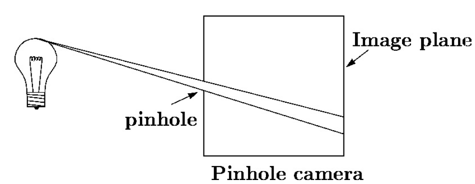
Refraction and dispersion



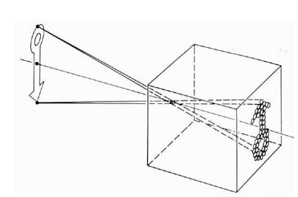
Diffraction

The pinhole camera

The first camera - “camera obscura” - known to Aristotle.

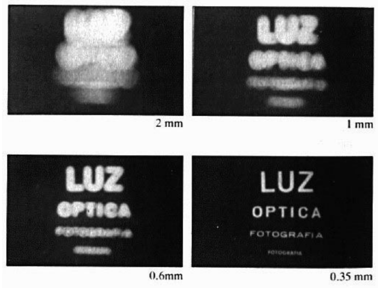


In 3D, we can visualize the blur induced by the pinhole (a.k.a., **aperture**):



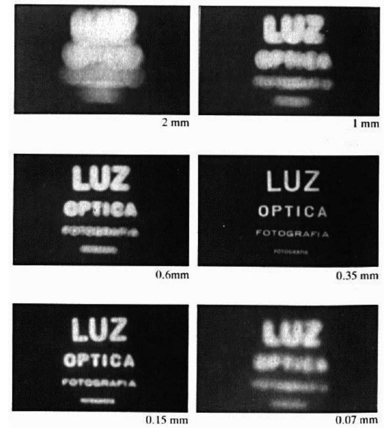
Q: How would we reduce blur?

Shrinking the pinhole



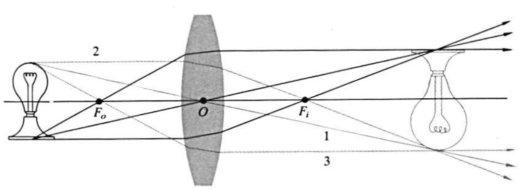
Q: What happens as we continue to shrink the aperture?

Shrinking the pinhole, cont'd



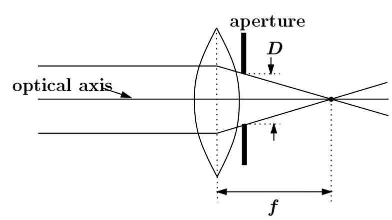
Lenses

Lenses focus a bundle of rays to one point => can have larger aperture.



Lenses

A lens images a bundle of parallel rays to a focal point at a distance, *f*, beyond the plane of the lens.



An aperture of diameter, D, restricts the extent of the bundle of refracted rays.

Note: *f* is a function of the index of refraction of the lens.

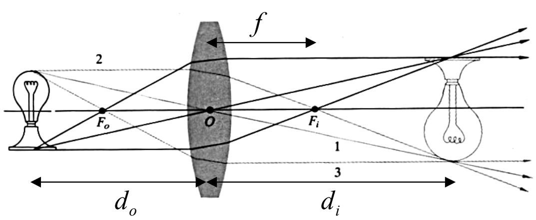
Lenses

For economical manufacture, lens surfaces are usually spherical.

A spherical lens is behaves ideally if we consider rays near the optical axis -- “paraxial rays.”

For a “thin” lens, we ignore lens thickness, and the paraxial approximation leads to the familiar Gaussian lens formula:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$



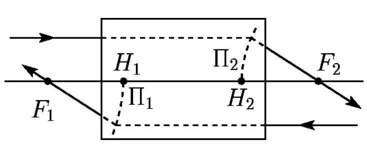
Cardinal points of a lens system

Most cameras do not consist of a single thin lens. Rather, they contain multiple lenses, some thick.

A system of lenses can be treated as a “black box” characterized by its cardinal points.

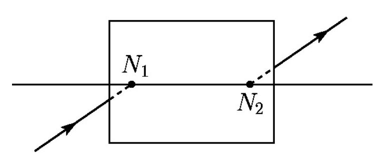
Focal and principal points

The **focal points**, **principal points**, and **principal planes** (well, surfaces actually) describe the paths of rays parallel to the optical axis.



Nodal points

The **nodal points** describe the paths of rays that are not refracted, but are translated down the optical axis.



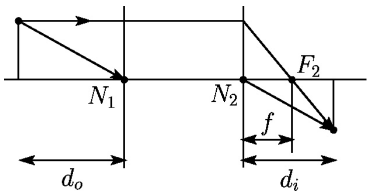
Cardinal points of a lens system

If:

- *the optical system is surrounded by air*
- *and the principal planes are assumed planar*

then

- *the nodal and principal points are the same*



The system still obeys Gauss's law, but all distances are now relative to the principal planes.

The principal and nodal points are, together, called the **cardinal points**.

Limitations of lens systems

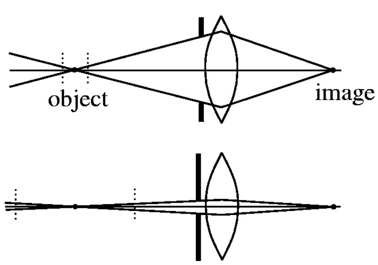
Q: What are some of the limitations of lenses?

Depth of field

Lens systems do have some limitations.

First, points that are not in the object plane will appear out of focus.

The **depth of field** is a measure of how far from the object plane points can be before appearing "too blurry."



Non-paraxial imaging

When we violate the paraxial assumption, we find that real imaging systems exhibit a number of imperfections.

We can set up the geometry of a *rotationally symmetric* lens system in terms of an object, aperture, and image:

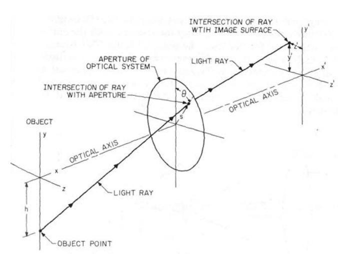


Figure 3.1 A ray from the point $y = h$, $(z = 0)$ in the object passes through the optical system aperture at a point defined by its polar coordinates, (s, θ), and intersects the image surface at y', z'.

We can then perform a Taylor series of the mapping from rays to image points:

$$
\begin{aligned}
y' &= A_1 s \cos\theta + A_2 h \\
&\quad + B_1 s^3 \cos\theta + B_2 s^2 h (2 + \cos 2\theta) + (3B_3 + B_4) s h^2 \cos\theta + B_5 h^3 \\
&\quad + C_1 s^5 \cos\theta + (C_2 + C_3 \cos 2\theta) s^4 h + (C_4 + C_6 \cos^2\theta) s^3 h^2 \cos\theta \\
&\quad + (C_7 + C_8 \cos 2\theta) s^2 h^3 + C_{10} s h^4 \cos\theta + C_{12} h^5 + D_1 s^7 \cos\theta + \cdots
\end{aligned}
$$

$$
\begin{aligned}
z' &= A_1 s \sin\theta \\
&\quad + B_1 s^3 \sin\theta + B_2 s^2 h \sin 2\theta + (B_3 + B_4) s h^2 \sin\theta \\
&\quad + C_1 s^5 \sin\theta + C_3 s^4 h \sin 2\theta + (C_5 + C_6 \cos^2\theta) s^3 h^2 \sin\theta \\
&\quad + C_9 s^2 h^3 \sin 2\theta + C_{11} s h^4 \sin\theta + D_1 s^7 \sin\theta + \cdots
\end{aligned}
$$

Third order aberrations

The first set of non-ideal terms beyond perfect imaging and depth of field form the basis for the **third order theory**.

Deviations from ideal optics are called the **primary** or **Seidel aberrations**:

- Spherical aberration
- Coma
- Astigmatism
- Petzval curvature
- Distortion

All of these aberrations can be reduced by stopping down the aperture, except distortion.

Distortion

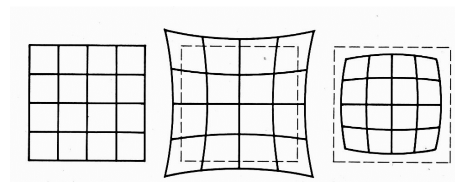
Distortion follows the form (replacing *h* with *r*):

$$r' = a_1 r + a_3 r^3 + a_5 r^5 + \cdots$$

Sometimes this is re-written as:

$$r' = r \cdot (a_1 + a_3 r^2 + a_5 r^4 + \cdots)$$

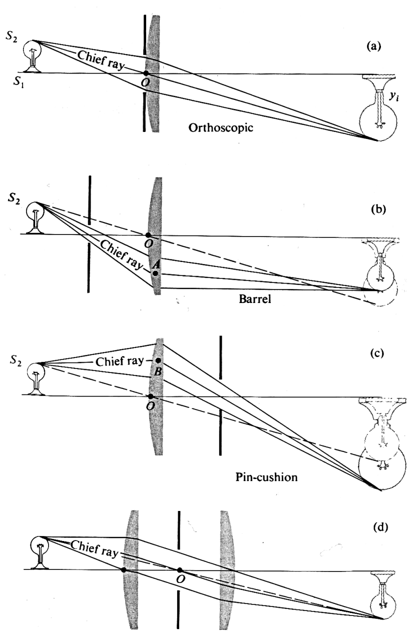
The effect is that non-radial lines curve out (barrel) or curve in (pin cushion).



No distortion　Pin cushion　Barrel

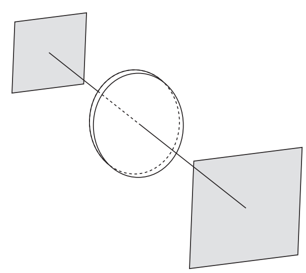
Distortion

Intuitively, distortion is caused by oblique rays hitting the edges of the lens.



Distortion

Q: Why is distortion radial?



Q: Why doesn't the Taylor series expansion have any even polynomial terms?

Chromatic aberration

Cause:

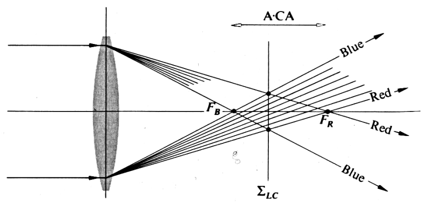
Index of refraction varies with wavelength (i.e., exhibits dispersion).

Effect:

Focus shifts with color, colored fringes on highlights

Ways of improving:

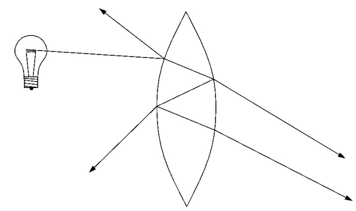
Achromatic designs



Flare

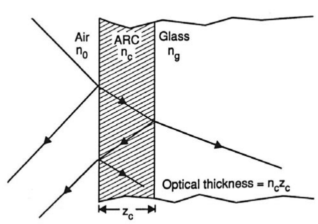
Light rays refract and reflect at the interfaces between air and the lens.

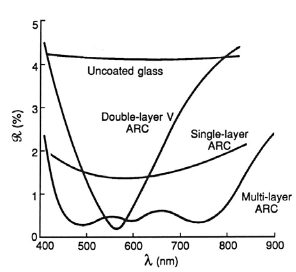
The "stray" light is not focused at the desired point in the image, resulting in ghosts or haziness, a phenomenon known as **lens flare**.



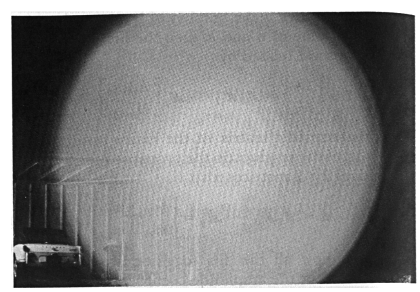
Optical coatings

Optical coatings are tuned to cancel out reflections at certain angles and wavelengths.

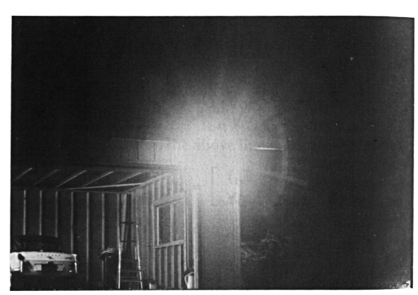




Single vs. multiple coatings



Single coating



Mutliple coatings

Vignetting

Light rays oblique to the lens will deliver less power per unit area (irradiance) due to:

- optical vignetting
- mechanical vignetting

Result: darkening at the edges of the image.

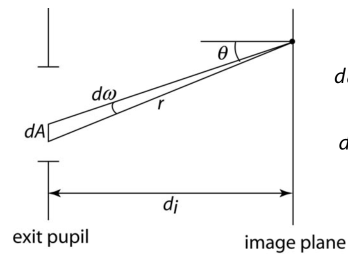
Optical vignetting

Optical vignetting is best explained in radiometric terms.

A sensor responds **irradiance** (power per unit area) which is defined in terms of **radiance** as:

$$dE = L\cos\theta d\omega$$
$$E = \int_H L\cos\theta d\omega$$

For a given image plane and **exit pupil**:



$$d\omega = \frac{dA_p}{r^2} = \frac{\cos\theta dA}{(d_i/\cos\theta)^2} = \frac{\cos^3\theta dA}{d_i^2}$$
$$dE = \frac{L\cos^4\theta dA}{d_i^2}$$

Thus:

$$E \approx L\frac{A}{d_i^2}\cos^4\theta$$

Optical vignetting, cont'd

We can rewrite this in terms of the diameter of the exit pupil:

$$\frac{A}{d_i^2} = \frac{\pi(D/2)^2}{d_i^2} = \frac{\pi}{4}\left(\frac{D}{d_i}\right)^2$$

In many cases, $d_o >> d_i$:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$
$$\frac{d_i}{f} = \frac{d_i}{d_o} + 1$$
$$\frac{d_i}{f} \approx 1 \Rightarrow d_i \approx f$$

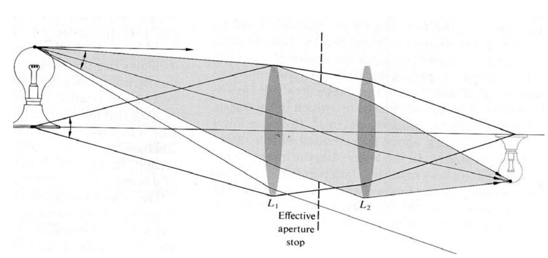
As a result:

$$E \approx L\left(\frac{D}{f}\right)^2\cos^4\theta$$

The term *f/D* is called the **f-number**.

Mechanical vignetting

Occlusion by apertures and lens extents results in mechanical vignetting.



Bibilography

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