

# Stereo

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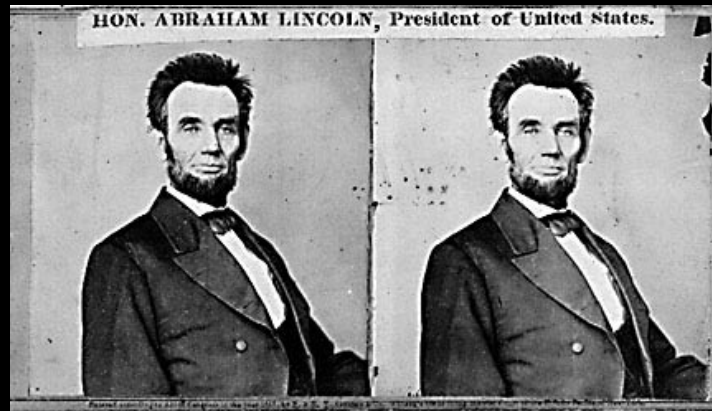
## Binocular Stereo

- Motivation
- Epipolar geometry
- Matching
- Depth estimation
- Rectification

## Calibration (finish up)

## Next Time

- Multiview stereo





Public Library, Stereoscopic Looking Room, Chicago, by Phillips, 1923



Teesta suspension bridge-Darjeeling, India



Mark Twain at Pool Table", no date, UCR Museum of Photography



Woman getting eye exam during immigration procedure at Ellis Island, c. 1905 - 1920 , UCR Museum of Photography

## Stereograms online

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### UCR stereographs

- <http://www.cmp.ucr.edu/site/exhibitions/stereo/>

### The Art of Stereo Photography

- <http://www.photostuff.co.uk/stereo.htm>

### History of Stereo Photography

- [http://www.rpi.edu/~ruiz/stereo\\_history/text/historystereog.html](http://www.rpi.edu/~ruiz/stereo_history/text/historystereog.html)

### Double Exposure

- <http://home.centurytel.net/s3dcor/index.html>

### Stereo Photography

- <http://www.shortcourses.com/book01/chapter09.htm>

### 3D Photography links

- <http://www.studyweb.com/links/5243.html>

### National Stereoscopic Association

- <http://204.248.144.203/3dLibrary/welcome.html>

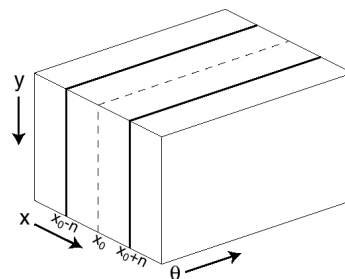
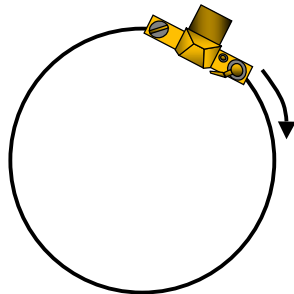
### Books on Stereo Photography

- <http://userwww.sfsu.edu/~hl/3d.biblio.html>

## Stereo Panoramas

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- [Ishiguro, Yamamoto, Tsuji 92]
- [Peleg and Ben-Ezra 99]
- [Shum, Kalai, Seitz 99]



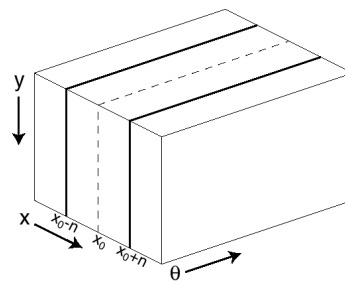
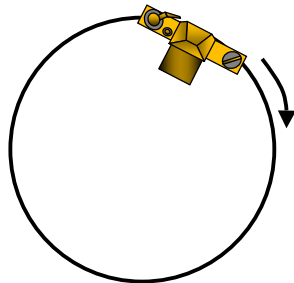
Interactive demo: <http://www.cs.columbia.edu/CAVE/>

## Depth from Stereo Panoramas



Disparity map result

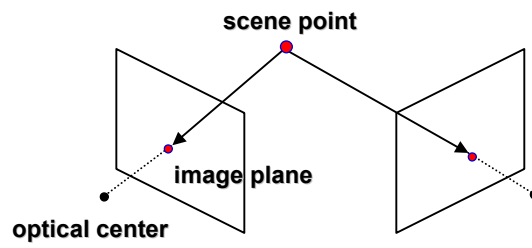
## Stereo Cyclographs [Seitz, 2001]





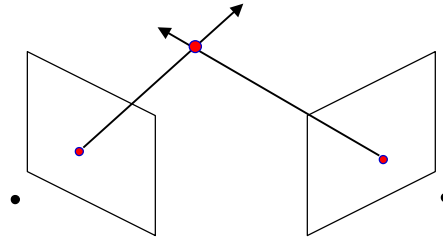
## Stereo

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## Stereo

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### Basic Principle: Triangulation

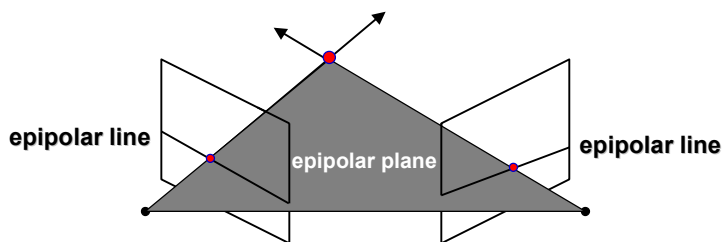
- Gives reconstruction as intersection of two rays
- Requires
  - calibration
  - *point correspondence*

## Stereo Correspondence

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### Determine Pixel Correspondence

- Pairs of points that correspond to same scene point

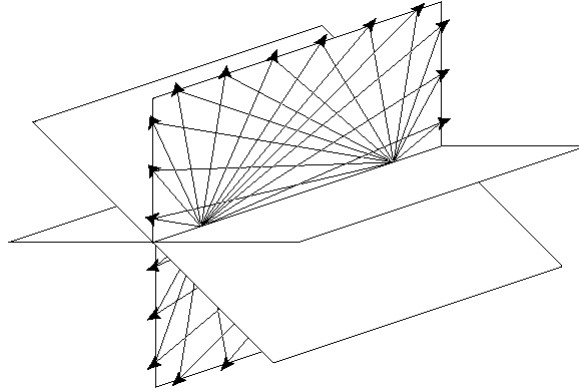


### Epipolar Constraint

- Reduces correspondence problem to 1D search along *conjugate epipolar lines*
- Java demo: <http://www.ai.sri.com/~luong/research/Meta3DViewer/EpipolarGeo.html>

## Epipolar Geometry

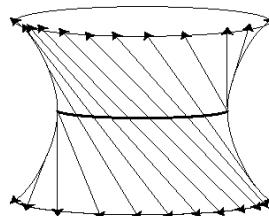
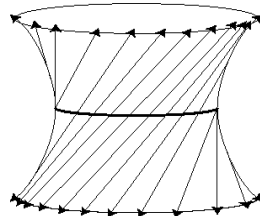
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All camera rays lie in a “pencil” of planes

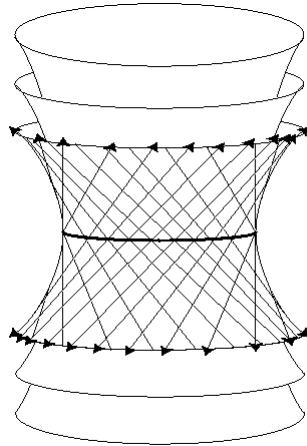
## Stereo Panoramas

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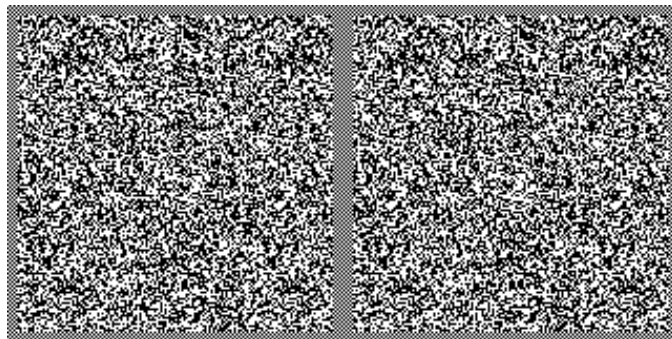
## Epipolar Geometry for Stereo Panoramas



## Stereo Matching

### Features vs. Pixels?

- Do we extract features prior to matching?



Julesz-style Random Dot Stereogram

## Stereo Matching Algorithms

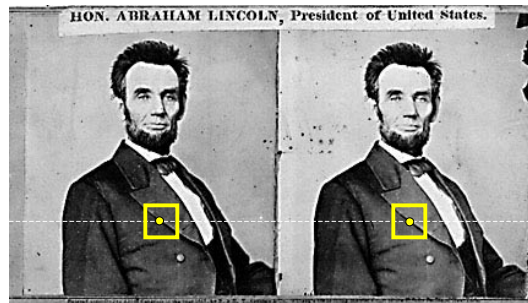
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### Match Pixels in Conjugate Epipolar Lines

- Assume color of point does not change
- Pitfalls
  - specularities
  - low-contrast regions
  - occlusions
  - image error
  - camera calibration error
- Numerous approaches
  - dynamic programming [Baker 81, Ohta 85]
  - smoothness functionals
  - more images (trinocular, N-ocular) [Okutomi 93]
  - graph cuts [Boykov 00]

## Your Basic Stereo Algorithm

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For each epipolar line

For each pixel in the left image

- compare with every pixel on same epipolar line in right image
- pick pixel with minimum match cost

Improvement: match *windows*

## Window Size

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W = 3



W = 20

### Smaller window

- more details
- more noise

### Larger window

- less noise
- less detail

### Better results with *adaptive window*

- T. Kanade and M. Okutomi, [A Stereo Matching Algorithm with an Adaptive Window: Theory and Experiment](#), Proc. International Conference on Robotics and Automation, 1991.
- D. Scharstein and R. Szeliski, [Stereo matching with nonlinear diffusion](#), International Journal of Computer Vision, 28(2):155-174, July 1998

## Stereo as Energy Minimization

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### Matching Cost Formulated as Energy

- “data” term penalizing bad matches

$$D(x, y, d) = |\mathbf{I}(x, y) - \mathbf{J}(x + d, y)|$$

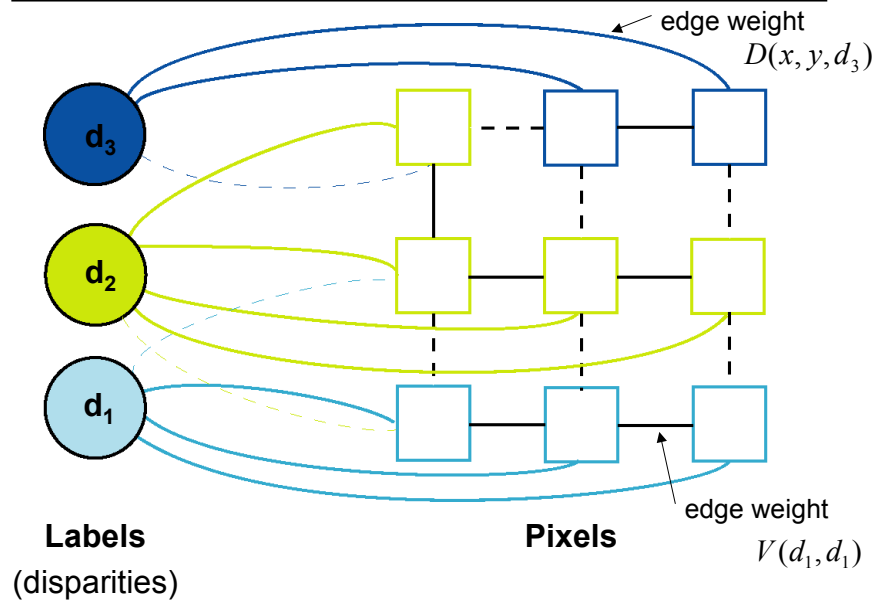
- “neighborhood term” encouraging spatial smoothness

$V(d_1, d_2)$  = cost of adjacent pixels with labels  $d_1$  and  $d_2$

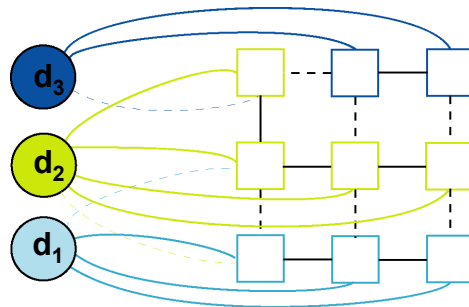
$$= |d_1 - d_2| \quad (\text{or something similar})$$

$$E = \sum_{(x,y)} D(x, y, d_{x,y}) + \sum_{\text{neighbors } (x_1,y_1), (x_2,y_2)} V(d_{x_1,y_1}, d_{x_2,y_2})$$

## Stereo as a Graph Problem [Boykov, 1999]



## Stereo as a Graph Problem [Boykov, 1999]

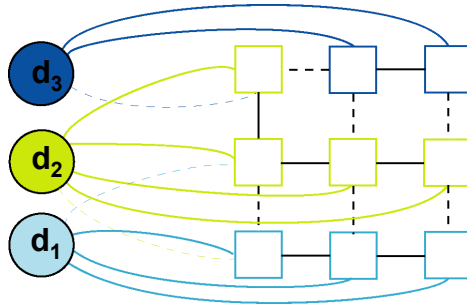


### Graph Cost

- Matching cost between images
- Neighborhood matching term
- Goal: figure out which labels are connected to which pixels

## Stereo Matching by Graph Cuts

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### Graph Cut

- Delete enough edges so that
  - each pixel is (transitively) connected to exactly one label node
- Cost of a cut: sum of deleted edge weights
- Finding min cost cut equivalent to finding global minimum of energy function

## Computing a multiway cut

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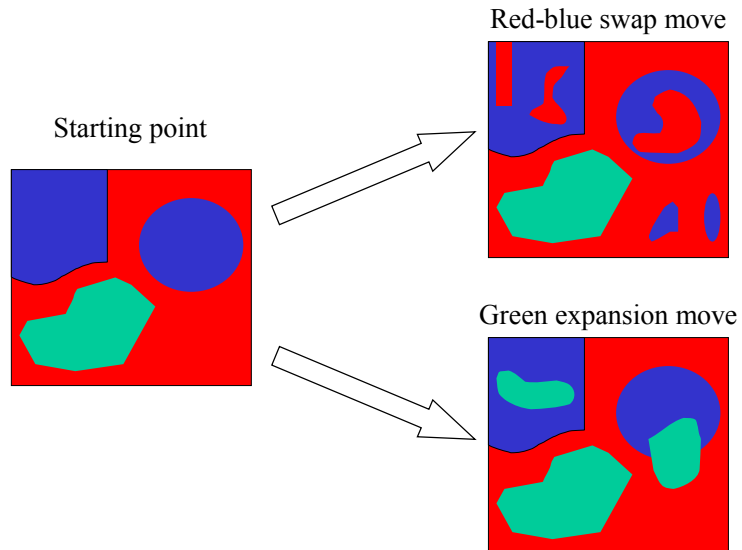
### With 2 labels: classical min-cut problem

- Solvable by standard flow algorithms
  - polynomial time in theory, nearly linear in practice
- More than 2 terminals: NP-hard  
[Dahlhaus *et al.*, STOC '92]

### Efficient approximation algorithms exist

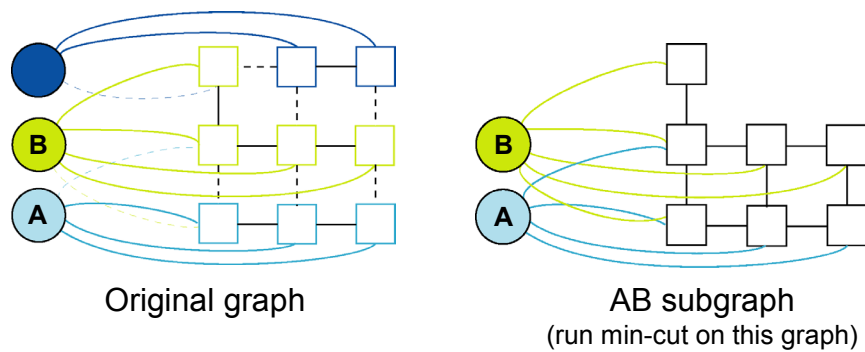
- Within a factor of 2 of optimal
- Computes local minimum in a strong sense
  - even very large moves will not improve the energy
- Yuri Boykov, Olga Veksler and Ramin Zabih, [Fast Approximate Energy Minimization via Graph Cuts](#), International Conference on Computer Vision, September 1999.

## Move Examples



## The Swap Move Algorithm

1. Start with an arbitrary labeling
2. Cycle through every label pair  $(A,B)$  in some order
  - 2.1 Find the lowest  $E$  labeling within a single  $AB$ -swap
  - 2.2 Go there if it's lower  $E$  than the current labeling
3. If  $E$  did not decrease in the cycle, we're done  
Otherwise, go to step 2



## The expansion move algorithm

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1. Start with an arbitrary labeling
2. Cycle through every label  $A$  in some order
  - 2.1 Find the lowest  $E$  labeling within a single  $A$ -expansion
  - 2.2 Go there if it's lower  $E$  than the current labeling
3. If  $E$  did not decrease in the cycle, we're done    Otherwise, go to step 2

## Stereo Results

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- Data from University of Tsukuba
- Similar results on other images without ground truth

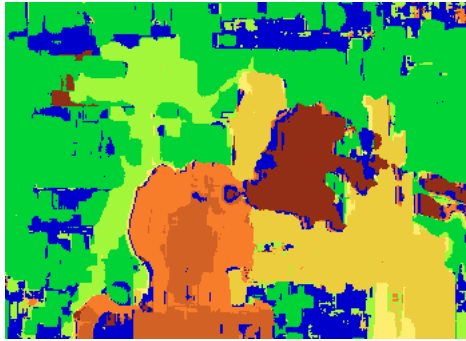


Scene



Ground truth

## Results with Window Correlation



Normalized correlation  
(best window size)



Ground truth

## Results with Graph Cuts



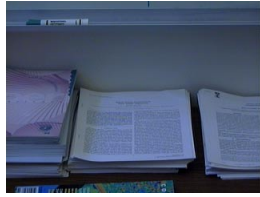
Graph Cuts  
(Potts model  $E$ ,  
expansion move algorithm)



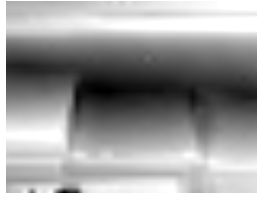
Ground truth



## Depth from Disparity



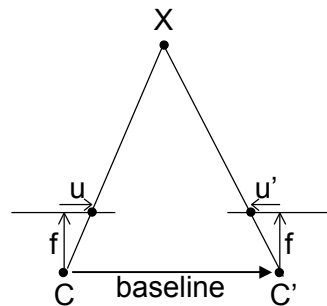
input image (1 of 2)



depth map  
[Szeliski & Kang '95]

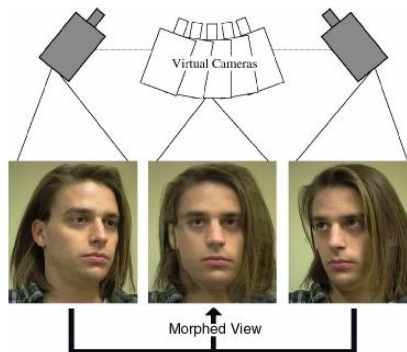


3D rendering



$$\text{disparity} = u - u' = \frac{\text{baseline} * f}{z}$$

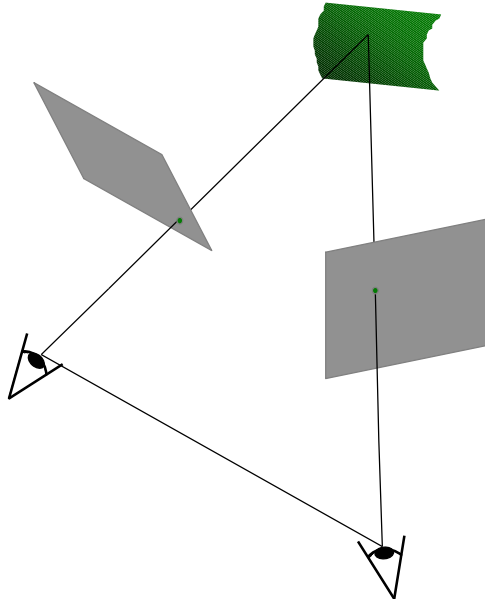
## Disparity-Based Rendering



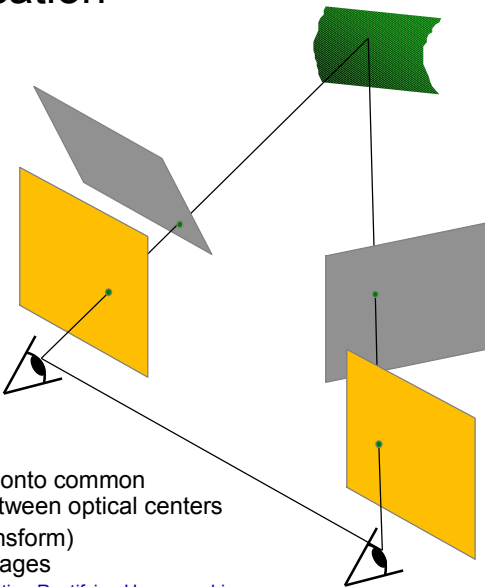
### Render new views from raw disparity

- S. M. Seitz and C. R. Dyer, [View Morphing](#), *Proc. SIGGRAPH 96*, 1996, pp. 21-30.
- L. McMillan and G. Bishop, [Plenoptic Modeling: An Image-Based Rendering System](#), *Proc. of SIGGRAPH 95*, 1995, pp. 39-46.

## Image Rectification



## Image Rectification



### Image Reprojection

- reproject image planes onto common plane parallel to line between optical centers
- a homography (3x3 transform) applied to both input images
- C. Loop and Z. Zhang. [Computing Rectifying Homographies for Stereo Vision](#). IEEE Conf. Computer Vision and Pattern Recognition, 1999.

## Stereo Reconstruction Pipeline

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### Steps

- Calibrate cameras
- Rectify images
- Compute disparity
- Estimate depth

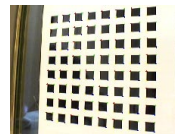
## Calibration from (unknown) Planes

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What's the image of a plane under perspective?

- a homography (3x3 projective transformation)

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \sim \begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \mathbf{H} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



- preserves lines, incidence, conics

**H** depends on camera parameters (**A**, **R**, **t**)

$$\mathbf{H} = \mathbf{A} [\mathbf{r}_1 \quad \mathbf{r}_2 \quad \mathbf{t}] \quad \text{where } \mathbf{A} = \begin{bmatrix} fa & c & u_c \\ 0 & f & v_c \\ 0 & 0 & 1 \end{bmatrix} \quad \mathbf{R} = [\mathbf{r}_1 \quad \mathbf{r}_2 \quad \mathbf{r}_3]$$

Given 3 homographies, can compute **A**, **R**, **t**

- Z. Zhang. [A flexible new technique for camera calibration](#). *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 22(11):1330-1334, 2000.
- <http://research.microsoft.com/~zhang/Calib/>

## Calibration from Planes

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### 1. Compute homography $\mathbf{H}^i$ for 3+ planes

- Doesn't require knowing 3D
- Does require mapping between at least 4 points on plane and in image (both expressed in 2D plane coordinates)

### 2. Solve for $\mathbf{A}$ , $\mathbf{R}$ , $\mathbf{t}$ from $\mathbf{H}^1$ , $\mathbf{H}^2$ , $\mathbf{H}^3$

- 1 plane if only  $f$  unknown
- 2 planes if  $(f, u_c, v_c)$  unknown
- 3+ planes for full  $\mathbf{K}$

### 3. Introduce radial distortion model

$$\hat{u} = u + u(\kappa_1 r^2 + \kappa_2 r^4)$$

$$\hat{v} = v + v(\kappa_1 r^2 + \kappa_2 r^4)$$

where

$$r = \sqrt{(u - u_c)^2 + (v - v_c)^2}$$

Solve for  $\mathbf{A}$ ,  $\mathbf{R}$ ,  $\mathbf{t}$ ,  $\kappa_1$ ,  $\kappa_2$

- nonlinear optimization (using Levenberg-Marquardt)