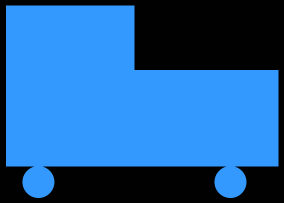
Specific Object Recognition: Matching in 2D



engine model

Is there an engine in the image?
If so, where is it located?

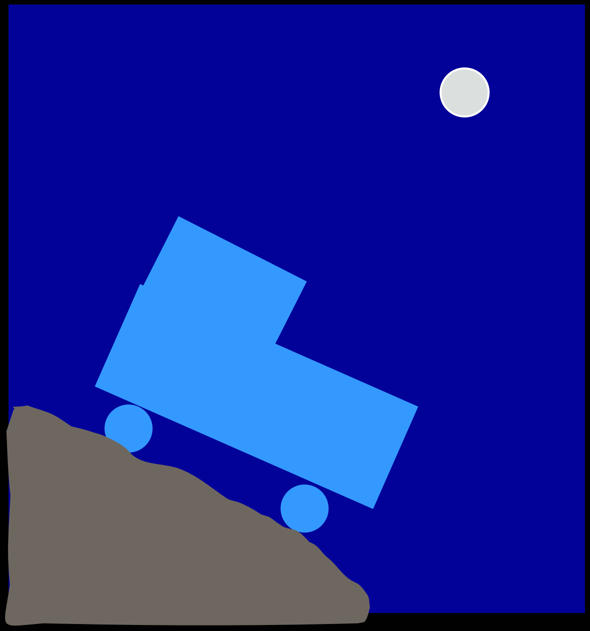
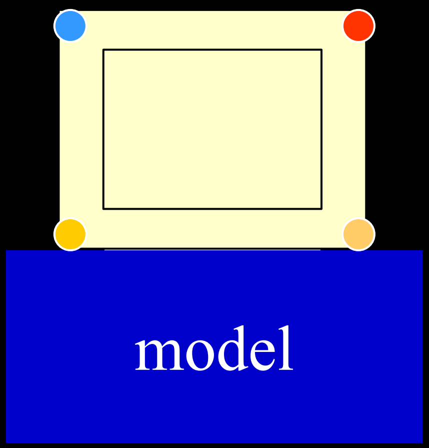


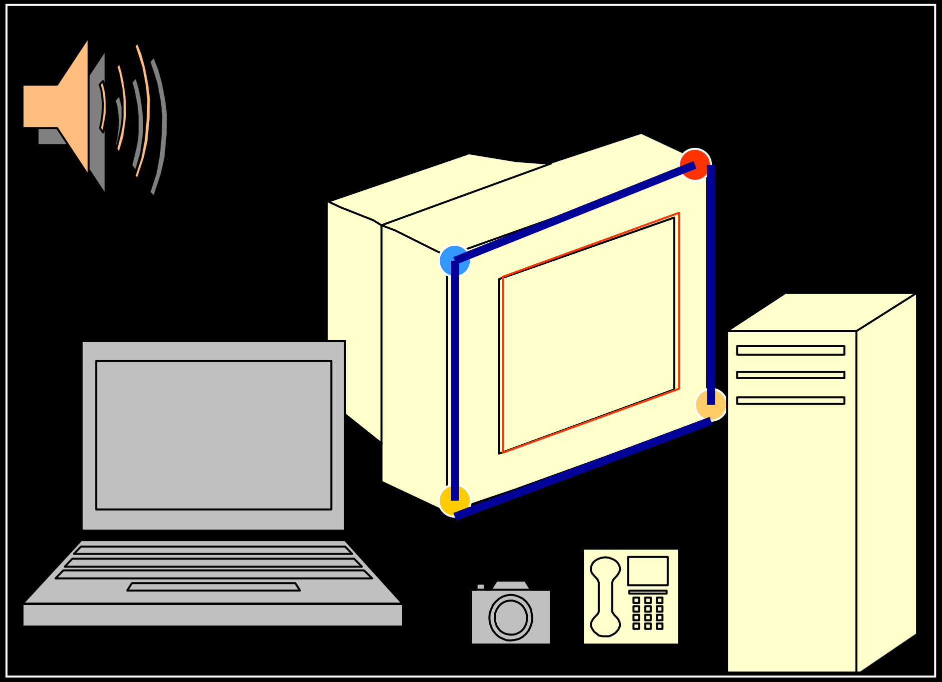
image containing an instance of the model

Alignment

- Use a geometric feature-based model of the object.
- Match features of the object to features in the image.
- Produce a hypothesis h (matching features)
- Compute an affine transformation T from h
- Apply T to the features of the model to map the model features to the image.
- Use a verification procedure to decide how well the model features line up with actual image features

Alignment





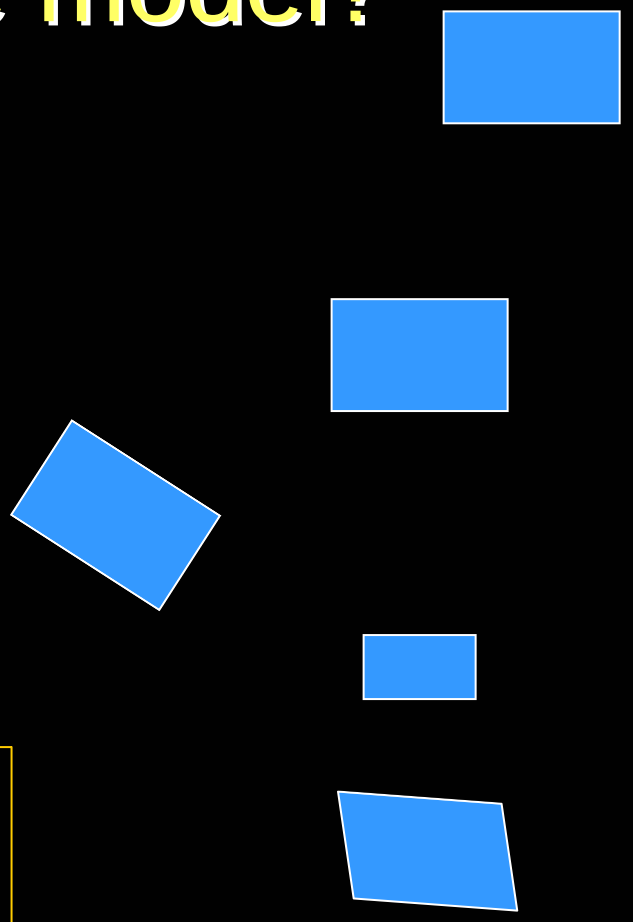
image

How can the object in the image differ from that in the model?

Most often used:
2D Affine Transformations

1. translation
2. rotation
3. scale
4. skew

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



Point Representation and Transformations

Normal Coordinates for a 2D Point

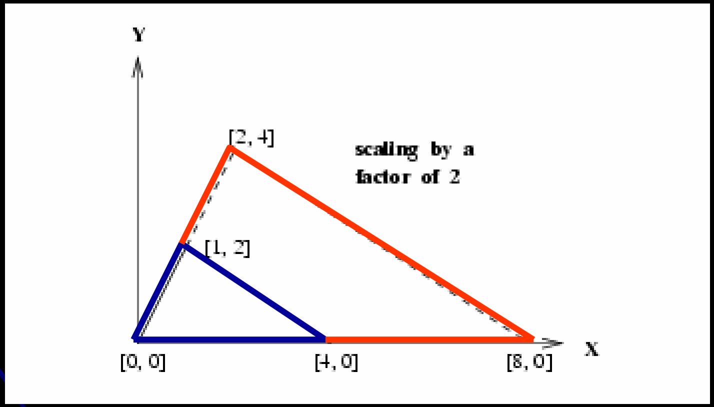
$$P = [x, y]^t = \begin{bmatrix} x \\ y \end{bmatrix}$$

Homogeneous Coordinates

$P = [sx, sy, s]^t$ where s is a scale factor

Scaling

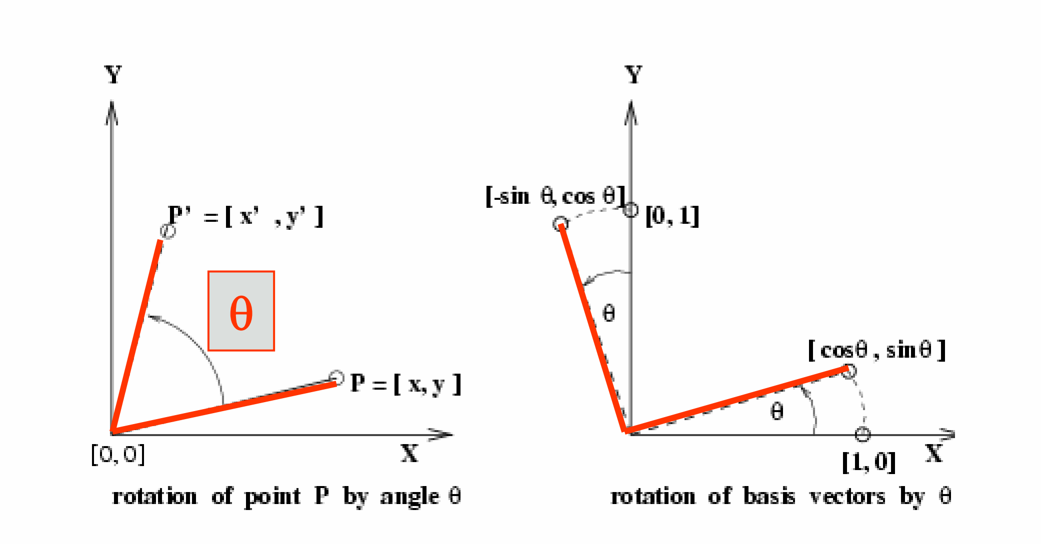
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} c_x & 0 \\ 0 & c_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_x * x \\ c_y * y \end{bmatrix}$$



scaling by a factor of 2 about (0,0)

Rotation

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \end{bmatrix}$$



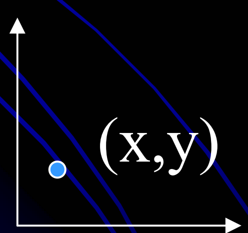
rotate point

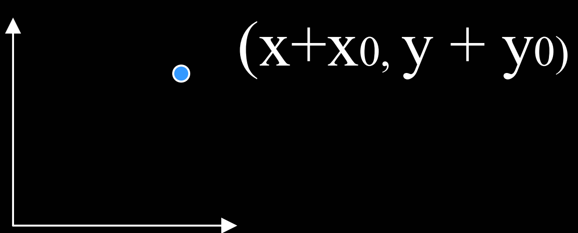
rotate axes

Translation

2 X 2 matrix doesn't work for translation!
Here's where we need homogeneous coordinates.

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x + x_0 \\ y + y_0 \\ 1 \end{pmatrix}$$

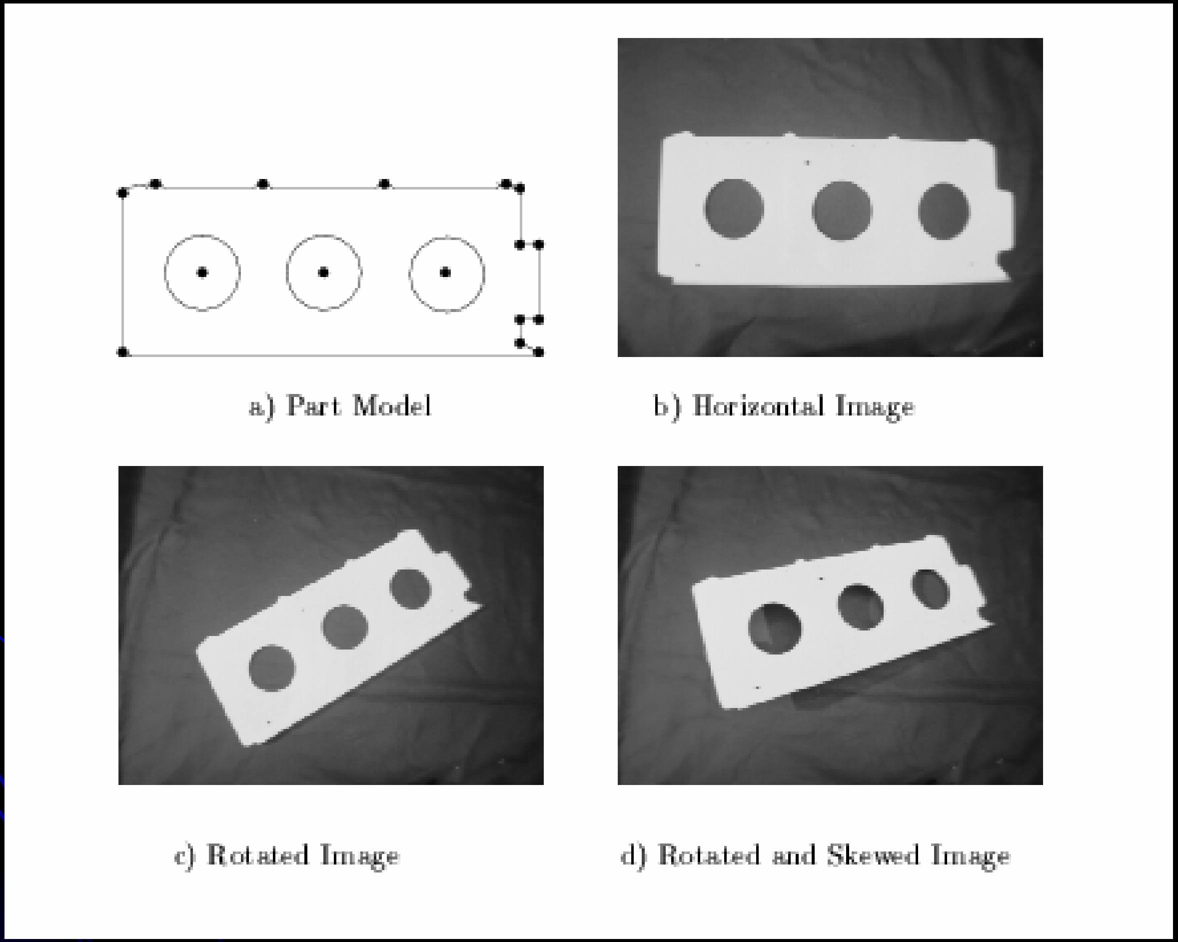




Rotation, Scaling and Translation

$$\underbrace{\begin{pmatrix} x_w \\ y_w \\ 1 \end{pmatrix} = \underset{T}{\begin{pmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{pmatrix}} \underset{S}{\begin{pmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{pmatrix}} \underset{R}{\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}}}_{T_R} \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix}$$

2D Model and 3 Matching Images of a Boeing Airplane Part



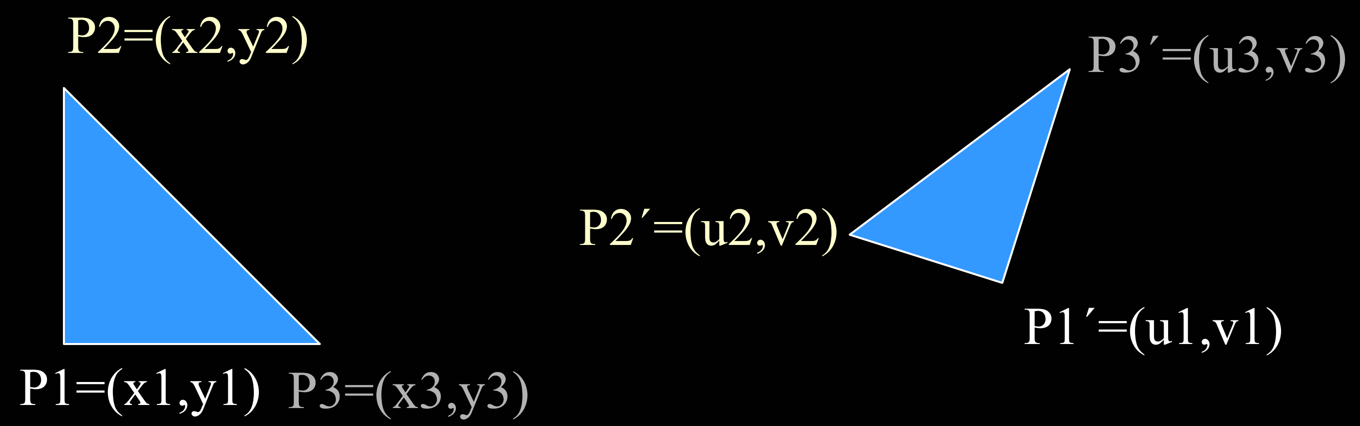
a) Part Model

b) Horizontal Image

c) Rotated Image

d) Rotated and Skewed Image

Computing Affine Transformations between Sets of Matching Points



Given 3 matching pairs of points, the affine transformation can be computed through solving a simple matrix equation.

$$\begin{pmatrix} u1 & u2 & u3 \\ v1 & v2 & v3 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x1 & x2 & x3 \\ y1 & y2 & y3 \\ 1 & 1 & 1 \end{pmatrix}$$

A More Robust Approach

Using only 3 points is dangerous, because if even one is off, the transformation can be far from correct.

Instead, use many (n =10 or more) pairs of matching control points to determine a least squares estimate of the six parameters of the affine transformation.

$$\text{Error}(a11, a12, a13, a21, a22, a23) = \sum_{j=1,n} \left((a11*xj + a12*yj + a13 - uj)^2 + (a21*xj + a22*yj + a23 - vj)^2 \right)$$

The Equations to Solve

$$\varepsilon(a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}) = \sum_{j=1}^{n}((a_{11}x_j + a_{12}y_j + a_{13} - u_j)^2 + (a_{21}x_j + a_{22}y_j + a_{23} - v_j)^2) \tag{11.16}$$

Taking the six partial derivatives of the error function with respect to each of the six variables and setting this expression to zero gives us the six equations represented in matrix form in Equation 11.17 .

$$\begin{bmatrix} \Sigma x_j{}^2 & \Sigma x_j y_j & \Sigma x_j & 0 & 0 & 0 \\ \Sigma x_j y_j & \Sigma y_j^2 & \Sigma y_j & 0 & 0 & 0 \\ \Sigma x_j & \Sigma y_j & \Sigma 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Sigma x_j{}^2 & \Sigma x_j y_j & \Sigma x_j \\ 0 & 0 & 0 & \Sigma x_j y_j & \Sigma y_j^2 & \Sigma y_j \\ 0 & 0 & 0 & \Sigma x_j & \Sigma y_j & \Sigma 1 \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{21} \\ a_{22} \\ a_{23} \end{bmatrix} = \begin{bmatrix} \Sigma u_j x_j \\ \Sigma u_j y_j \\ \Sigma u_j \\ \Sigma v_j x_j \\ \Sigma v_j y_j \\ \Sigma v_j \end{bmatrix} \tag{11.17}$$

What is this for?

Many 2D matching techniques use it.

1. Local-Feature Focus Method

2. Pose Clustering

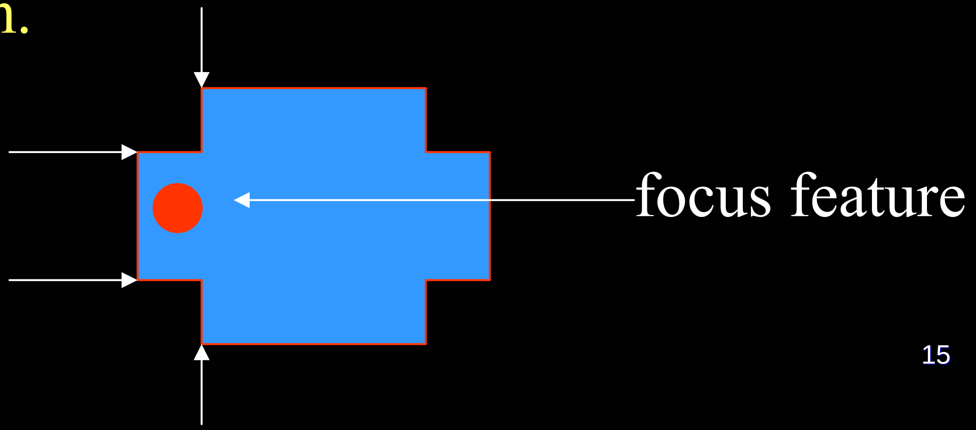
3. Geometric Hashing

Local-Feature-Focus Method

- Each model has a set of features (interesting points).

  - The focus features are the particularly detectable features, usually representing several different areas of the model.

  - Each focus feature has a set of nearby features that can be used, along with the focus feature, to compute the transformation.



LFF Algorithm

Let G be the set of detected image features.
Let Fm be focus features of the model.
Let S(f) be the nearby features for feature f.

for each focus feature Fm
  for each image feature Gi of the same type as Fm

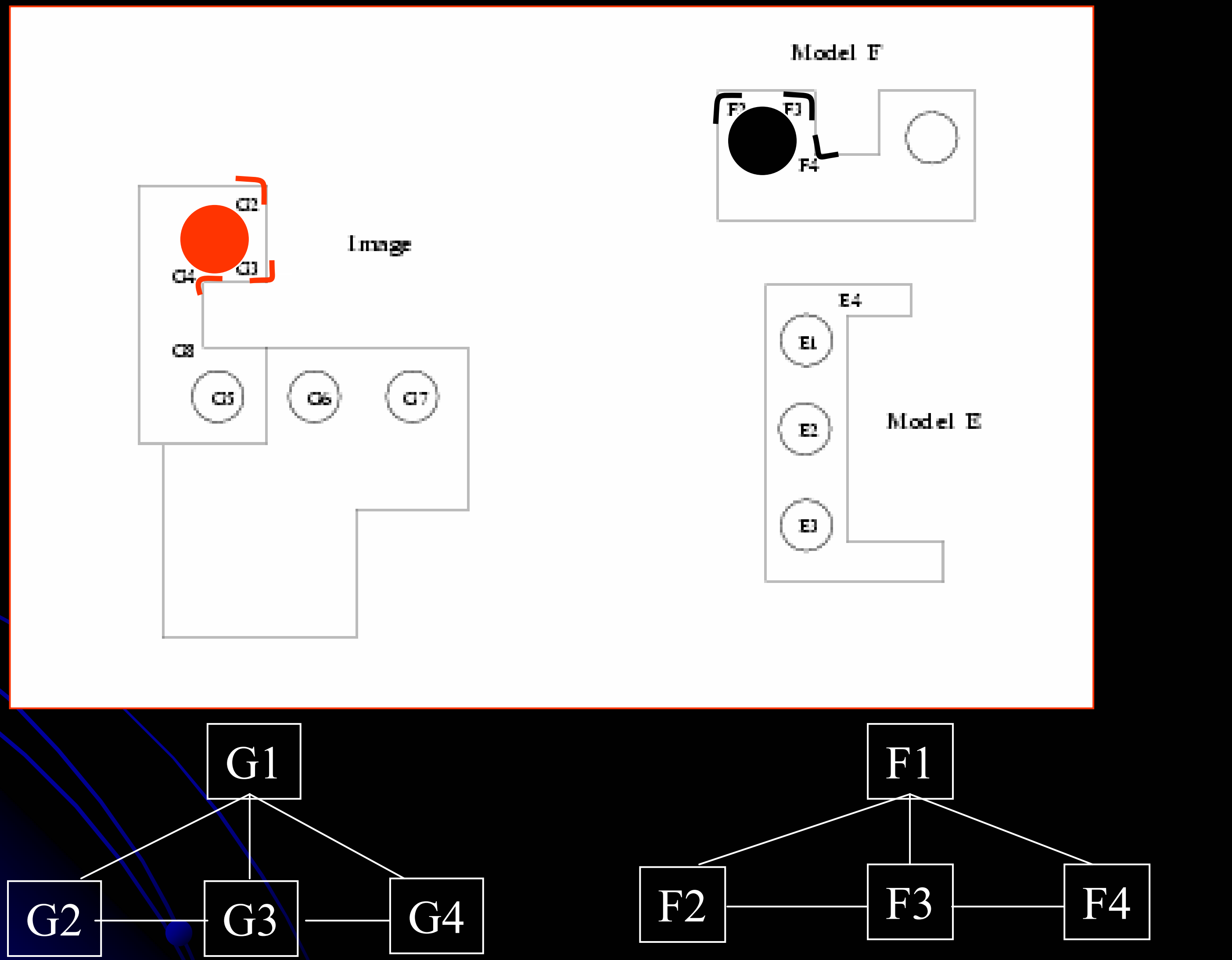
1. find the maximal subgraph Sm of S(Fm) that matches a subgraph Si of S(Gi).

2. Compute transformation T that maps the points of each feature of Sm to the corresponding one of Si.

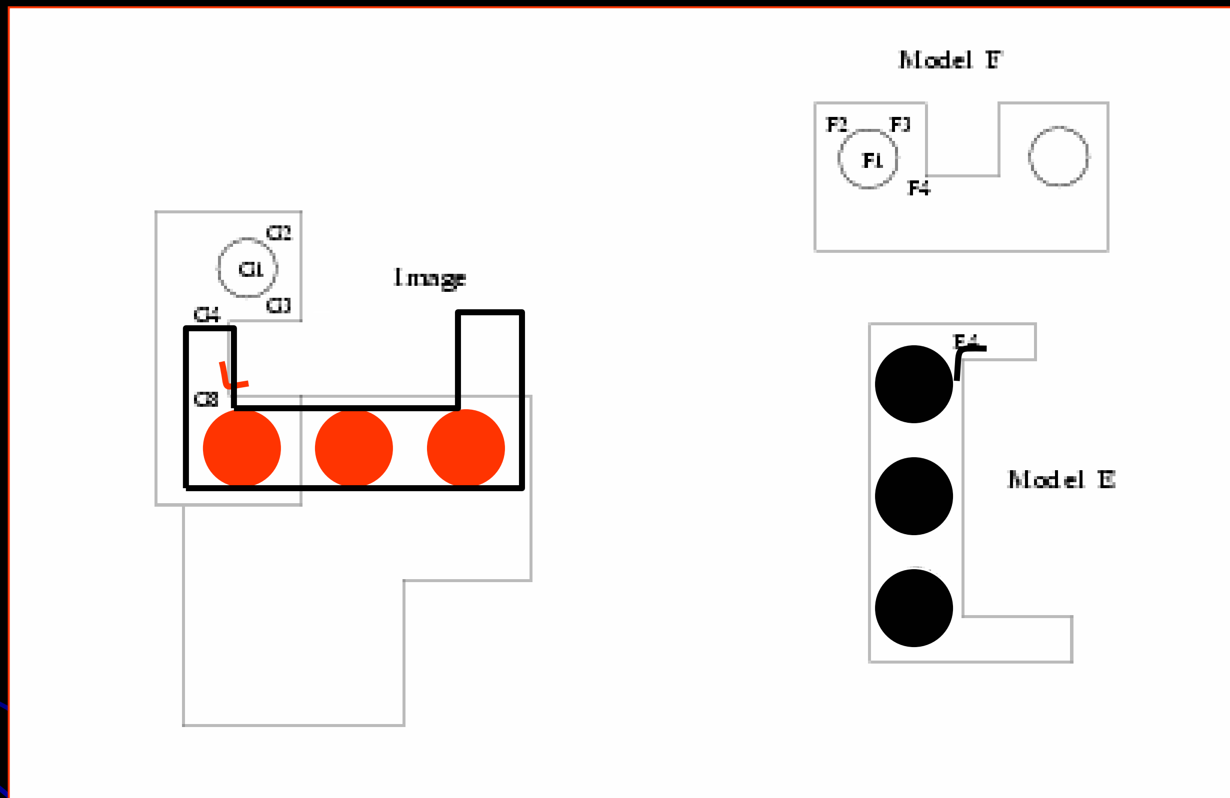
3. Apply T to the line segments of the model.

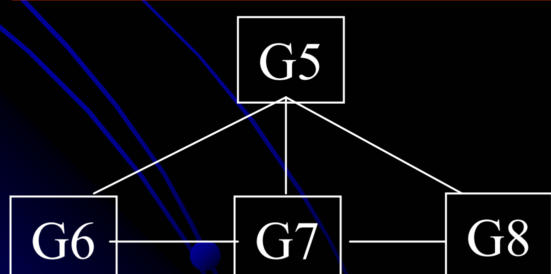
4. If enough transformed segments find evidence in the image, return(T)

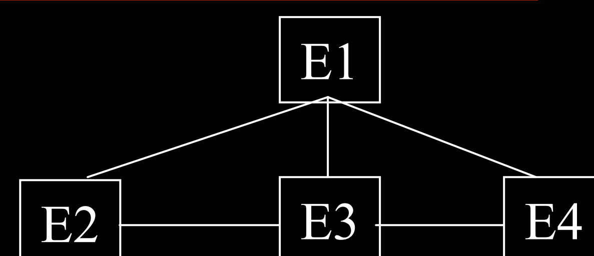
Example Match 1: Good Match



Example Match 2: Poor Match







Pose Clustering

Let T be a transformation aligning model M with image object O

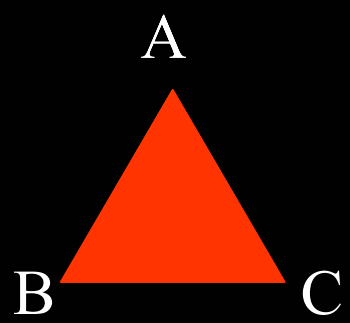
The pose of object O is its location and orientation, defined by T.

The idea of pose clustering is to compute lots of possible pose transformations, each based on 2 points from the model and 2 hypothesized corresponding points from the image.*

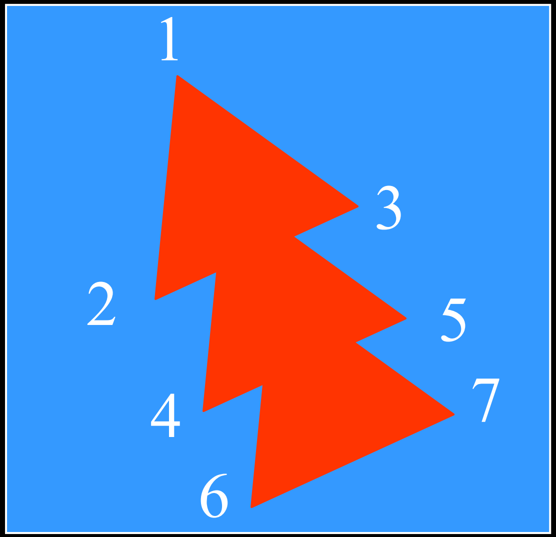
Then cluster all the transformations in pose space and try to verify the large clusters.

* This is not a full affine transformation, just RST.

Pose Clustering



Model

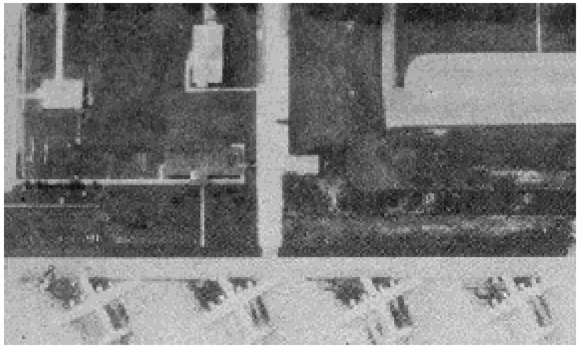


Image

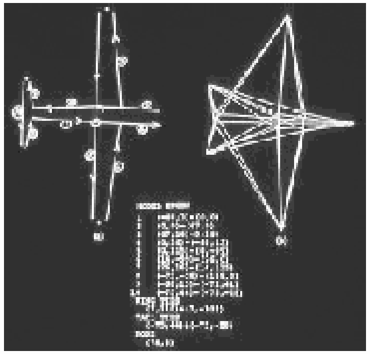
Correct Match: mapping = { (1,A), (2,B), (3,C) }

There will be some votes for (B,C) -> (4,5), (B,C) -> (6,7) etc.

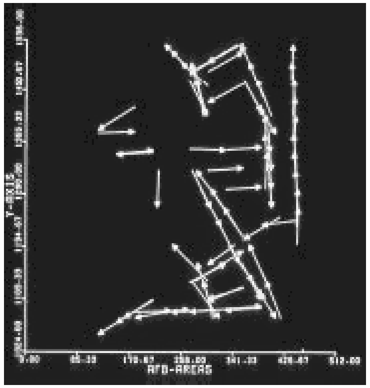
Pose Clustering Applied to Detecting a Particular Airplane



(a) original airfield image



(b) model of object



(c) detections matching model

Geometric Hashing

- This method was developed for the case where there is a whole database of models to try to find in an image.

- It trades:

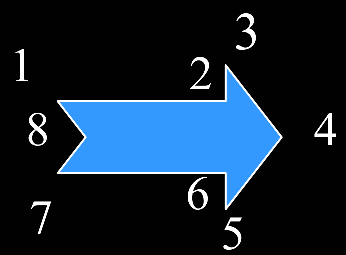
  a large amount of offline preprocessing and
  a large amount of space

- for potentially fast online

  object recognition
  pose detection

Theory Behind Geometric Hashing

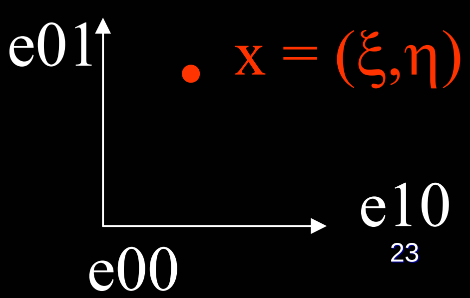
- A model M is a an ordered set of feature points.



M = <P1,P2,P3,P4,P5,P6,P7,P8>

- An affine basis is any subset E={e00,e01,e10} of noncollinear points of M.

- For basis E, any point $x \in M$ can be represented in affine coordinates (ξ,η).



$$\mathbf{x = \xi(e10 - e00) + \eta(e01 - e00) + e00}$$

Affine Transform

If x is represented in affine coordinates (ξ,η).

$$\mathbf{x = \xi(e10 - e00) + \eta(e01- e00) + e00}$$

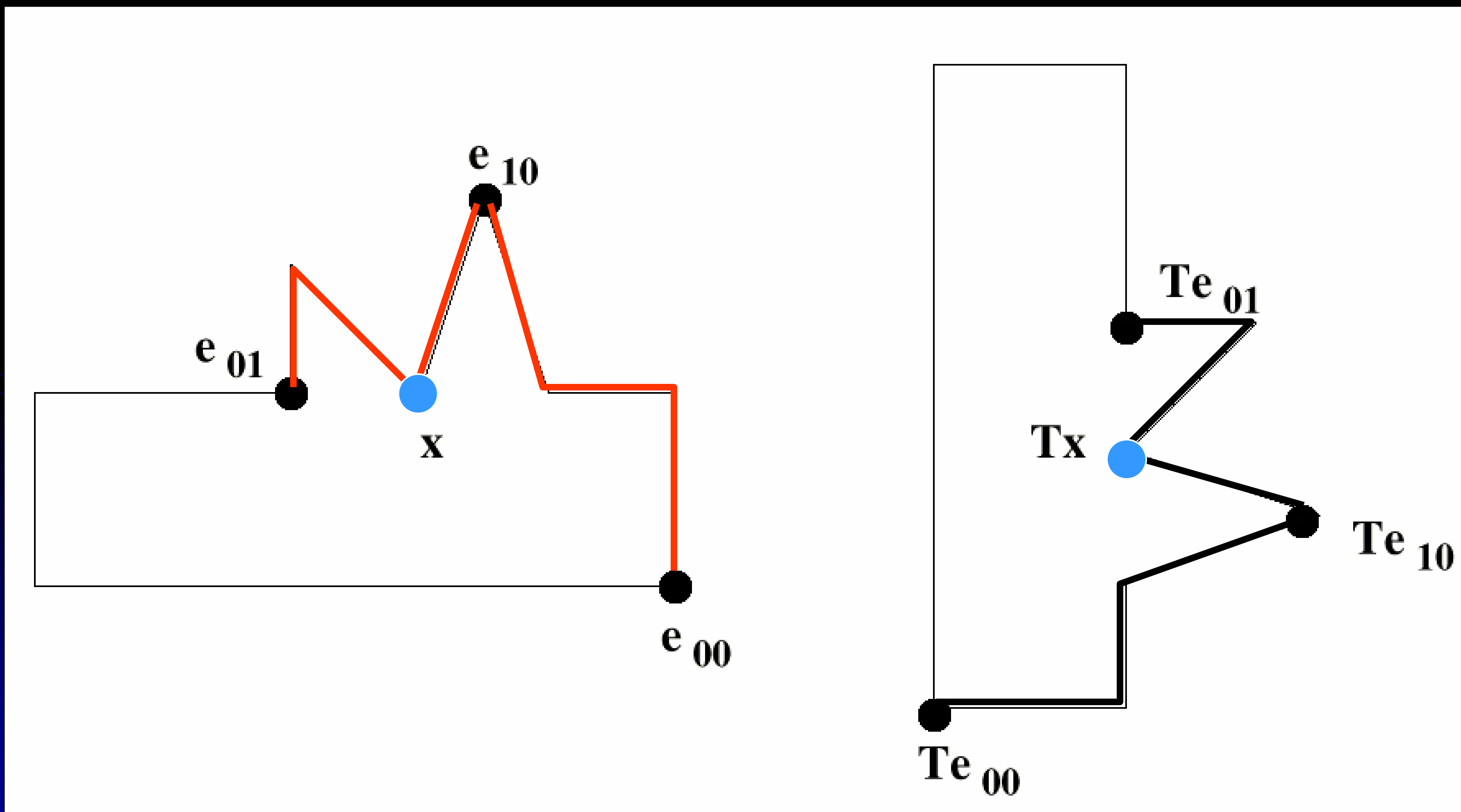
and we apply affine transform T to point x, we get

$$\mathbf{Tx = \xi(Te10 - Te00) + \eta(Te01-Te00) + Te00}$$

In both cases, x has the same coordinates (ξ,η).

Example

original object　　transformed object

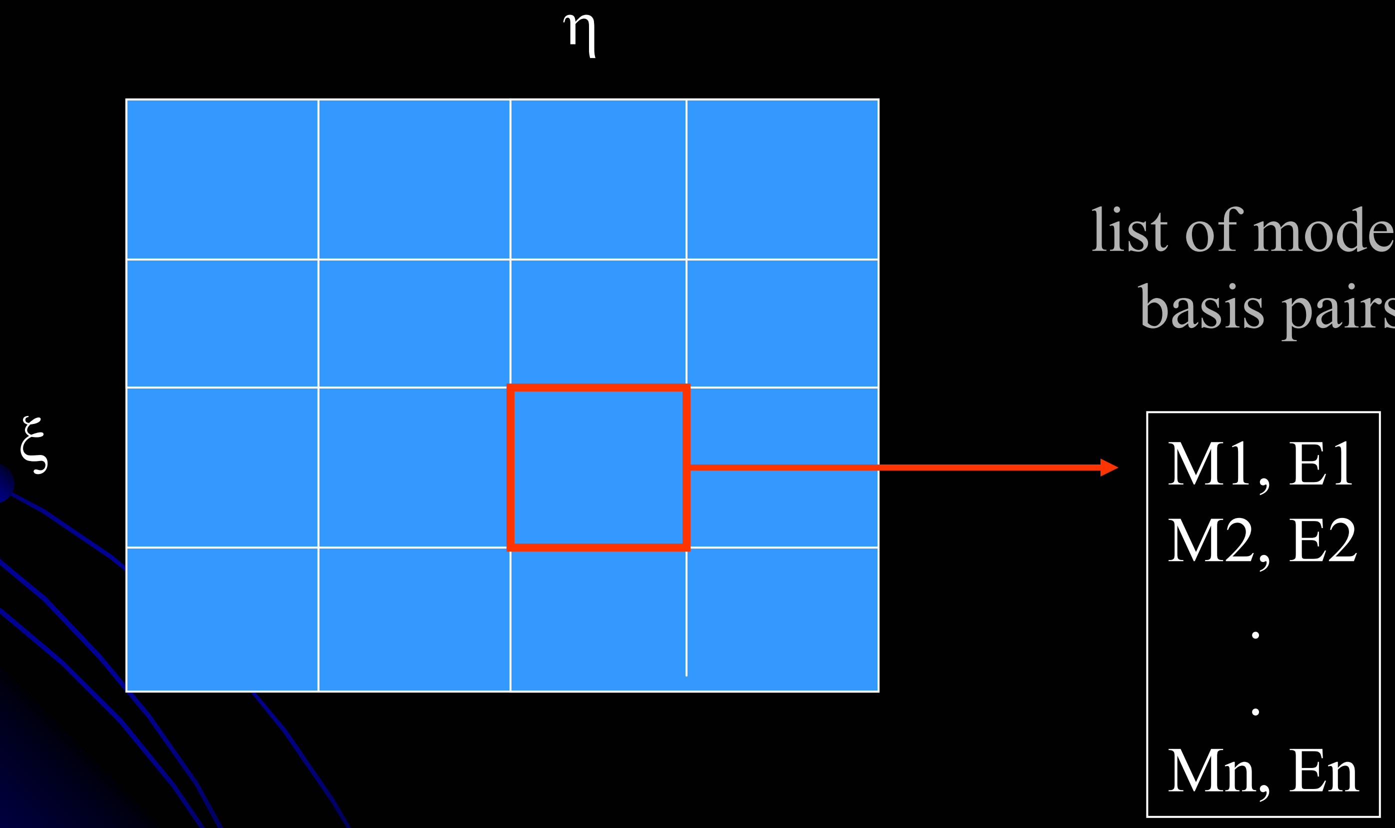


Offline Preprocessing

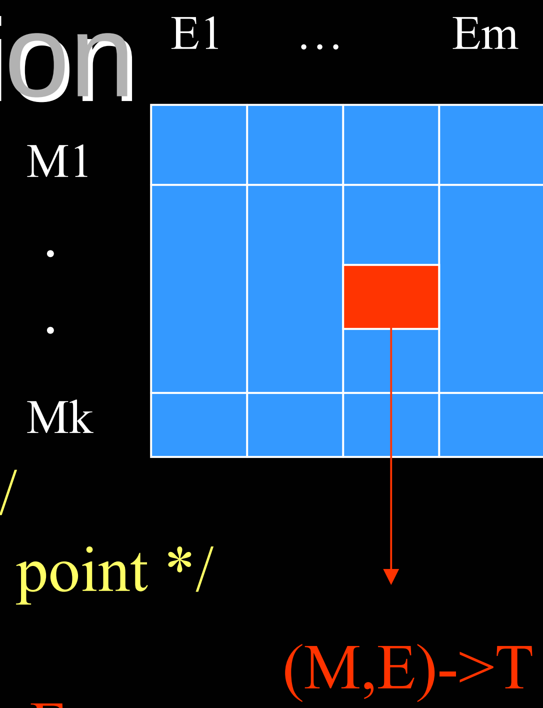
```
For each model M
  {
   Extract feature point set FM

   for each noncollinear triple E of FM  (basis)
        for each other point x of FM
            {
             calculate (ξ,η) for x with respect to E
             store (M,E) in hash table H at index (ξ,η)
             }
  }
```

Hash Table



Online Recognition




```
initialize accumulator A to all zero
extract feature points from image
for each basis triple F      /* one basis */
   for each other point v   /* each image point */
      {
       calculate (ξ,η) for v with respect to F
       retrieve list L from hash table at index (ξ,η)
       for each pair (M,E) of L
           A[M,E] = A[M,E] + 1
       }
  find peaks in accumulator array A
  for each peak (M,E) in A
      calculate and try to verify T э:  F = TE
```

Verification

How well does the transformed model line up with the image.

- compare positions of feature points
- compare full line or curve segments

Whole segments work better, allow less halucination, but there's a higher cost in execution time.

2D Matching Mechanisms

- We can formalize the recognition problem as finding a mapping from model structures to image structures.

- Then we can look at different paradigms for solving it.

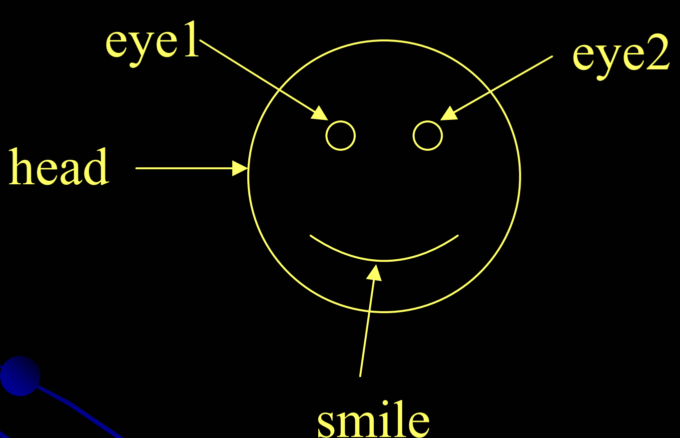
  - interpretation tree search
  - discrete relaxation
  - relational distance
  - continuous relaxation

Formalism

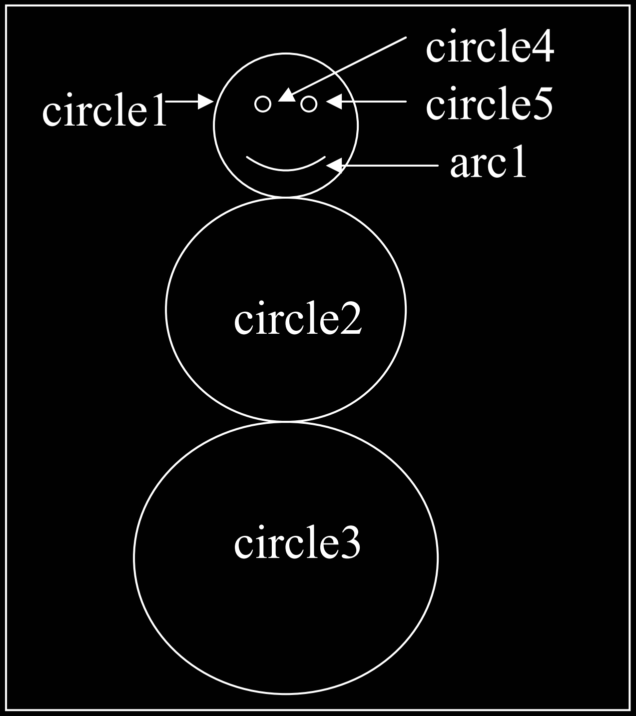
- A part (unit) is a structure in the scene, such as a region or segment or corner.
- A label is a symbol assigned to identify the part.
- An N-ary relation is a set of N-tuples defined over a set of parts or a set of labels.
- An assignment is a mapping from parts to labels.

Example

model



image



What are the relationships?

What is the best assignment
of model labels to image features?

Consistent Labeling Definition

Given:

1. a set of units P
2. a set of labels for those units L
3. a relation RP over set P
4. a relation RL over set L

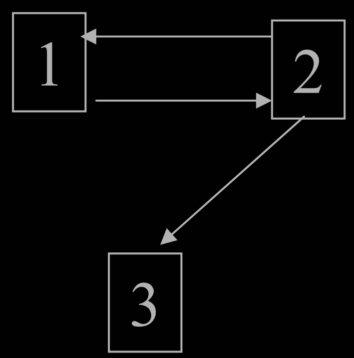
A **consistent labeling** f is a mapping **f: P -> L** satisfying

if $(pi, pj) \in RP$, then $(f(pi), f(pj)) \in RL$

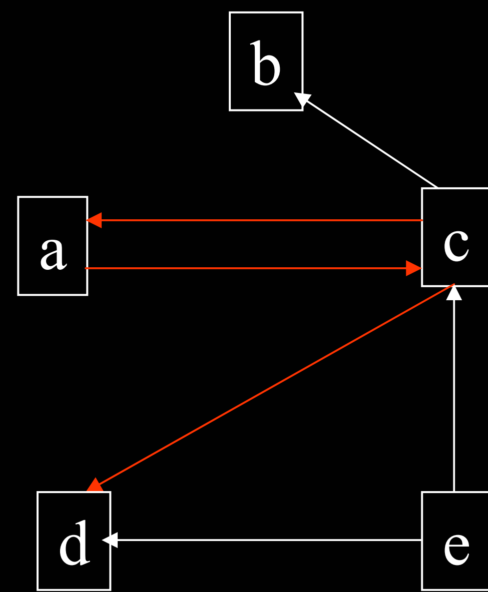
which means that a consistent labeling preserves relationships.

Abstract Example

binary relation RP



binary relation RL



P = {1,2,3}

RP={(1,2),(2,1),(2,3)}

L = {a,b,c,d,e}

RL = {(a,c),(c,a),(c,b),
(c,d),(e,c),(e,d)}

One consistent labeling is {(1,a),(2,c),(3,d)}

House Example

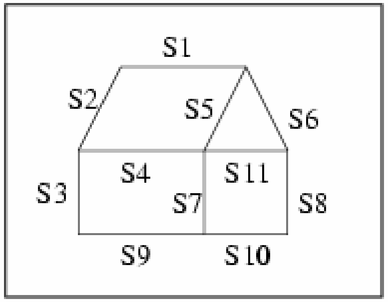


Image 1 **P**

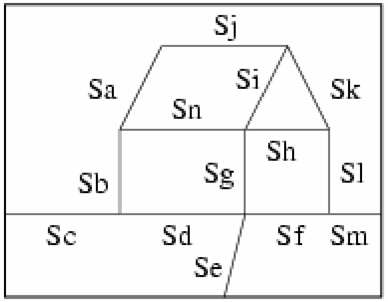


Image 2 **L**

RP and RL are connection relations.

P = {S1,S2,S3,S4,S5,S6,S7,S8,S9,S10,S11}.

L = {Sa,Sb,Sc,Sd,Se,Sf,Sg,Sh,Si,Sj,Sk,Sl,Sm}.

R_P = { (S1,S2), (S1,S5), (S1,S6), (S2,S3), (S2,S4), (S3,S4), (S3,S9), (S4,S5), (S4,S7), (S4,S11), (S5,S6), (S5,S7), (S5,S11), (S6,S8), (S6,S11), (S7,S9), (S7,S10), (S7,S11), (S8,S10), (S8,S11), (S9,S10) }.

R_L = { (Sa,Sb), (Sa,Sj), (Sa,Sn), (Sb,Sc), (Sb,Sd), (Sb,Sn), (Sc,Sd), (Sd,Se), (Sd,Sf), (Sd,Sg), (Se,Sf), (Se,Sg), (Sf,Sg), (Sf,Sl), (Sf,Sm), (Sg,Sh), (Sg,Si), (Sg,Sn), (Sh,Si), (Sh,Sk), (Sh,Sl), (Sh,Sn), (Si,Sj), (Si,Sk), (Si,Sn), (Sj,Sk), (Sk,Sl), (Sl,Sm) }.

f(S1)=Sj
f(S2)=Sa
f(S3)=Sb

f(S4)=Sn
f(S5)=Si
f(S6)=Sk

f(S7)=Sg
f(S8) = Sl
f(S9)=Sd

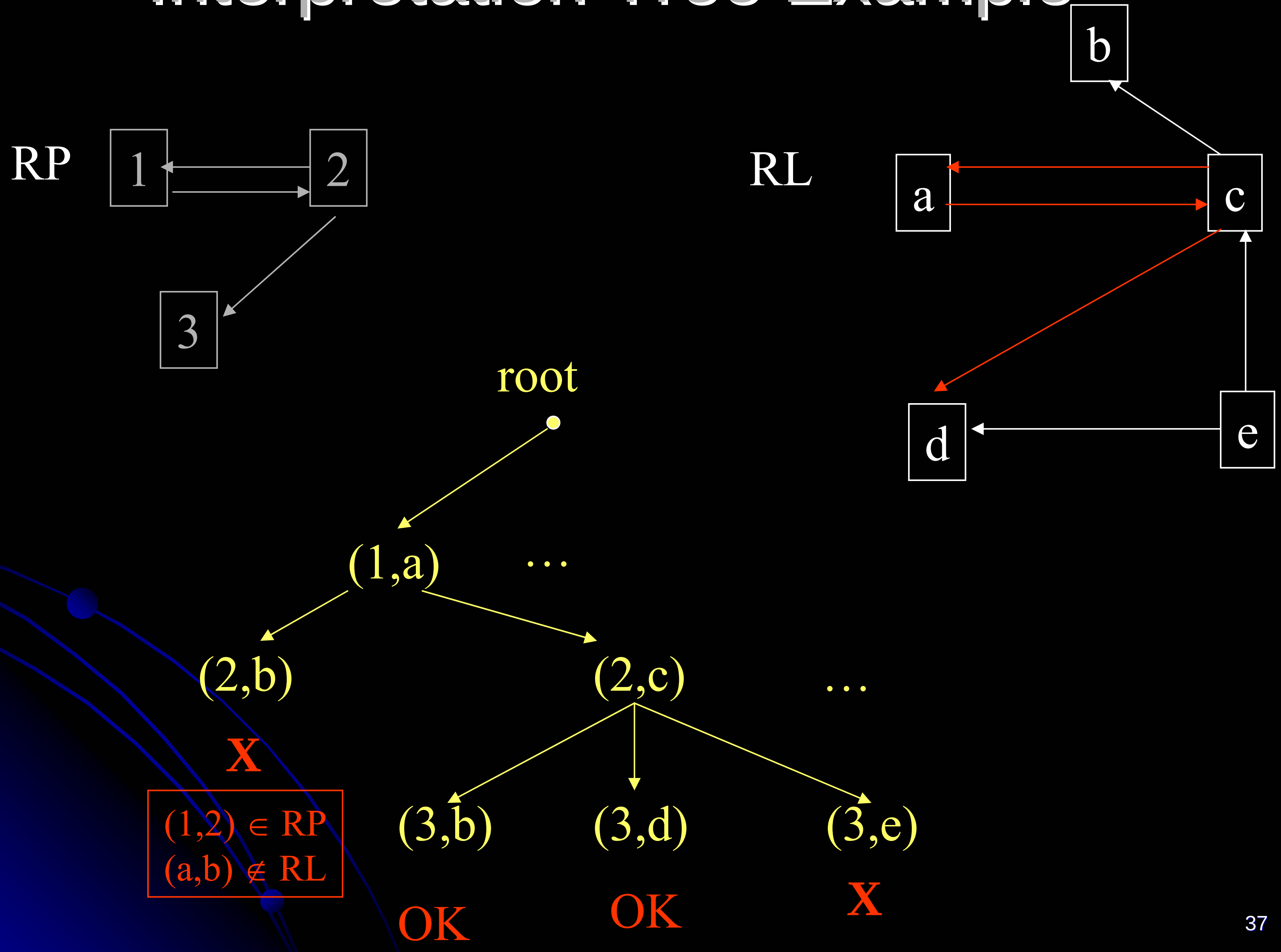
f(S10)=Sf
f(S11)=Sh

1. Interpretation Tree

- An interpretation tree is a tree that represents all assignments of labels to parts.
- Each path from the root node to a leaf represents a (partial) assignment of labels to parts.
- Every path terminates as either

  1. a complete consistent labeling
  2. a failed partial assignment

Interpretation Tree Example



2. Discrete Relaxation

- Discrete relaxation is an alternative to (or addition to) the interpretation tree search.
- Relaxation is an iterative technique with polynomial time complexity.
- Relaxation uses local constraints at each iteration.
- It can be implemented on parallel machines.

How Discrete Relaxation Works

1. Each unit is assigned a set of initial possible labels.

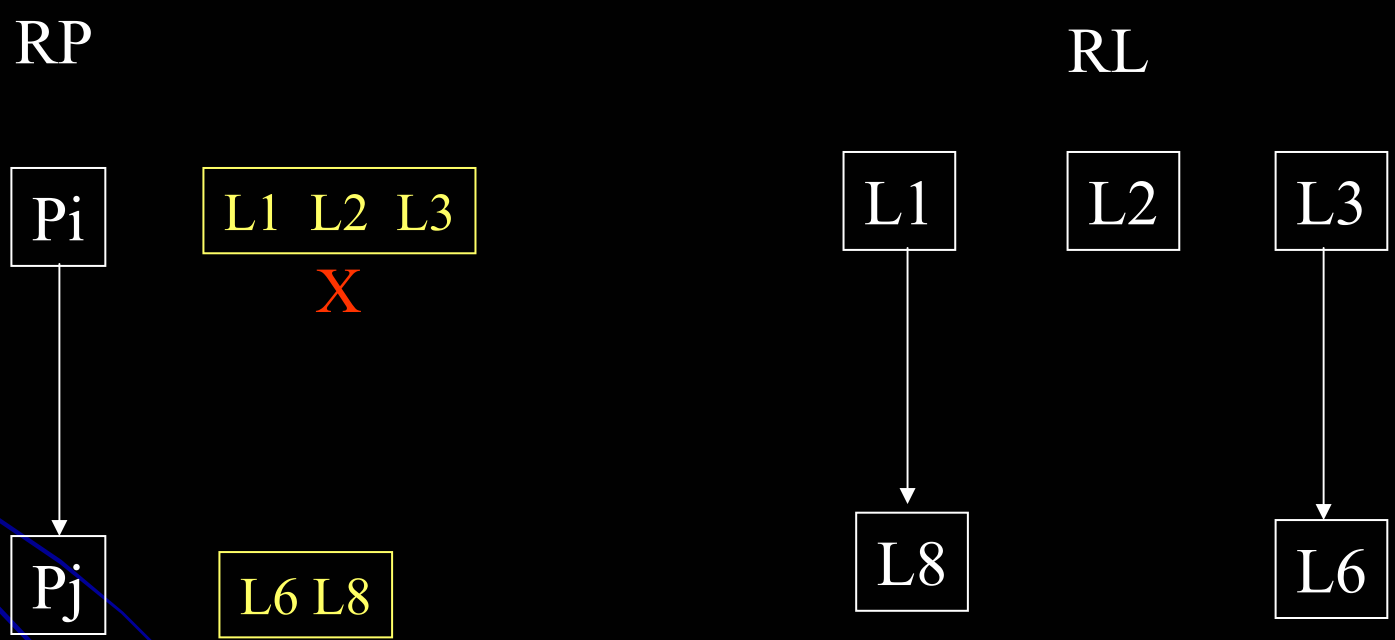
2. All relations are checked to see if some pairs of labels are impossible for certain pairs of units.

3. Inconsistent labels are removed from the label sets.

4. If any labels have been filtered out
   then another pass is executed
   else the relaxation part is done.

5. If there is more than one labeling left, a tree search can be used to find each of them.

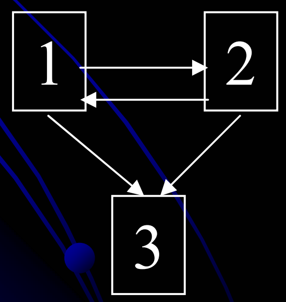
Example of Discrete Relaxation

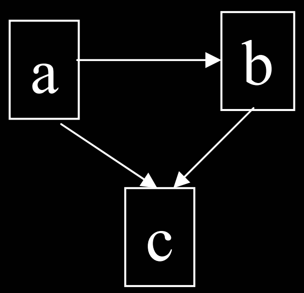


There is no label in Pj's label set that is connected to L2 in Pi's label set. L2 is inconsistent and filtered out.

3. Relational Distance Matching

- A fully consistent labeling is unrealistic.
- An image may have missing and extra features; required relationships may not always hold.
- Instead of looking for a consistent labeling, we can look for the best mapping from P to L, the one that preserves the most relationships.





Preliminary Definitions

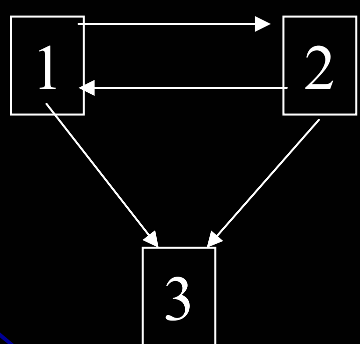
Def: A relational description DP is a sequence of relations over a set of primitives P.

- Let DA = {R1,…,RI} be a relational description over A.
- Let DB = {S1,…,SI} be a relational description over B.
- Let f be a 1-1, onto mapping from A to B.
- For any relation R, the composition R°f is given by

R°f = {(b1,…,bn) | (a1,…,an) is in R and f(ai)=(bi), i=1,n}

Example of Composition

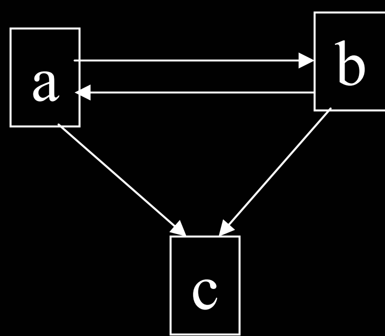
$R \circ f = \{(b1, \ldots, bn) \mid (a1, \ldots, an) \text{ is in } R \text{ and } f(ai)=(bi),\ i=1,n\}$



R

1	a
2	b
3	c

f



R°f

R°f is an isomorphic copy of R with nodes renamed by f.

Relational Distance Definition

Let DA be a relational description over set A,
DB be a relational description over set B,
and f : A -> B.

- The **structural error of f** for Ri in DA and Si in DB is

$$E_S^i(f) = | Ri \circ f - Si | + | Si \circ f^{-1} - Ri |$$

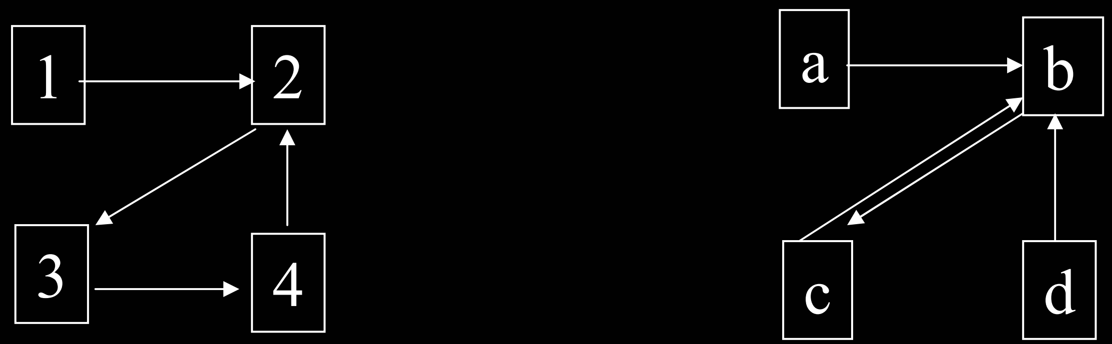
- The **total error of f** with respect to DA and DB is

$$E(f) = \sum_{i=1}^{I} E_S^i(f)$$

- The **relational distance** GD(DA,DB) is given by

$$GD(DA,DB) = \min_{f : A \rightarrow B,\ f \text{ 1-1 and onto}} E(f)$$

Example



What is the best mapping?

What is the error of the best mapping?

Example
Let f = {(1,a),(2,b),(3,c),(4,d)}

$| R \circ f - S | = |\{(a,b)(b,c)(c,d)(d,b)\} - \{(a,b)(b,c)(c,b)(d,b)\}|$
$= |\{(c,d)\}| = 1$

$|S \circ f^{-1} - R| = |\{(1,2)(2,3)(3,2)(4,2)\} - \{(1,2)(2,3)(3,4)(4,2)\}|$
$= |\{(3,2)\}| = 1$

$E(f) = 1+1 = 2$

Is there a better mapping?

Variations

- Different weights on different relations
- Normalize error by dividing by total possible
- Attributed relational distance for attributed relations
- Penalizing for NIL mappings

Implementation

- Relational distance requires finding the lowest cost mapping from object features to image features.
- It is typically done using a branch and bound tree search.
- The search keeps track of the error after each object part is assigned an image feature.
- When the error becomes higher than the best mapping found so far, it backs up.
- It can also use discrete relaxation or forward checking (see Russel AI book on Constraint Satisfaction) to prune the search.

4. Continuous Relaxation

- In discrete relaxation, a label for a unit is either possible or not.
- In continuous relaxation, each (unit, label) pair has a probability.
- Every label for unit i has a prior probability.
- A set of compatibility coefficients $C = \{c_{ij}\}$ gives the influence that the label of unit i has on the label of unit j.
- The relationship R is replaced by a set of unit/label compatibilities where $r_{ij}(l,l')$ is the compatibility of label l for part i with label l' for part j.
- An iterative process updates the probability of each label for each unit in terms of its previous probability and the compatibilities of its current labels and those of other units that influence it.

Continuous Relaxation Updates

$$pr_i^0(l) = pr_i(l)$$

Initialize probability of label l for part i to the *a priori* probability.

$$q_i^k(l) = \sum_{\{j \mid (i,j) \in R_P\}} c_{ij} [\sum_{l' \in L_j} r_{ij}(l, l') pr_j^k(l')]$$

At step k, compute a multiplicative factor based on looking at every other part j, how much i and j constrain one another, the possible labels for part j, their current probabilities and their compatability with label l.

$$pr_i^{k+1}(l) = \frac{pr_i^k(l)(1 + q_i^k(l))}{\sum_{l' \in L_i} pr_i^k(l')(1 + q_i^k(l'))}$$

Recognition by Appearance

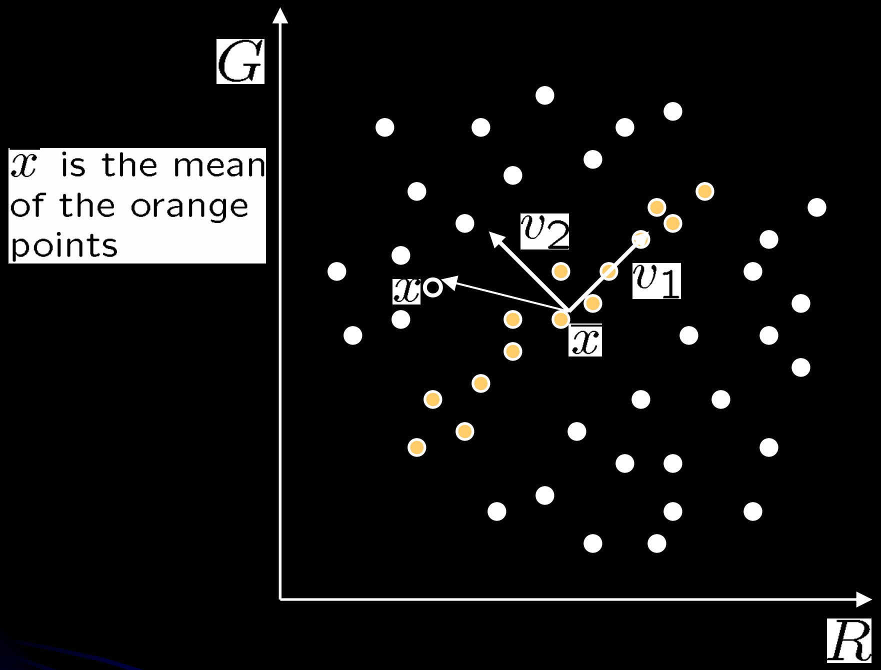
- Appearance-based recognition is a competing paradigm to features and alignment.

- No features are extracted.

- Images are represented by basis functions (eigenvectors) and their coefficients.

- Matching is performed on this compressed image representation.

Eigenvectors and Eigenvalues



Consider the sum squared distance of a point **x** to all of the orange points:

$$SSD(\mathbf{v}) = \sum_{\text{orange point } \mathbf{x}} \|(\mathbf{x} - \bar{\mathbf{x}})^{\mathbf{T}} \cdot \mathbf{v}\|$$

What unit vector **v** minimizes SSD?

$$\mathbf{v}_2 = min_{\mathbf{v}} \ \{SSD(\mathbf{v})\}$$

What unit vector **v** maximizes SSD?

$$\mathbf{v}_1 = max_{\mathbf{v}} \ \{SSD(\mathbf{v})\}$$

$$\begin{aligned}
SSD(\mathbf{v}) &= \sum_{\mathbf{x}} \|(\mathbf{x} - \bar{\mathbf{x}})^{\mathbf{T}} \cdot \mathbf{v}\| \\
&= \sum_{\mathbf{x}} \mathbf{v}^{\mathbf{T}}(\mathbf{x} - \bar{\mathbf{x}})(\mathbf{x} - \bar{\mathbf{x}})^{\mathbf{T}}\mathbf{v} \\
&= \mathbf{v}^{\mathbf{T}} \left[\sum_{\mathbf{x}} (\mathbf{x} - \bar{\mathbf{x}})(\mathbf{x} - \bar{\mathbf{x}})^{\mathbf{T}}\right] \mathbf{v} \\
&= \mathbf{v}^{\mathbf{T}} \mathrm{A} \mathbf{v} \quad \text{where } \mathrm{A} = \sum_{\mathbf{x}} (\mathbf{x} - \bar{\mathbf{x}})(\mathbf{x} - \bar{\mathbf{x}})^{\mathbf{T}}
\end{aligned}$$

Solution: $\mathbf{v_1}$ is eigenvector of **A** with *largest* eigenvalue
$\mathbf{v_2}$ is eigenvector of **A** with *smallest* eigenvalue

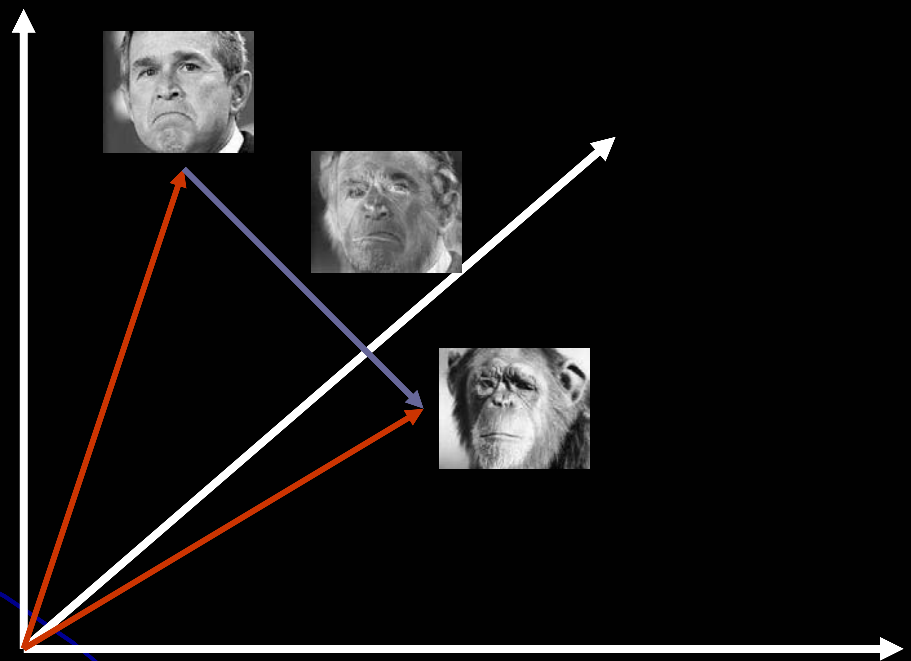
Principle component analysis

- Suppose each data point is N-dimensional
  - Same procedure applies:

$$\begin{aligned} SSD(\mathbf{v}) &= \sum_{\mathbf{x}} \|(\mathbf{x}-\overline{\mathbf{x}})^{\mathbf{T}} \cdot \mathbf{v}\| \\ &= \mathbf{v}^{\mathbf{T}}\mathbf{A}\mathbf{v} \quad \text{where } \mathbf{A} = \sum_{\mathbf{x}} (\mathbf{x}-\overline{\mathbf{x}})(\mathbf{x}-\overline{\mathbf{x}})^{\mathbf{T}} \end{aligned}$$

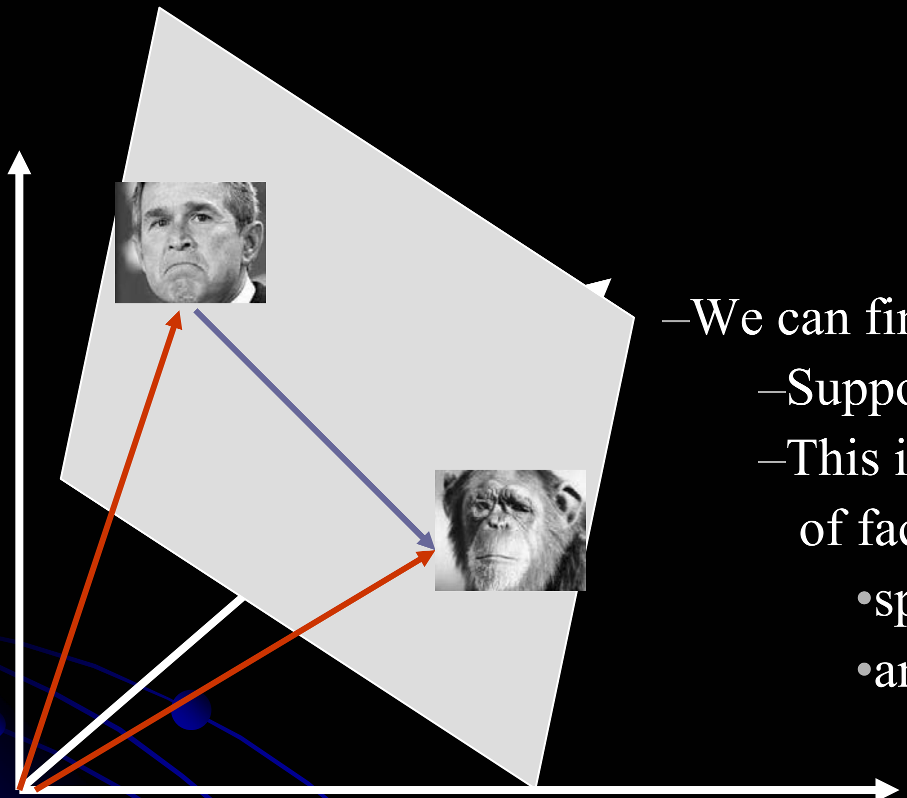
  - The eigenvectors of **A** define a new coordinate system
    - eigenvector with largest eigenvalue captures the most variation among training vectors **x**
    - eigenvector with smallest eigenvalue has least variation
  - We can compress the data by only using the top few eigenvectors

The space of faces



- An image is a point in a high-dimensional space
  - An N x M image is a point in R^{NM}
  - We can define vectors in this space

Dimensionality reduction



– We can find the best subspace using PCA
  – Suppose it is K dimensional
  – This is like fitting a "hyper-plane" to the set of faces
    - spanned by vectors $\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_K}$
    - any face $\mathbf{x} \approx a_1\mathbf{v_1} + a_2\mathbf{v_2} + , \ldots, + a_K\mathbf{v_K}$

The set of faces is a "subspace" of the set of images.

Turk and Pentland's Eigenfaces: Training

- Let F1, F2,…, FM be a set of training face images.
  Let F be their mean and $\Phi_i = F_i - F$.

- Use principal components to compute the eigenvectors and eigenvalues of the covariance matrix.

$$C = (1/M) \sum_{i=1}^{M} \Phi_i \Phi_i^T$$

- Choose the vector u of most significant M eigenvectors to use as the basis.

- Each face is represented as a linear combination of eigenfaces

u = (u1, u2, u3, u4, u5); F27 = a1*u1 + a2*u2 + … + a5*u5

Matching

unknown face image I → convert to its eigenface representation → $\Omega = (\Omega 1, \Omega 2, \ldots, \Omega m)$

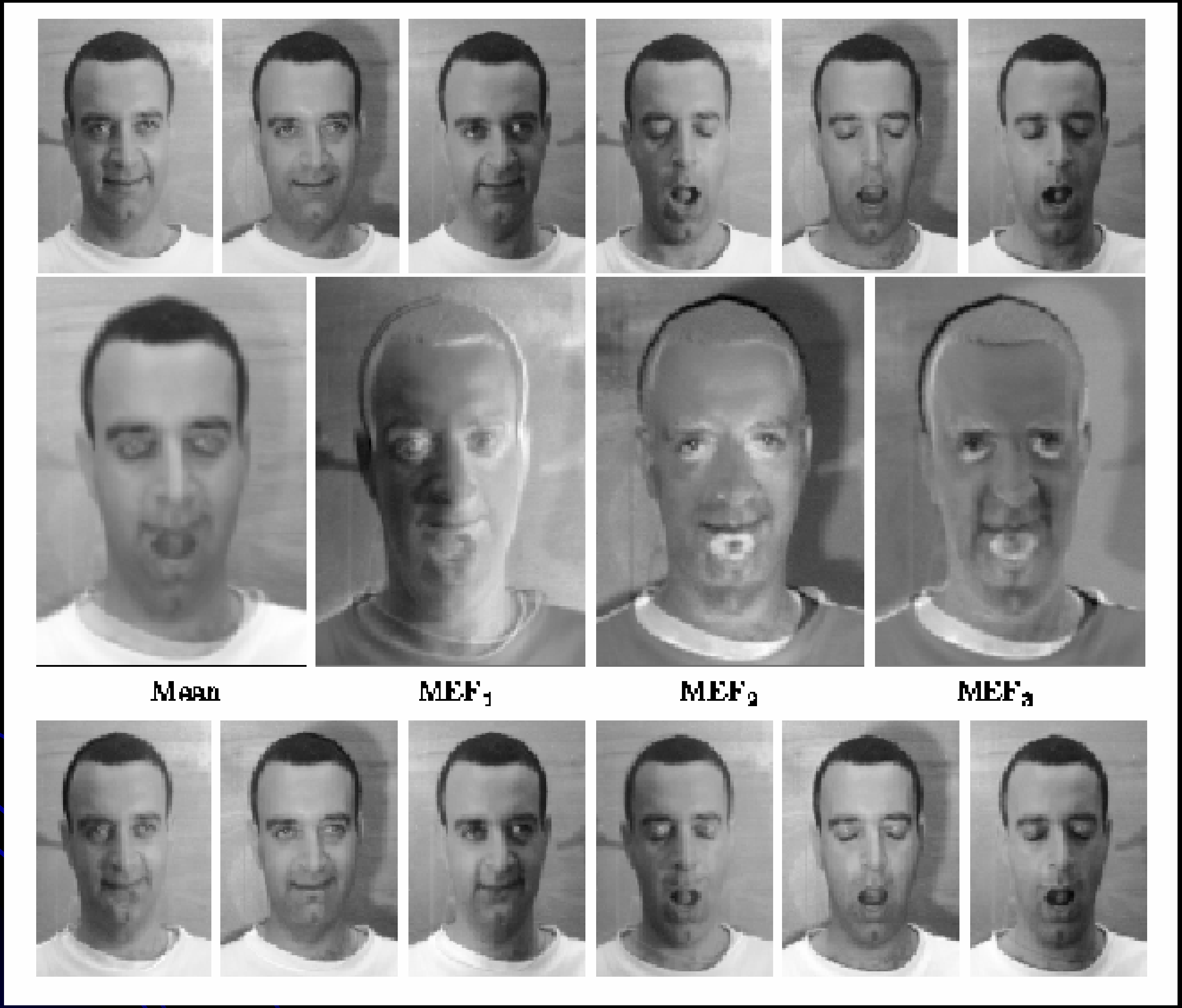
Find the face class k that minimizes

$$\varepsilon k = \| \Omega - \Omega k \|$$

training images

mean image

linear approxi-mations

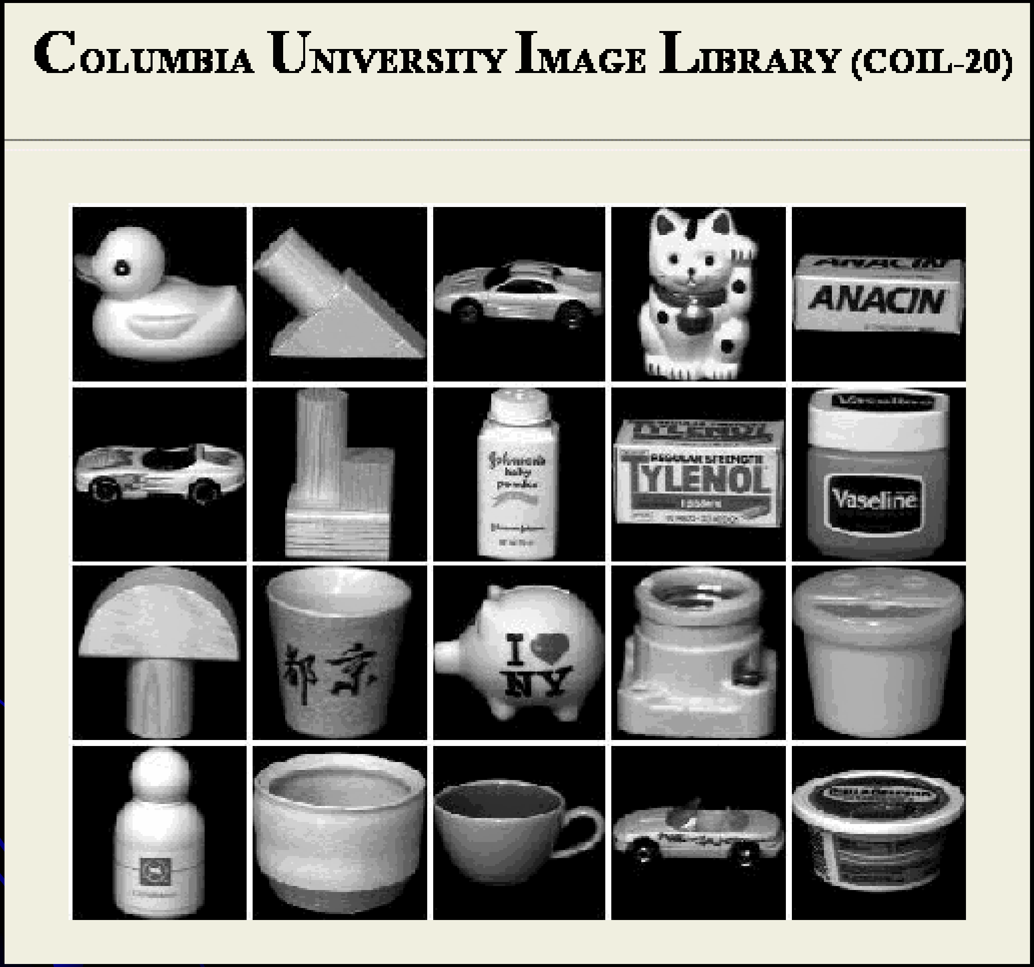


3 eigen-images

Extension to 3D Objects

- Murase and Nayar (1994, 1995) extended this idea to 3D objects.
- The training set had multiple views of each object, on a dark background.
- The views included multiple (discrete) rotations of the object on a turntable and also multiple (discrete) illuminations.
- The system could be used first to identify the object and then to determine its (approximate) pose and illumination.

Sample Objects
Columbia Object Recognition Database



Significance of this work

- The extension to 3D objects was an important contribution.
- Instead of using brute force search, the authors observed that

  All the views of a single object, when transformed into the eigenvector space became points on a manifold in that space.
- Using this, they developed fast algorithms to find the closest object manifold to an unknown input image.
- Recognition with pose finding took less than a second.

Appearance-Based Recognition

- Training images must be representative of the instances of objects to be recognized.
- The object must be well-framed.
- Positions and sizes must be controlled.
- Dimensionality reduction is needed.
- It is not powerful enough to handle general scenes without prior segmentation into relevant objects.

* • The newer systems that use “parts” from interest operators are an answer to these restrictions.

Summary

- 2D object recognition for specific objects (usually industrial) is done by alignment.

  -Affine transformations are usually powerful enough to handle objects that are mainly two-dimensional.

  -Matching can be performed by many different “graph-matching” methodoloties.

  -Verification is a necessary part of the procedure.

- Appearance-based recognition is another 2D recognition paradigm that came from the need for recognizing specific instances of a general object class, motivated by face recognition.