3D Sensing

- 3D Shape from X
- Perspective Geometry
- Camera Model
- Camera Calibration
- General Stereo Triangulation
- 3D Reconstruction

3D Shape from X

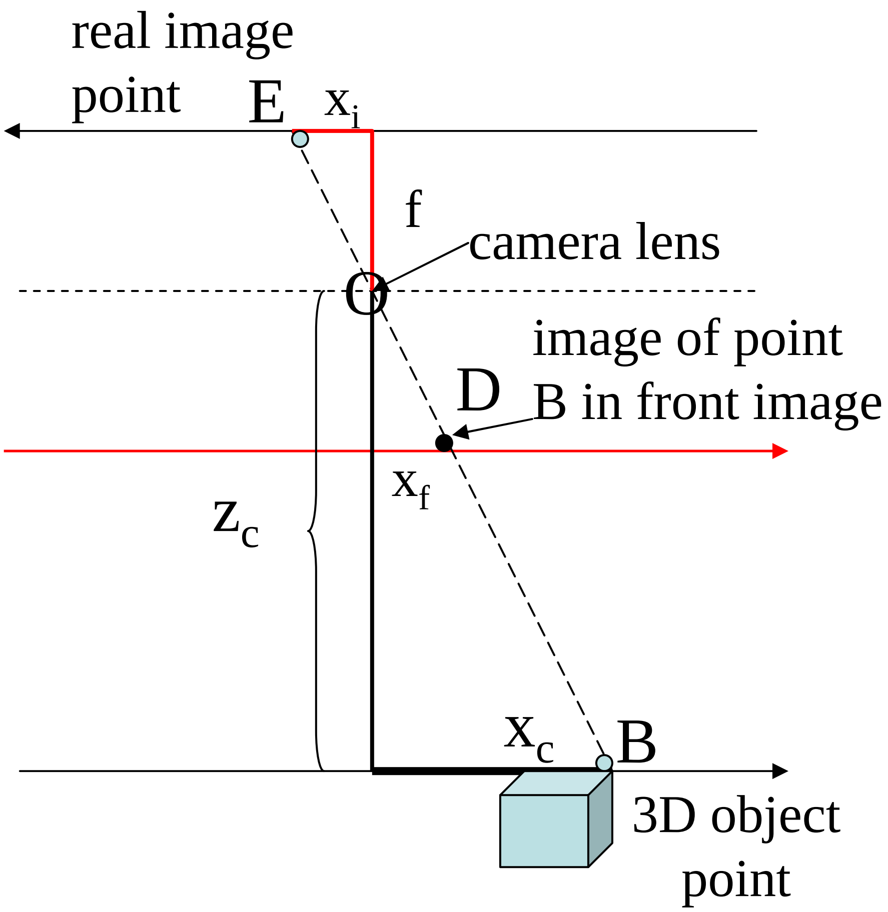
- shading
- silhouette
- texture

mainly research

- stereo
- light striping
- motion

used in practice

Perspective Imaging Model: 1D



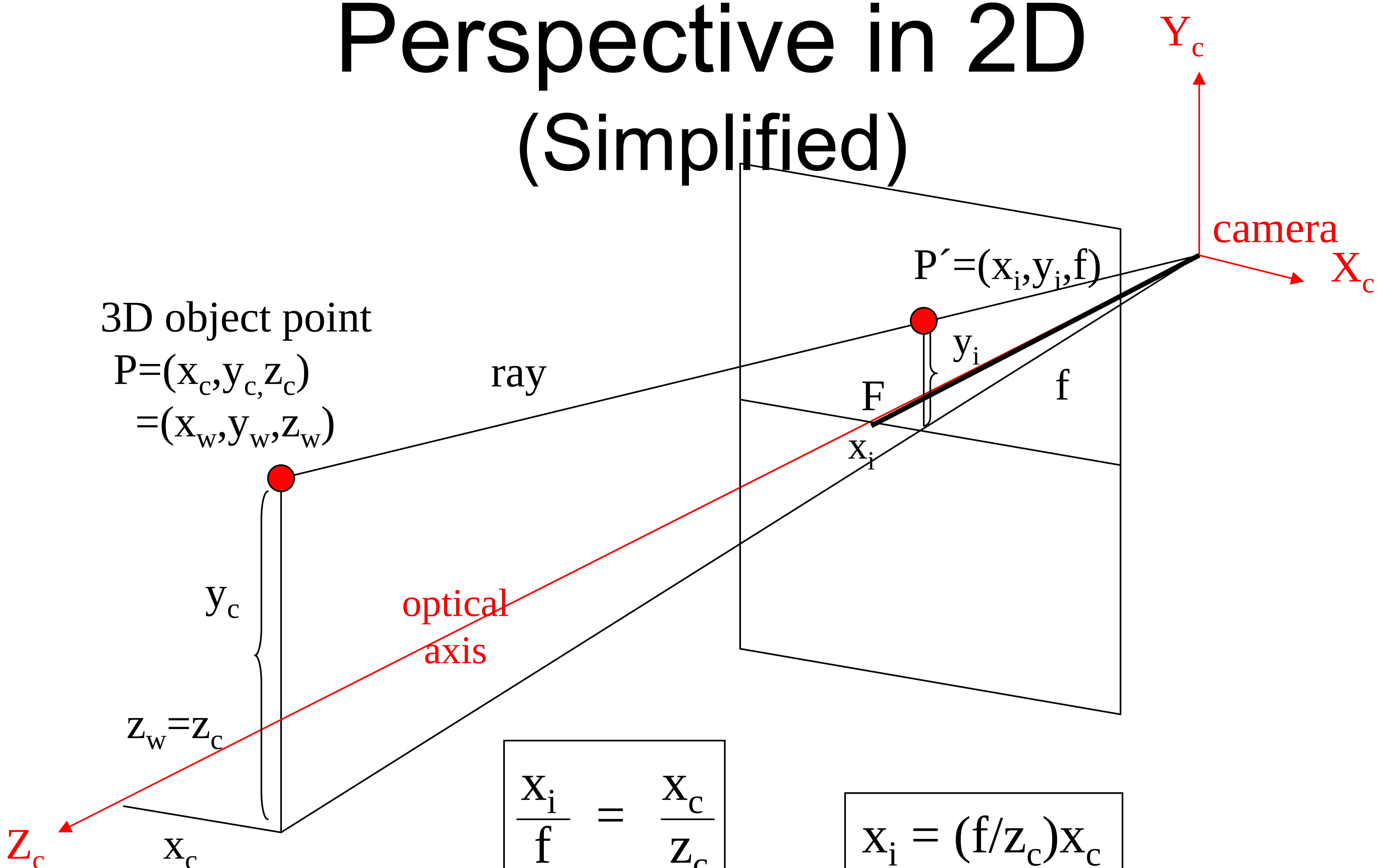
This is the axis of the real image plane.

O is the center of projection.

This is the axis of the front image plane, which we use.

$$\frac{x_i}{f} = \frac{x_c}{z_c}$$

Perspective in 2D
(Simplified)



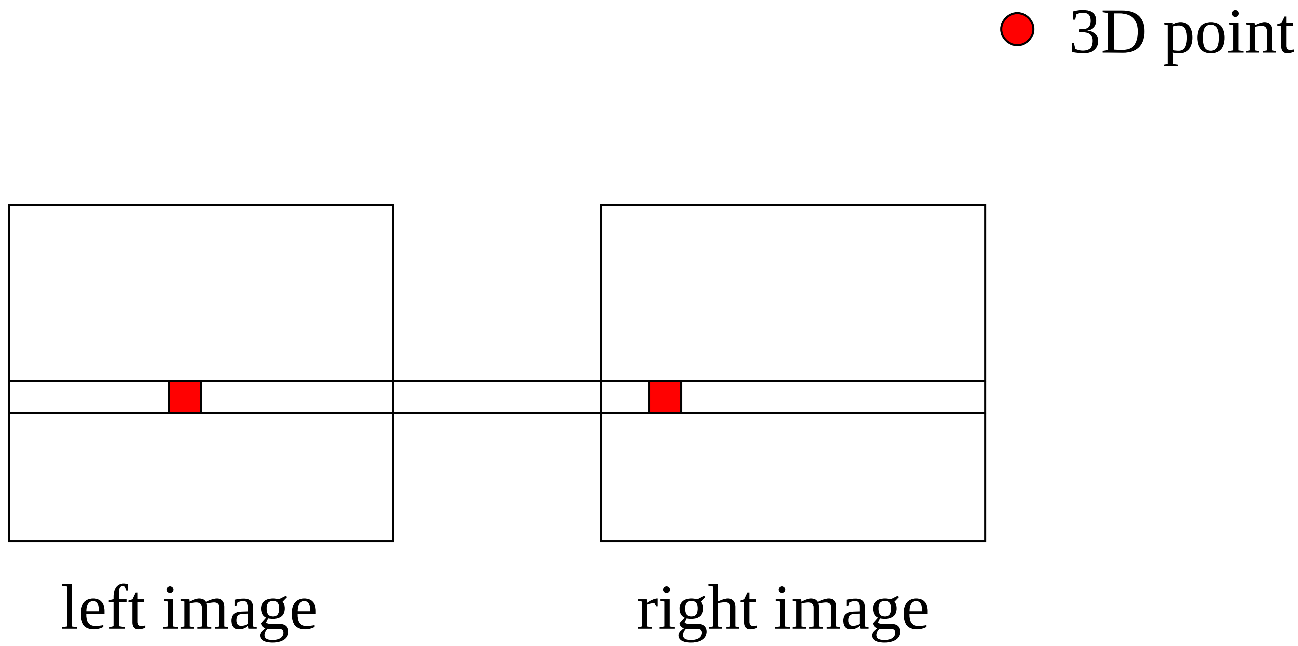
$$\frac{x_i}{f} = \frac{x_c}{z_c}$$

$$\frac{y_i}{f} = \frac{y_c}{z_c}$$

$$x_i = (f/z_c)x_c$$
$$yi = (f/z_c)y_c$$

Here camera coordinates equal world coordinates.

3D from Stereo

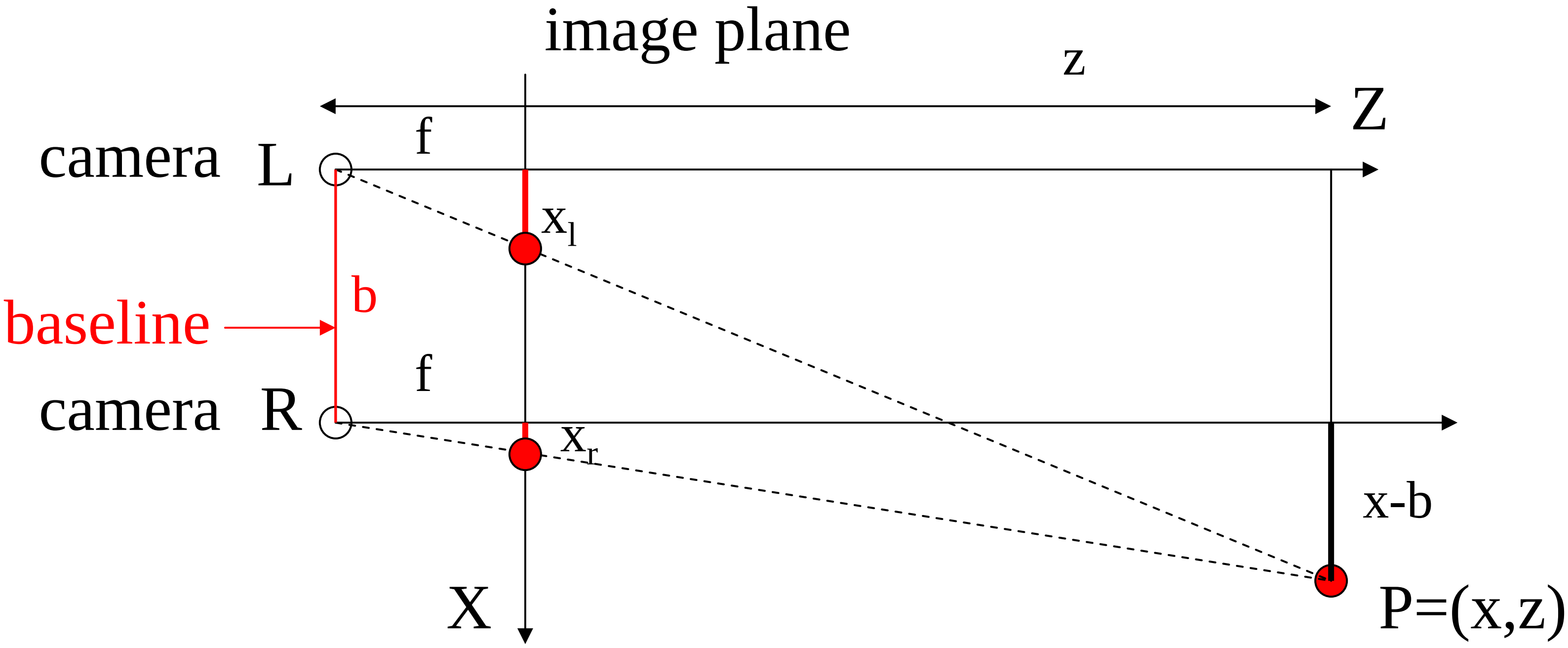


disparity: the difference in image location of the same 3D point when projected under perspective to two different cameras.

$$d = x_{left} - x_{right}$$

Depth Perception from Stereo

Simple Model: Parallel Optic Axes



$$\frac{z}{f} = \frac{x}{x_l}$$

$$\frac{z}{f} = \frac{x-b}{x_r}$$

$$\frac{z}{f} = \frac{y}{y_l} = \frac{y}{y_r}$$

y-axis is perpendicular to the page.

Resultant Depth Calculation

For stereo cameras with parallel optical axes, focal length f, baseline b, corresponding image points (x_l,y_l) and (x_r,y_r) with disparity d:

$$z = f*b / (x_l - x_r) = f*b/d$$

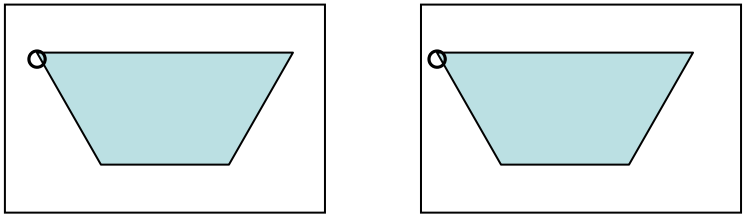
$$x = x_l*z/f \quad \text{or} \quad b + x_r*z/f$$

$$y = y_l*z/f \quad \text{or} \quad y_r*z/f$$

This method of determining depth from disparity is called **triangulation**.

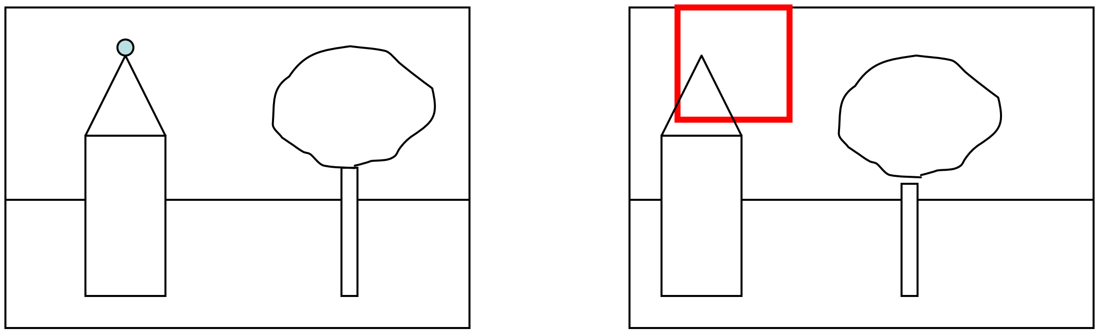
Finding Correspondences

- If the correspondence is correct, triangulation works **VERY** well.
- But correspondence finding is not perfectly solved. (What methods have we studied?)
- For some very specific applications, it can be solved for those specific kind of images, e.g. windshield of a car.



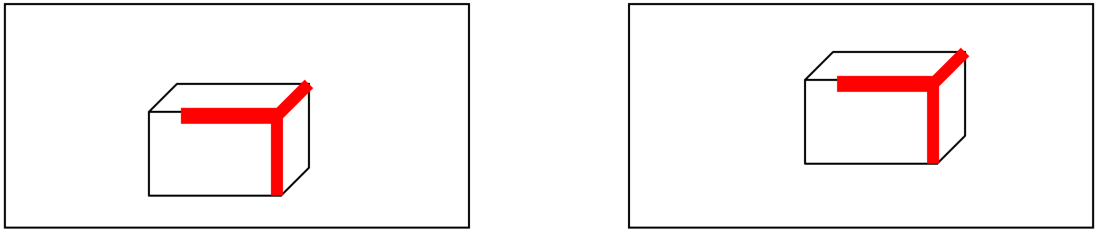
3 Main Matching Methods

1. Cross correlation using small windows.



dense

2. Symbolic feature matching, usually using segments/corners.



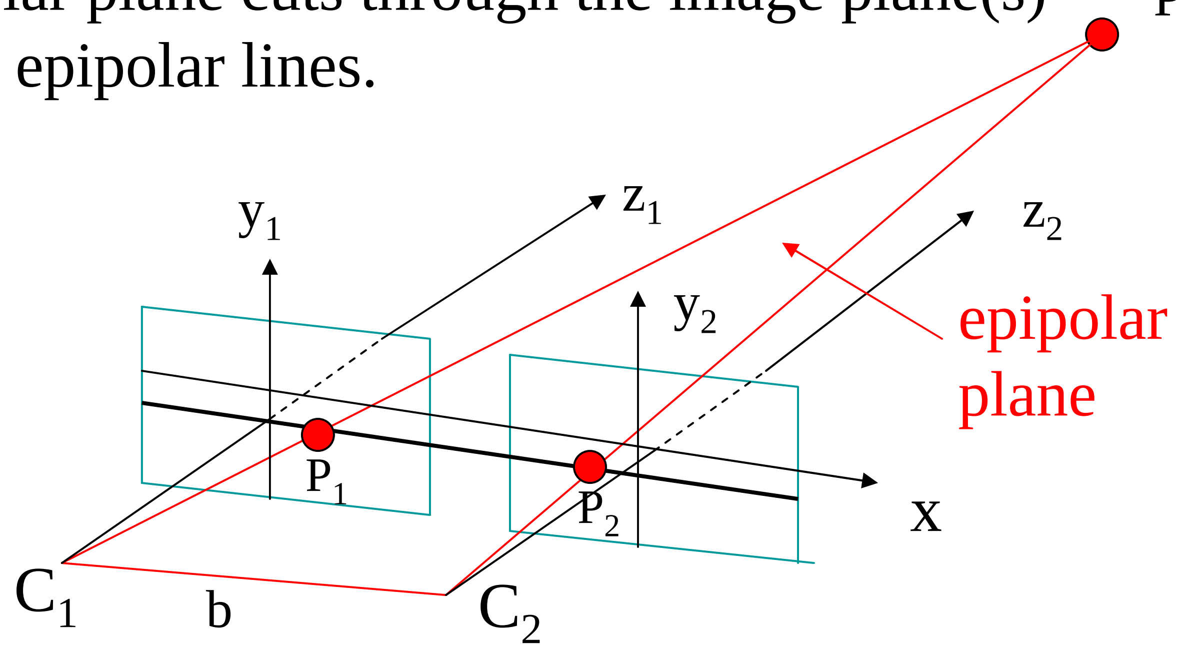
sparse

3. Use the newer interest operators, ie. SIFT.

sparse

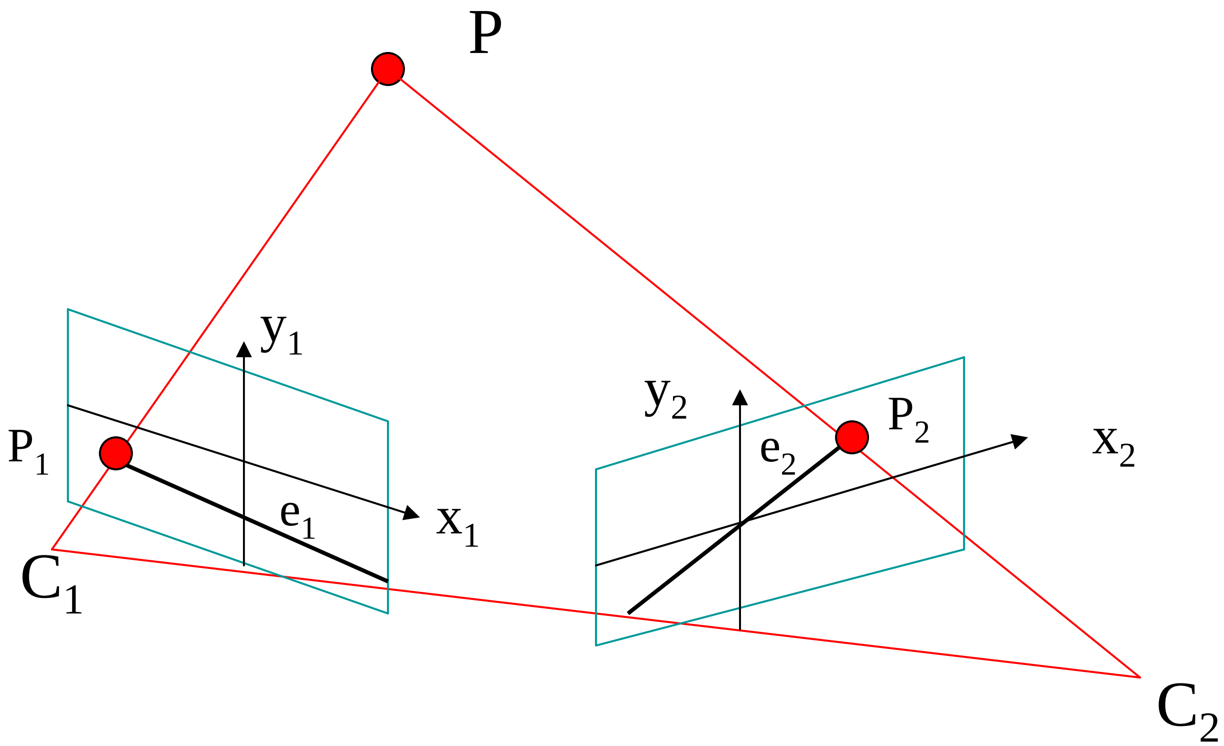
Epipolar Geometry Constraint: 1. Normal Pair of Images

The epipolar plane cuts through the image plane(s) forming 2 epipolar lines.



The match for P_1 (or P_2) in the other image, must lie on the same epipolar line.

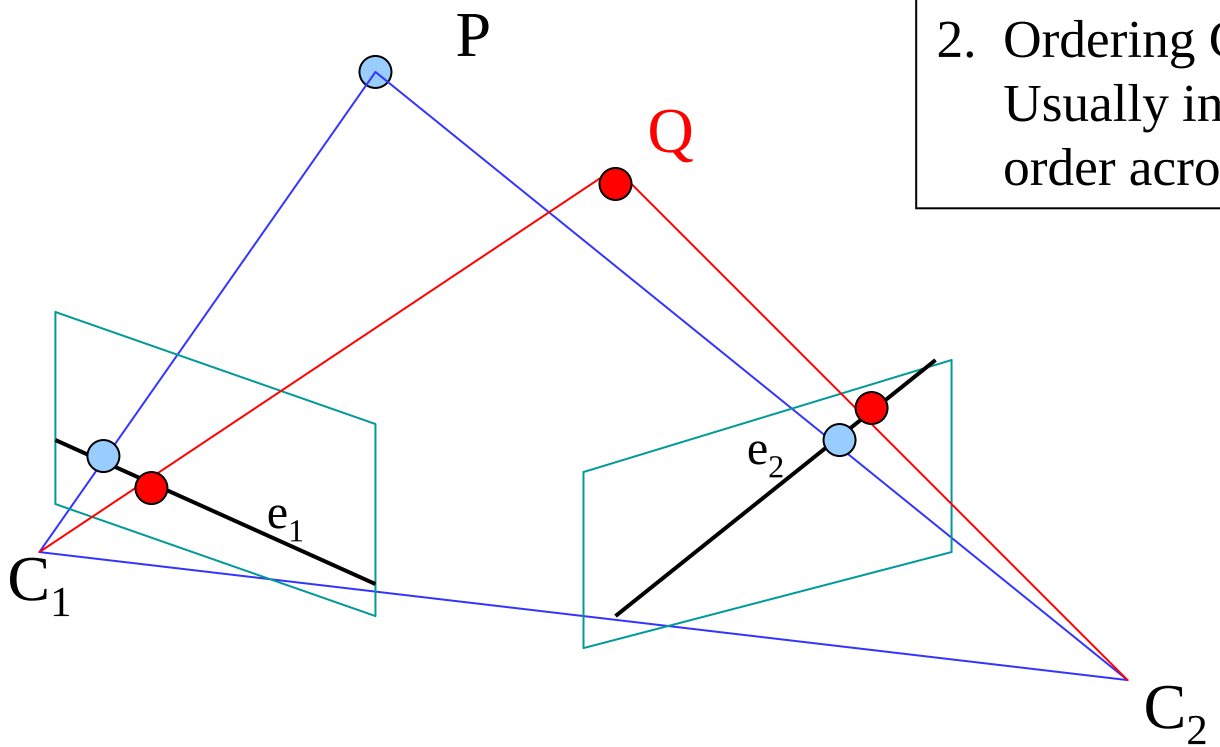
Epipolar Geometry: General Case



Constraints

1. Epipolar Constraint: Matching points lie on corresponding epipolar lines.

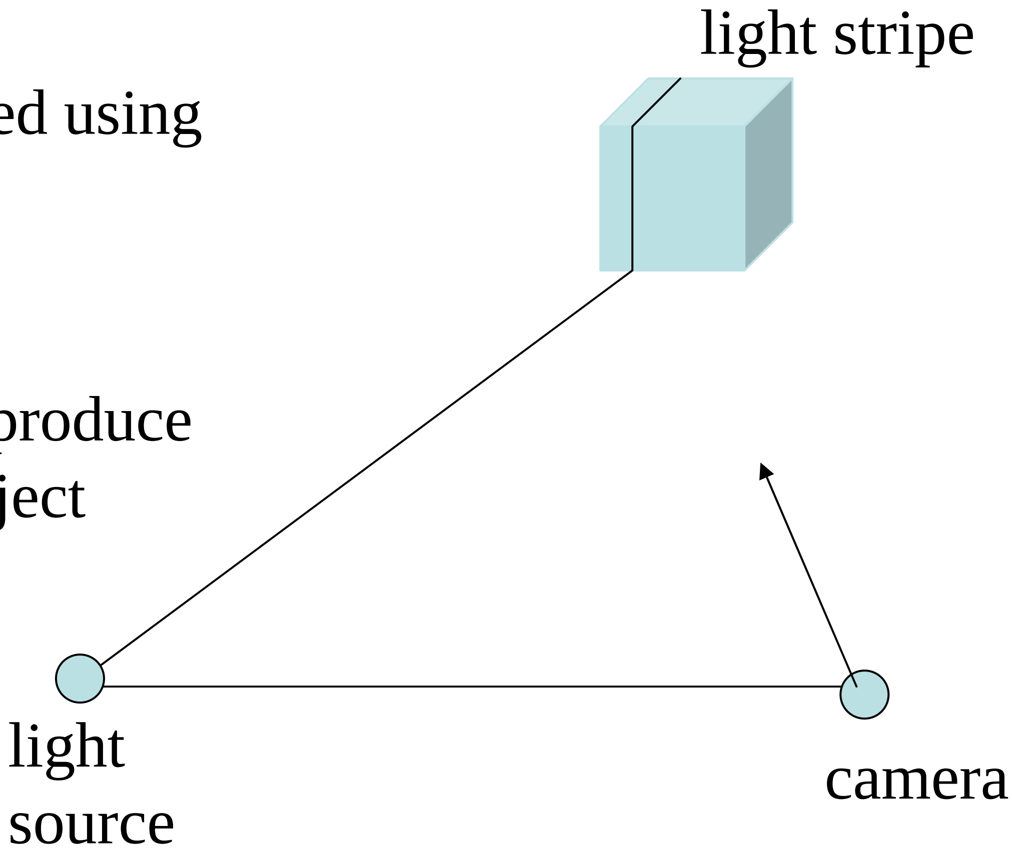
2. Ordering Constraint: Usually in the same order across the lines.



Structured Light

3D data can also be derived using

- a single camera
- a light source that can produce stripe(s) on the 3D object



Structured Light
3D Computation

3D data can also be derived using

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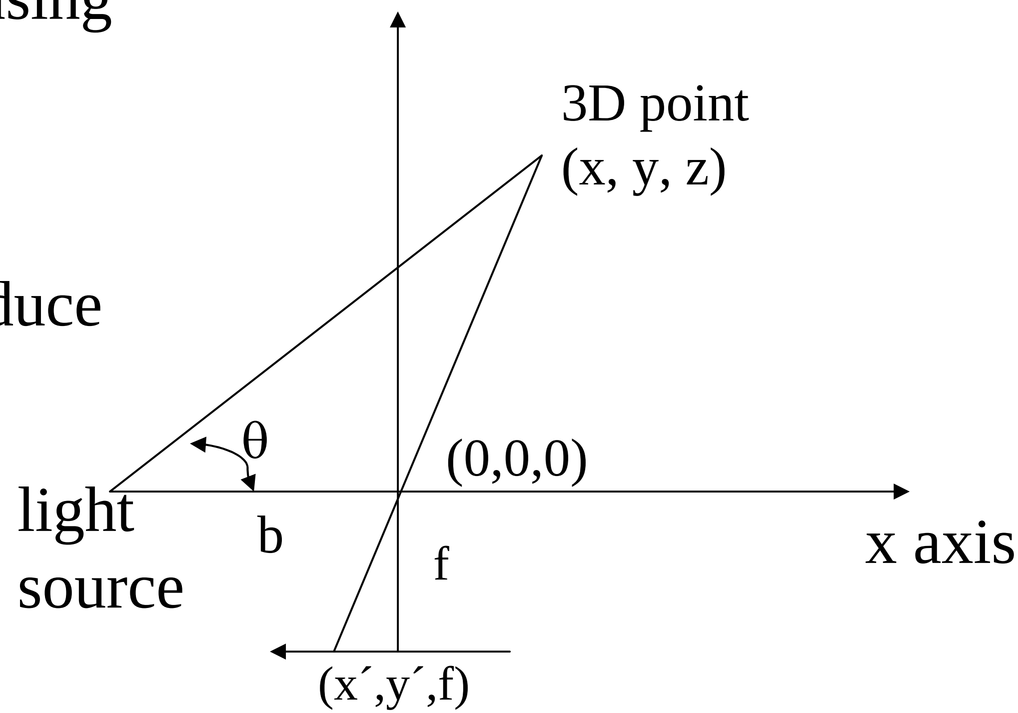
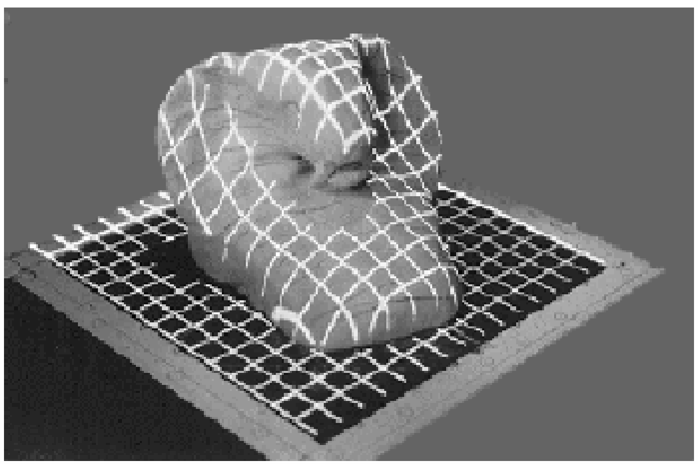
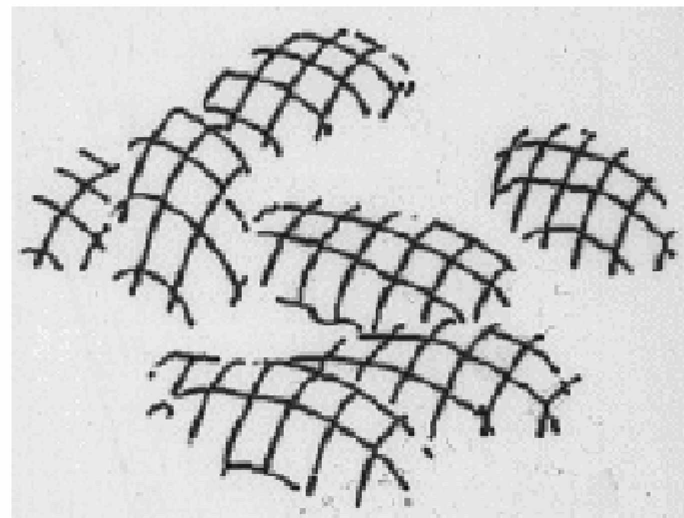
- a light source that can produce stripe(s) on the 3D object



$$\underset{\text{3D}}{[x\ y\ z]} = \frac{b}{f \cot \theta - x'} \underset{\text{image}}{[x'\ y'\ f]}$$

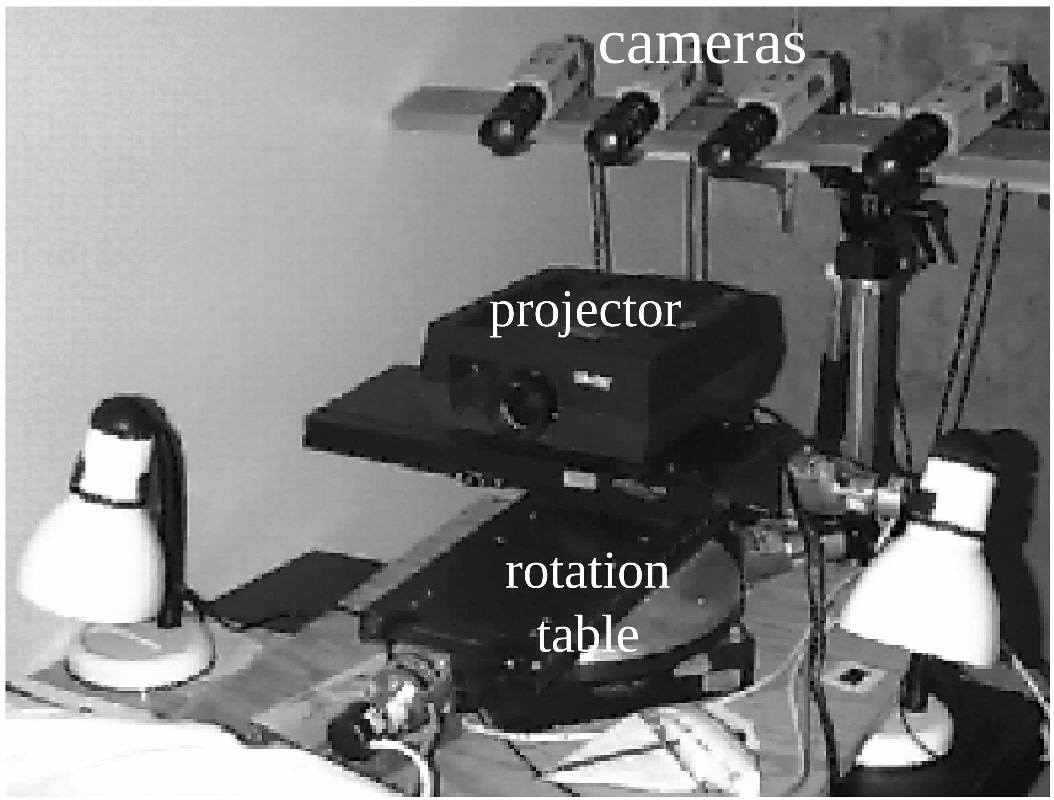
Depth from Multiple Light Stripes

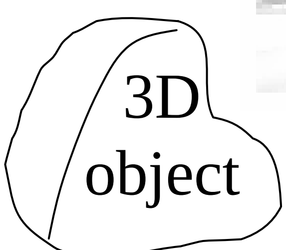




What are these objects?

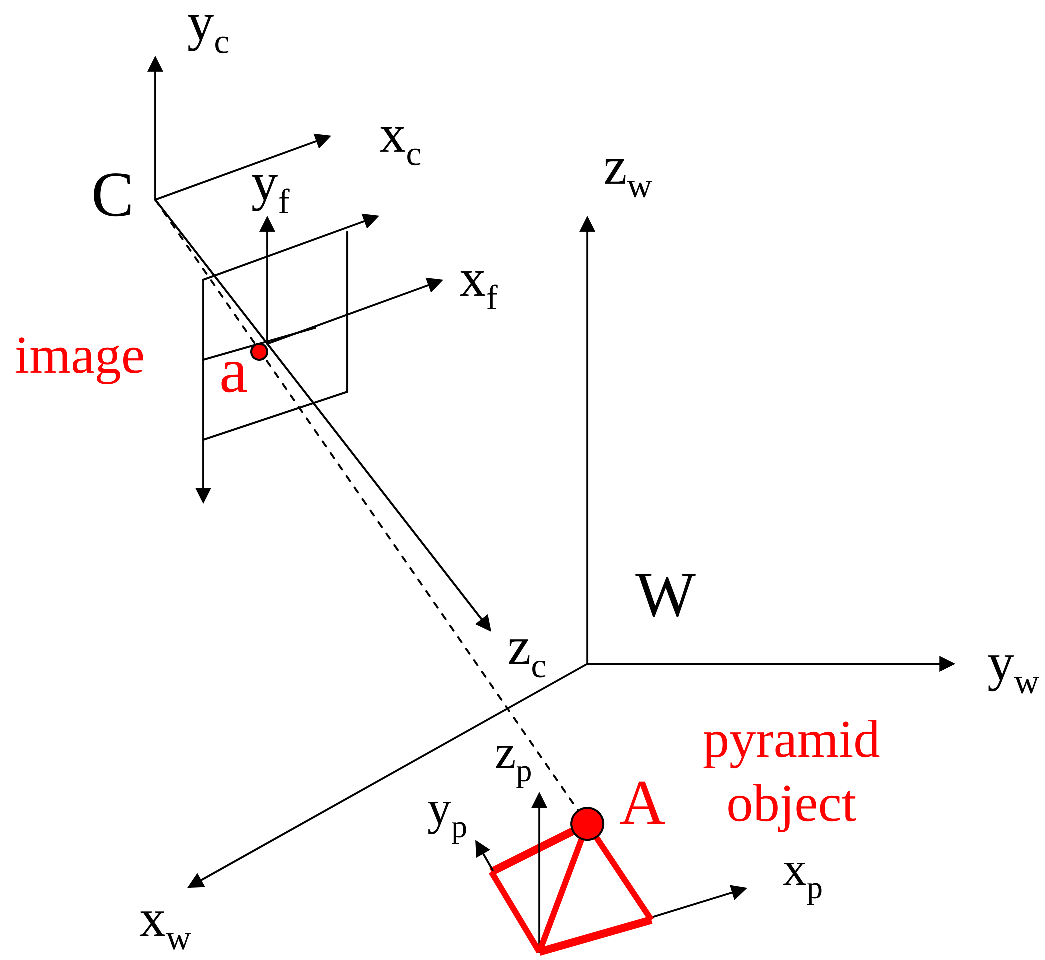
Our (former) System
4-camera light-striping stereo





Camera Model: Recall there are 5 Different Frames of Reference

- **Object**
- **World**
- **Camera**
- **Real Image**
- **Pixel Image**



The Camera Model

How do we get an image point IP from a world point P?

$$\begin{bmatrix} s\,IP_r \\ s\,IP_c \\ s \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & 1 \end{bmatrix} \begin{bmatrix} P_x \\ P_y \\ P_z \\ 1 \end{bmatrix}$$

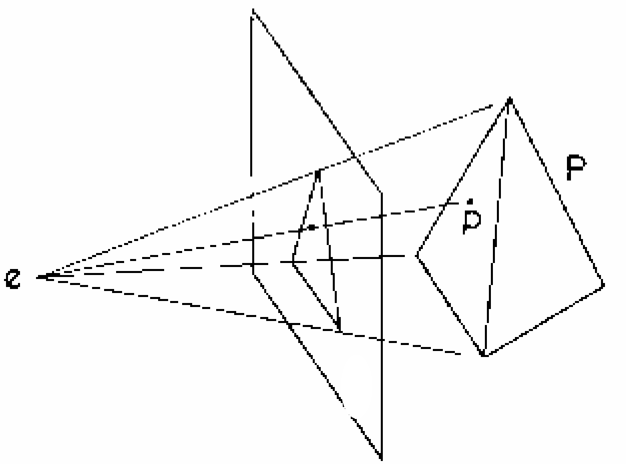
image point — camera matrix C — world point

What's in C?

The camera model handles the rigid body transformation from world coordinates to camera coordinates plus the perspective transformation to image coordinates.

1.	CP	=	T R	WP
2.	FP	=	π(f)	CP

Why is there not a scale factor here?



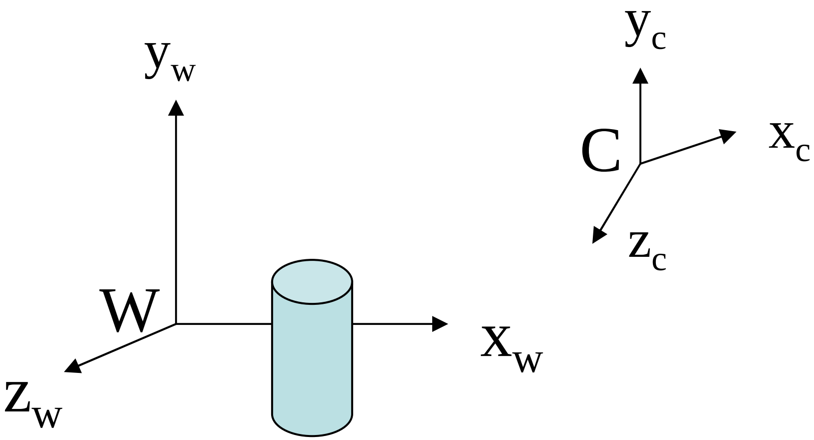
$$\begin{pmatrix} s\,FP_x \\ s\,FP_y \\ s\,FP_z \\ s \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/f & 0 \end{pmatrix} \begin{pmatrix} CP_x \\ CP_y \\ CP_z \\ 1 \end{pmatrix}$$

image point | perspective transformation | 3D point in camera coordinates

Camera Calibration

- In order work in 3D, we need to know the parameters of the particular camera setup.

- Solving for the camera parameters is called calibration.

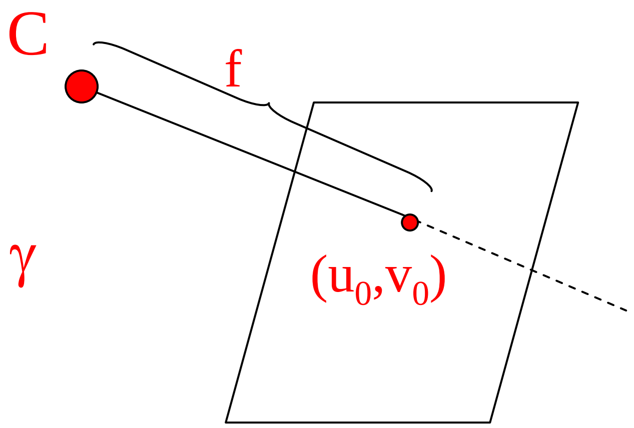


- intrinsic parameters are of the camera device

- extrinsic parameters are where the camera sits in the world

Intrinsic Parameters

- principal point (u_0,v_0)
- scale factors (d_x,d_y)
- aspect ratio distortion factor γ
- focal length f
- lens distortion factor κ
  (models radial lens distortion)



Extrinsic Parameters

- translation parameters

  $t = [t_x \ \ t_y \ \ t_z]$

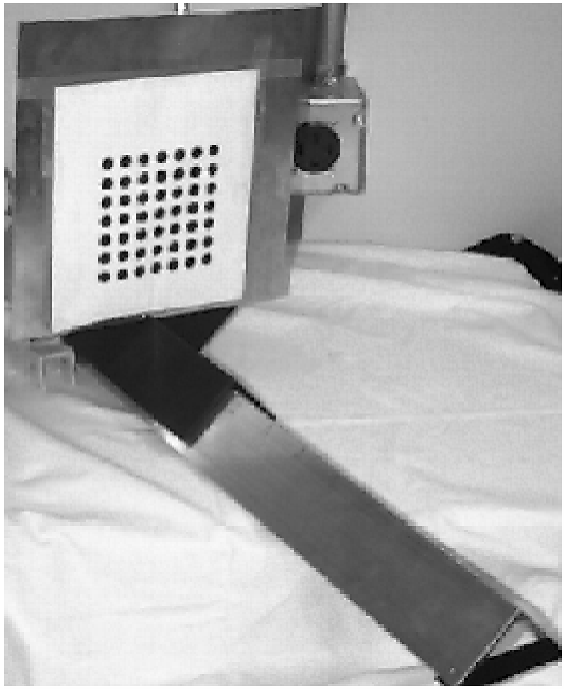
- rotation matrix

$$R = \begin{pmatrix} r_{11} & r_{12} & r_{13} & 0 \\ r_{21} & r_{22} & r_{23} & 0 \\ r_{31} & r_{32} & r_{33} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Are there really nine parameters?

Calibration Object

The idea is to snap images at different depths and get a lot of 2D-3D point correspondences.



The Tsai Procedure

- The Tsai procedure was developed by Roger Tsai at IBM Research and is most widely used.

- Several images are taken of the calibration object yielding point correspondences at different distances.

- Tsai's algorithm requires n > 5 correspondences

$$\{(x_i, y_i, z_i), (u_i, v_i)) \mid i = 1,\ldots,n\}$$

between (real) image points and 3D points.

In this* version of Tsai's algorithm,

• The real-valued (u,v) are computed from their pixel positions (r,c):

$$u = \gamma\, d_x\, (c - u_0) \quad v = -d_y\, (r - v_0)$$

where

- (u_0,v_0) is the center of the image

- d_x and d_y are the center-to-center (real) distances between pixels and come from the camera's specs

- γ is a scale factor learned from previous trials

* This version is for single-plane calibration.

Tsai's Procedure

1. Given the n point correspondences $((x_i,y_i,z_i), (u_i,v_i))$

   Compute matrix A with rows a_i

   $$a_i = (v_i*x_i, v_i*y_i, -u_i*x_i, -u_i*v_i, v_i)$$

These are known quantities which will be used to solve for intermediate values, which will then be used to solve for the parameters sought.

Intermediate Unknowns

2. The vector of unknowns is $\mu = (\mu_1, \mu_2, \mu_3, \mu_4, \mu_5)$:

$\mu_1 = r_{11}/t_y$ $\mu_2 = r_{12}/t_y$ $\mu_3 = r_{21}/t_y$ $\mu_4 = r_{22}/t_y$ $\mu_5 = t_x/t_y$

where the r's and t's are unknown rotation and translation parameters.

3. Let vector **b** = $(u_1, u_2, \ldots, u_n)$ contain the u image coordinates.

4. Solve the system of linear equations

$$\mathbf{A}\,\mu = \mathbf{b}$$

for unknown parameter vector μ.

Use μ to solve for ty, tx, and 4 rotation parameters

5. Let $U = \mu_1^2 + \mu_2^2 + \mu_3^2 + \mu_4^2$. Use U to calculate t_y^2 .

$$t_y^2 = \begin{cases} \frac{U - [U^2 - 4(\mu_1\mu_4 - \mu_2\mu_3)^2]^{1/2}}{2(\mu_1\mu_4 - \mu_2\mu_3)^2} & \text{if } (\mu_1\mu_4 - \mu_2\mu_3) \neq 0 \\ \frac{1}{\mu_1^2 + \mu_2^2} & \text{if } (\mu_1^2 + \mu_2^2) \neq 0 \\ \frac{1}{\mu_3^2 + \mu_4^2} & \text{if } (\mu_3^2 + \mu_4^2) \neq 0 \end{cases}$$

6. Try the positive square root $t_y = (t^2)^{1/2}$ and use it to compute translation and rotation parameters.

$$
\begin{aligned}
r_{11} &= \mu_1 t_y \\
r_{12} &= \mu_2 t_y \\
r_{21} &= \mu_3 t_y \\
r_{22} &= \mu_4 t_y \\
t_x &= \mu_5 t_y
\end{aligned}
$$

Now we know
2 translation parameters and
4 rotation parameters.

except…

Determine true sign of t_y and compute remaining rotation parameters.

7. Select an object point P whose image coordinates (u,v) are far from the image center.

8. Use P's coordinates and the translation and rotation parameters so far to estimate the image point that corresponds to P.

   If its coordinates have the same signs as (u,v), then keep t_y, else negate it.

9. Use the first 4 rotation parameters to calculate the remaining 5.

Calculating the remaining 5 rotation parameters:

$$
\begin{aligned}
r_{13} &= (1 - r_{11}^2 - r_{12}^2)^{1/2} \\
r_{23} &= (1 - r_{21}^2 - r_{22}^2)^{1/2} \\
r_{31} &= \frac{1 - r_{11}^2 - r_{12} r_{21}}{r_{13}} \\
r_{32} &= \frac{1 - r_{21} r_{12} - r_{22}^2}{r_{23}} \\
r_{33} &= (1 - r_{31} r_{13} - r_{32} r_{23})^{1/2}
\end{aligned}
$$

Solve another linear system.

10. We have t_x and t_y and the 9 rotation parameters.
Next step is to find t_z and f.

Form a matrix A´ whose rows are:

$$a_i' = (r_{21}*x_i + r_{22}*y_i + t_y, \quad v_i)$$

and a vector b´ whose rows are:

$$b_i' = (r_{31}*x_i + r_{32}*y_i) * v_i$$

11. Solve $A'*\mathbf{v} = \mathbf{b}'$ for $\mathbf{v} = (f, t_z)$.

Almost there

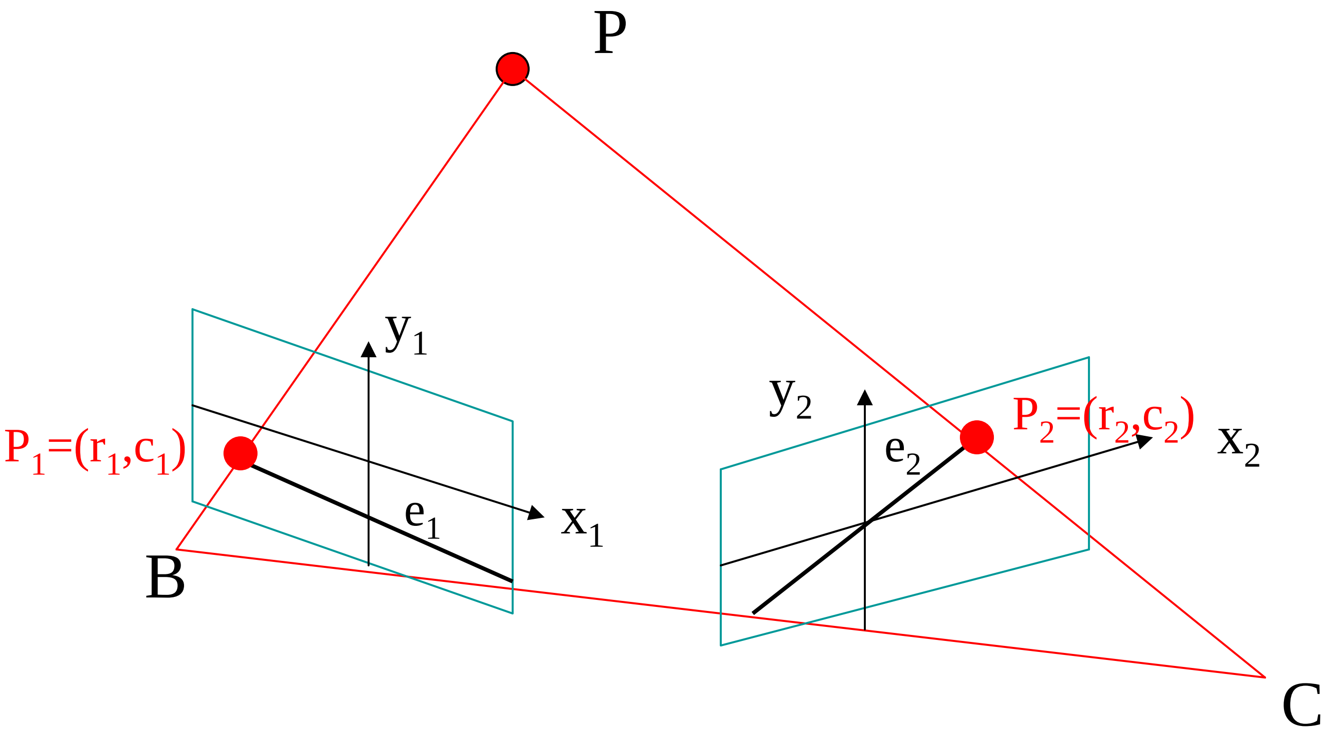
12. If f is negative, change signs (see text).

13. Compute the lens distortion factor κ and improve the estimates for f and t_z by solving a nonlinear system of equations by a nonlinear regression.

14. All parameters have been computed.

Use them in 3D data acquisition systems.

We use them for general stereo.



For a correspondence (r_1,c_1) in image 1 to (r2,c2) in image 2:

1. Both cameras were calibrated. Both camera matrices are then known. From the two camera equations B and C we get

   4 linear equations in 3 unknowns.

$r_1 = (b_{11} - b_{31}*r_1)\mathbf{x} + (b_{12} - b_{32}*r_1)\mathbf{y} + (b_{13}-b_{33}*r_1)\mathbf{z}$
$c_1 = (b_{21} - b_{31}*c1)\mathbf{x} + (b_{22} - b_{32}*c_1)\mathbf{y} + (b_{23}-b_{33}*c_1)\mathbf{z}$

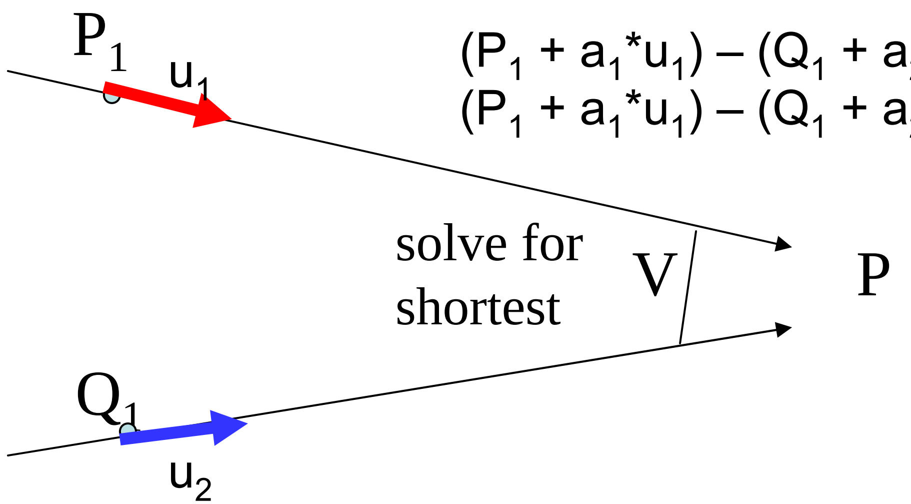
$r_2 = (c_{11} - c_{31}*r_{2)}\mathbf{x} + (c_{12} - c_{32}*r_2)\mathbf{y} + (c_{13} - c_{33}*r_2)\mathbf{z}$
$c_2 = (c_{21} - c_{31}*c_2)\mathbf{x} + (c_{22} - c_{32}*c_2)\mathbf{y} + (c_{23} - c_{33}*c_2)\mathbf{z}$

Direct solution uses 3 equations, won't give reliable results.

Solve by computing the closest approach of the two skew rays.

$$V = (P_1 + a_1{*}u_1) - (Q_1 + a_2{*}u_2)$$

$$(P_1 + a_1{*}u_1) - (Q_1 + a_2{*}u_2) \cdot u_1 = 0$$
$$(P_1 + a_1{*}u_1) - (Q_1 + a_2{*}u_2) \cdot u_2 = 0$$



If the rays intersected perfectly in 3D, the intersection would be P. Instead, we solve for the shortest line segment connecting the two rays and let P be its midpoint.

Surface Modeling and Display from Range and Color Data

Kari	Pulli	UW
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John	McDonald	UW
Linda	Shapiro	UW
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UW = University of Washington
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Introduction

Goal

- develop robust algorithms for constructing 3D models from range & color data
- use those models to produce realistic renderings of the scanned objects



Surface Reconstuction

Step 1: Data acquisition

Obtain range data that covers the object. Filter, remove background.

Step 2: Registration

Register the range maps into a common coordinate system.

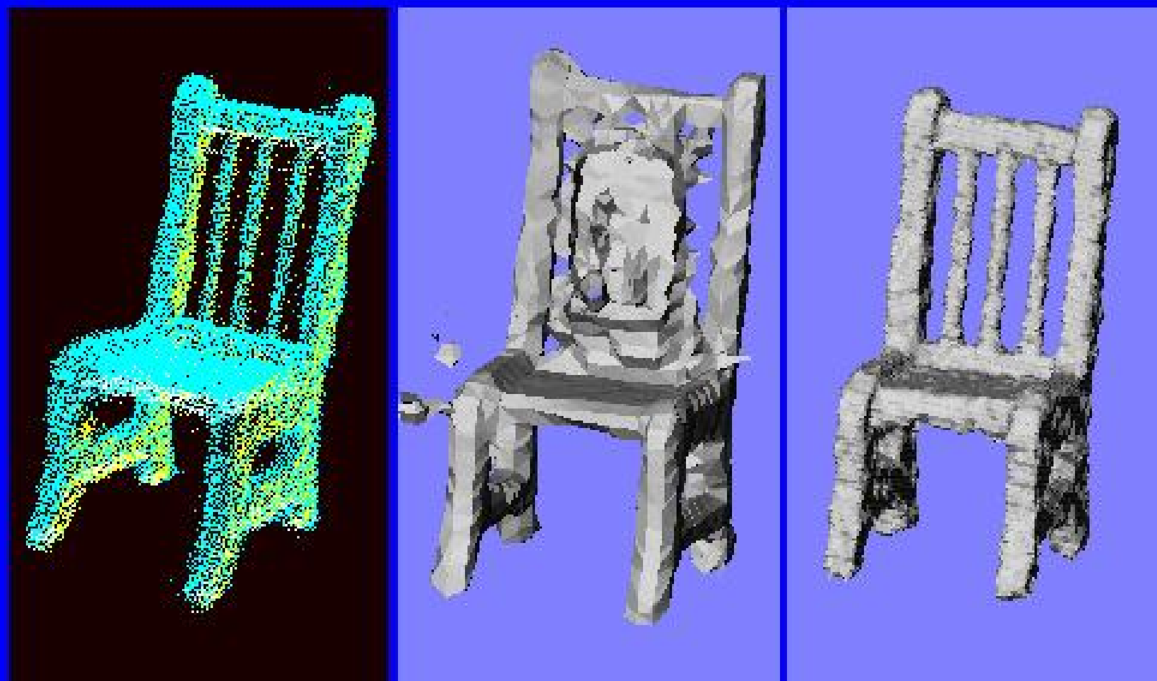
Step 3: Integration

Integrate the registered range data into a single surface representation.

Step 4: Optimization

Fit the surface more accurately to the data, simplify the representation.

Problem

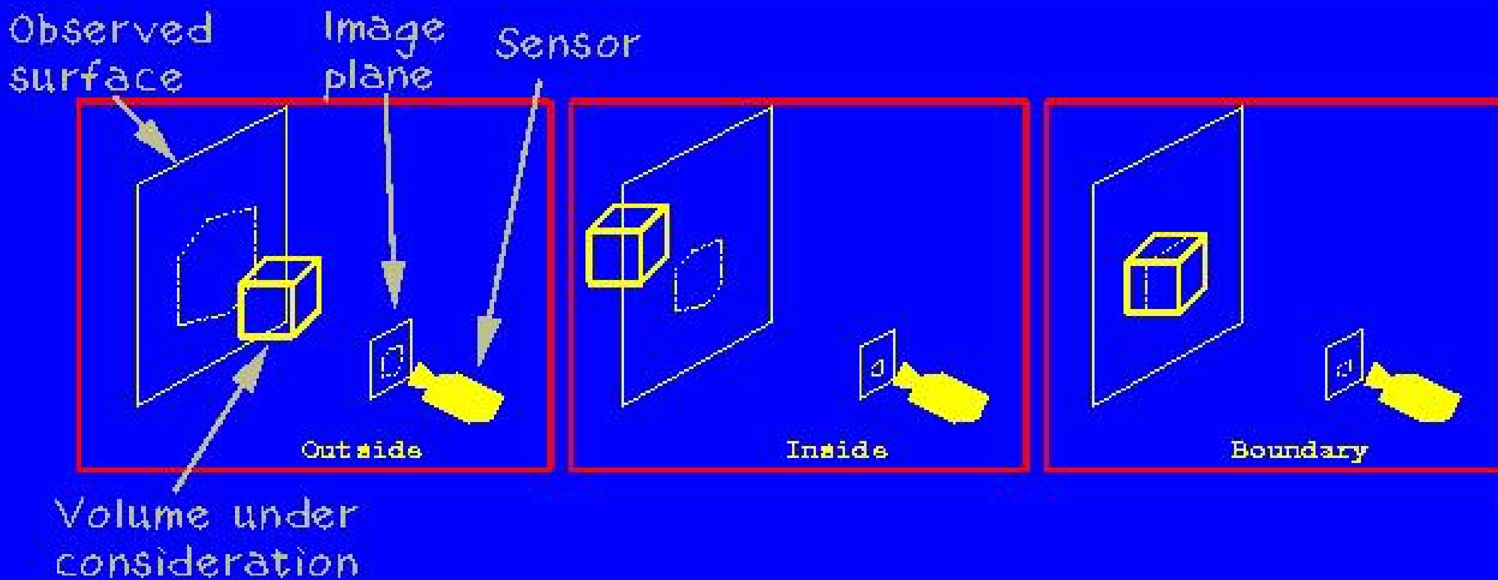


Noisy registered data

Signed distance fn & marching cubes

Hierarchical & directional space carving

Carve space in cubes



Label cubes

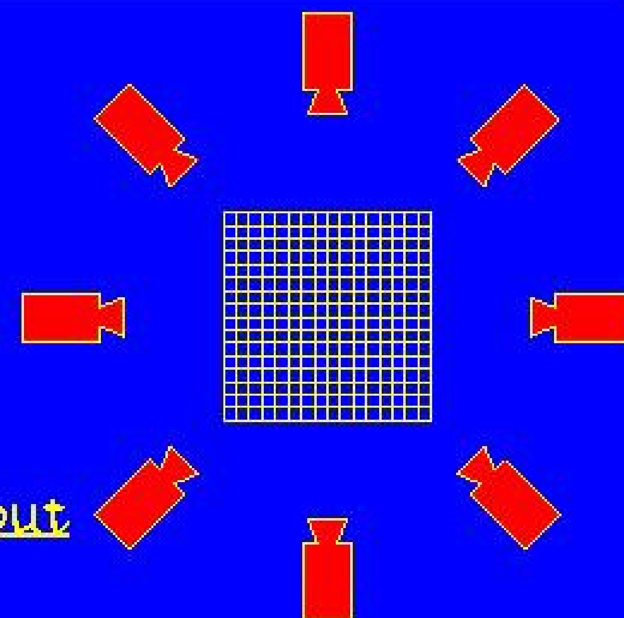
- Project cube to image plane (hexagon)
- Test against data in the hexagon

Several views

Processing order:

FOR EACH cube

  FOR EACH view



Rules:

any view thinks cube's out
 => it's out

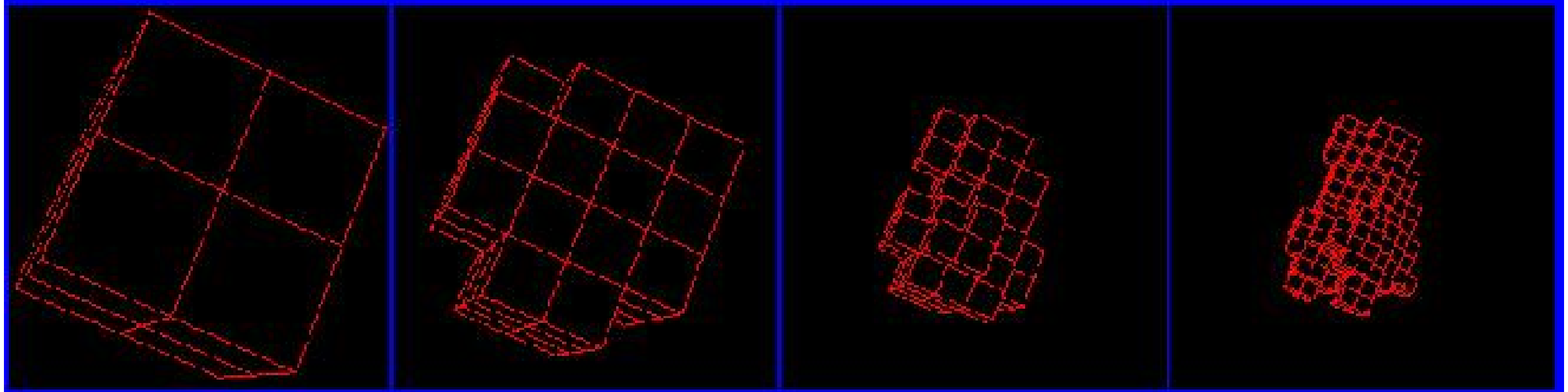
every view thinks cube's in
 => it's in

else
 => it's at boundary

Hierarchical space carving

- Big cubes => fast, poor results
- Small cubes => slow, more accurate results
- Combination = octrees

RULES:
- cube's out => done
- cube's in => done
- else => recurse



Hierarchical space carving

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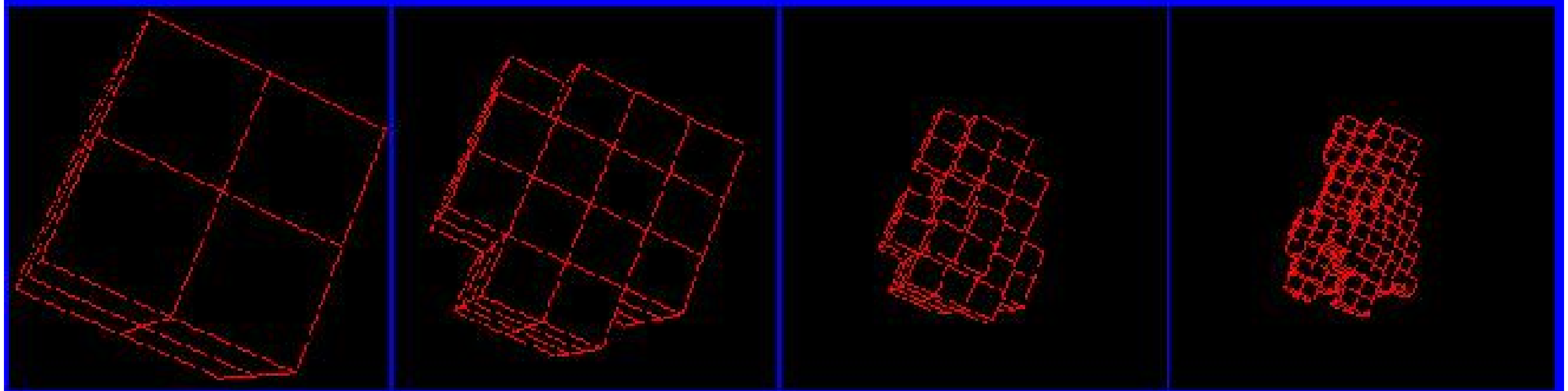
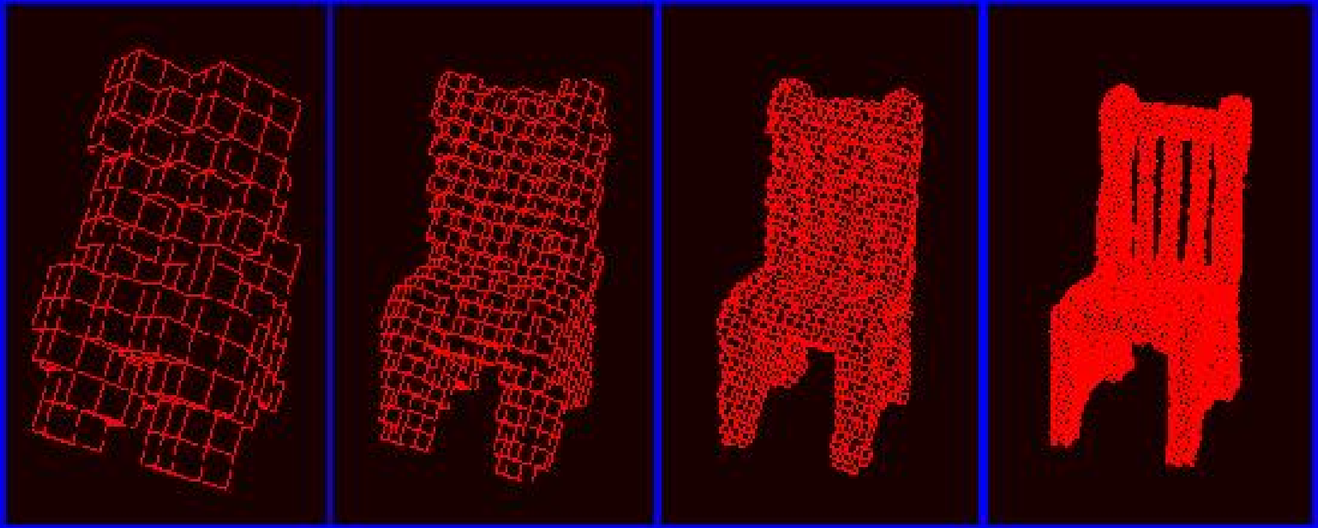
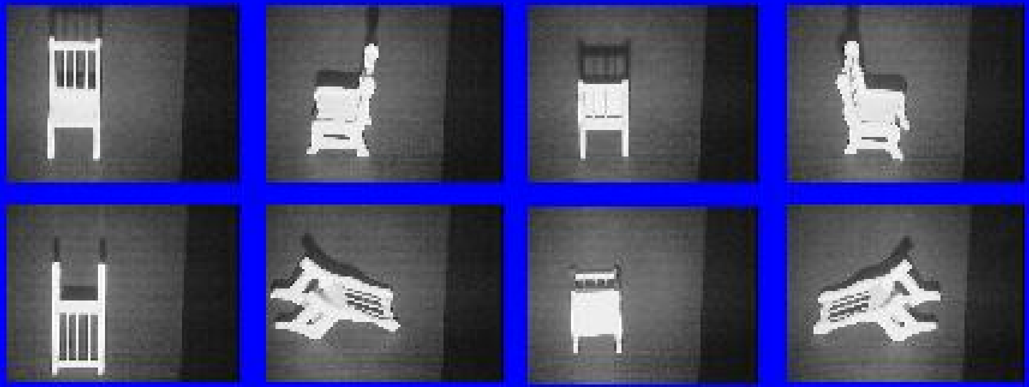
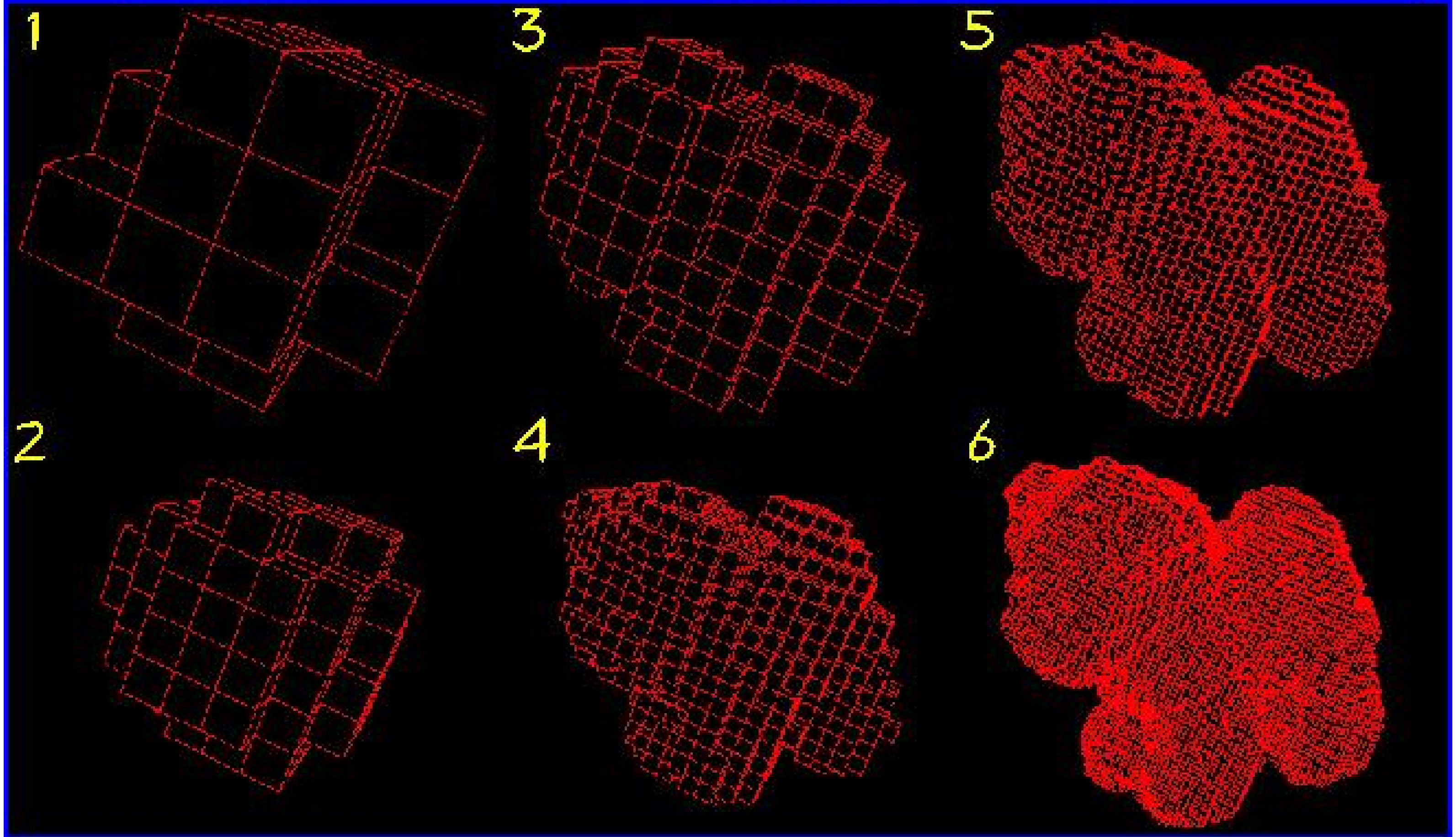


The rest of the chair

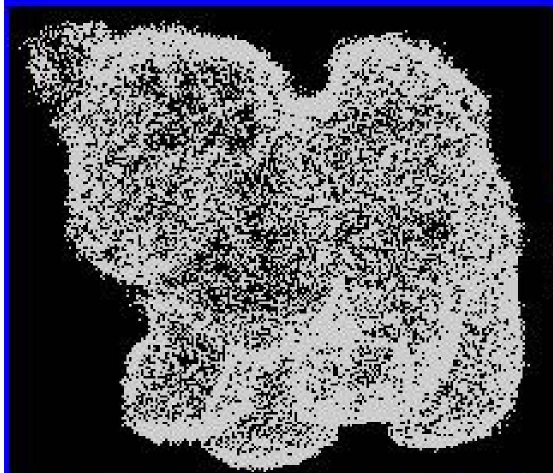




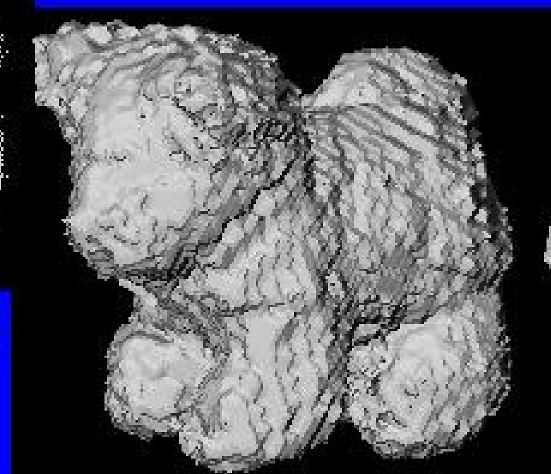
Same for a husky pup



Optimizing the dog mesh



Registered points



Initial mesh

Optimized mesh

View dependent texturing

Our viewer