

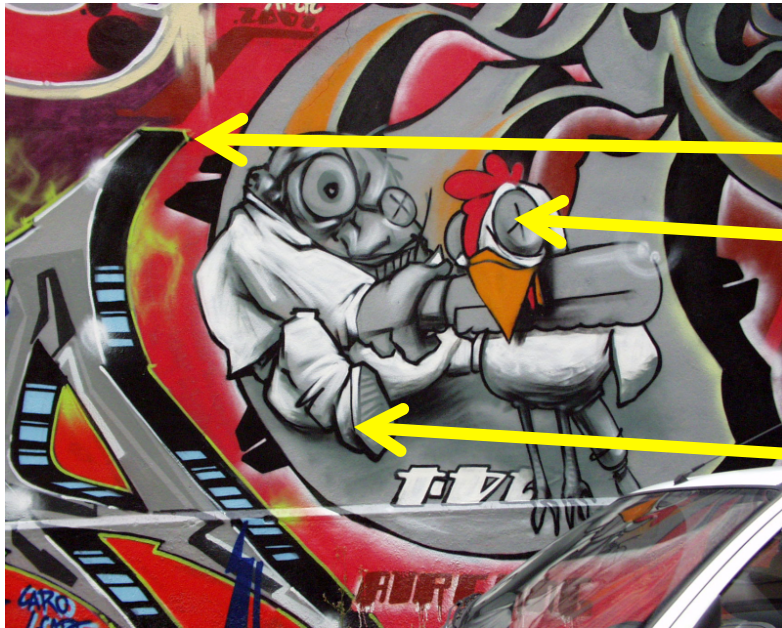
# Interest points

CSE 576

Ali Farhadi

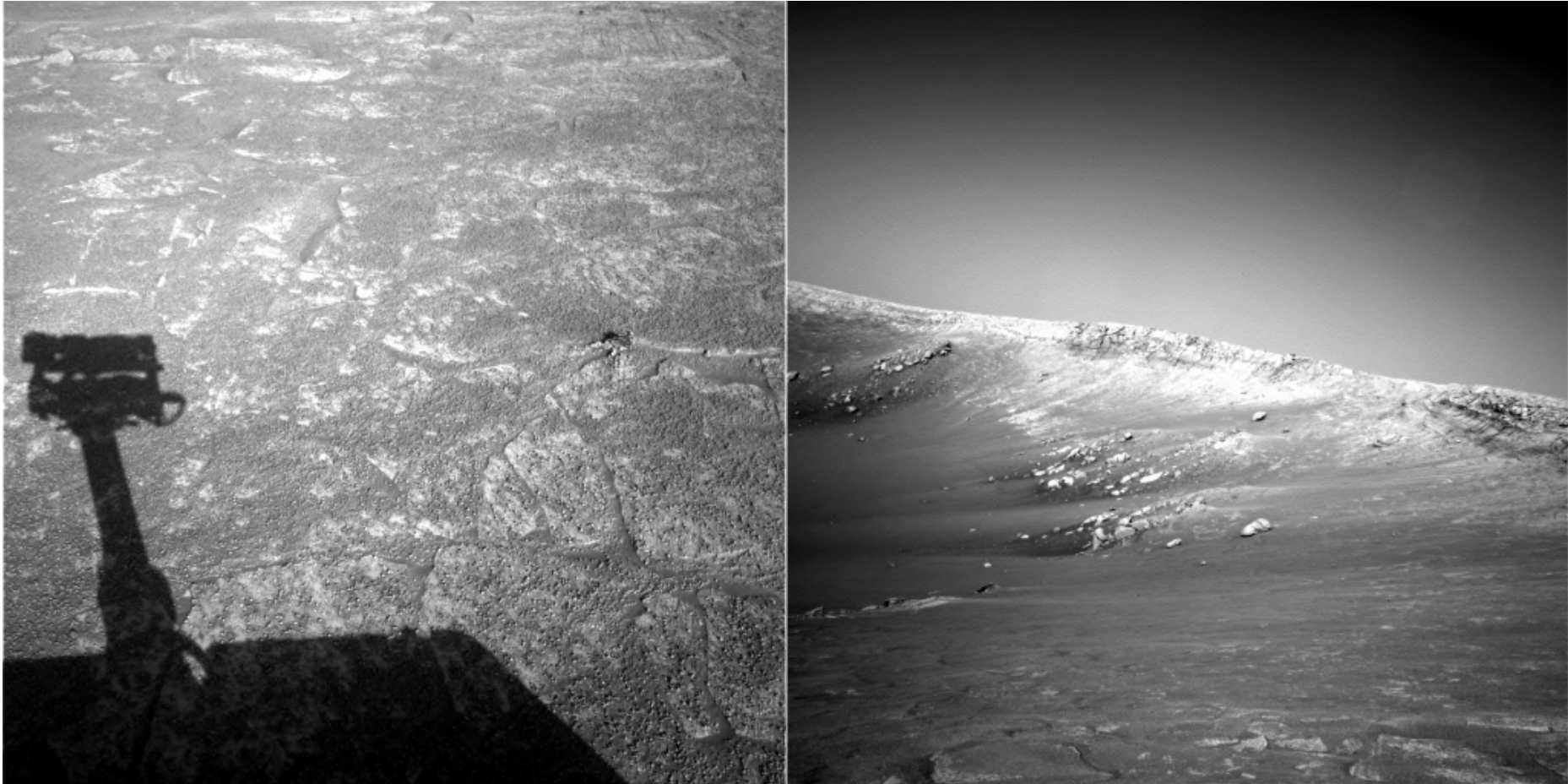
Many slides from Steve Seitz, Larry Zitnick

# How can we find corresponding points?



# Not always easy

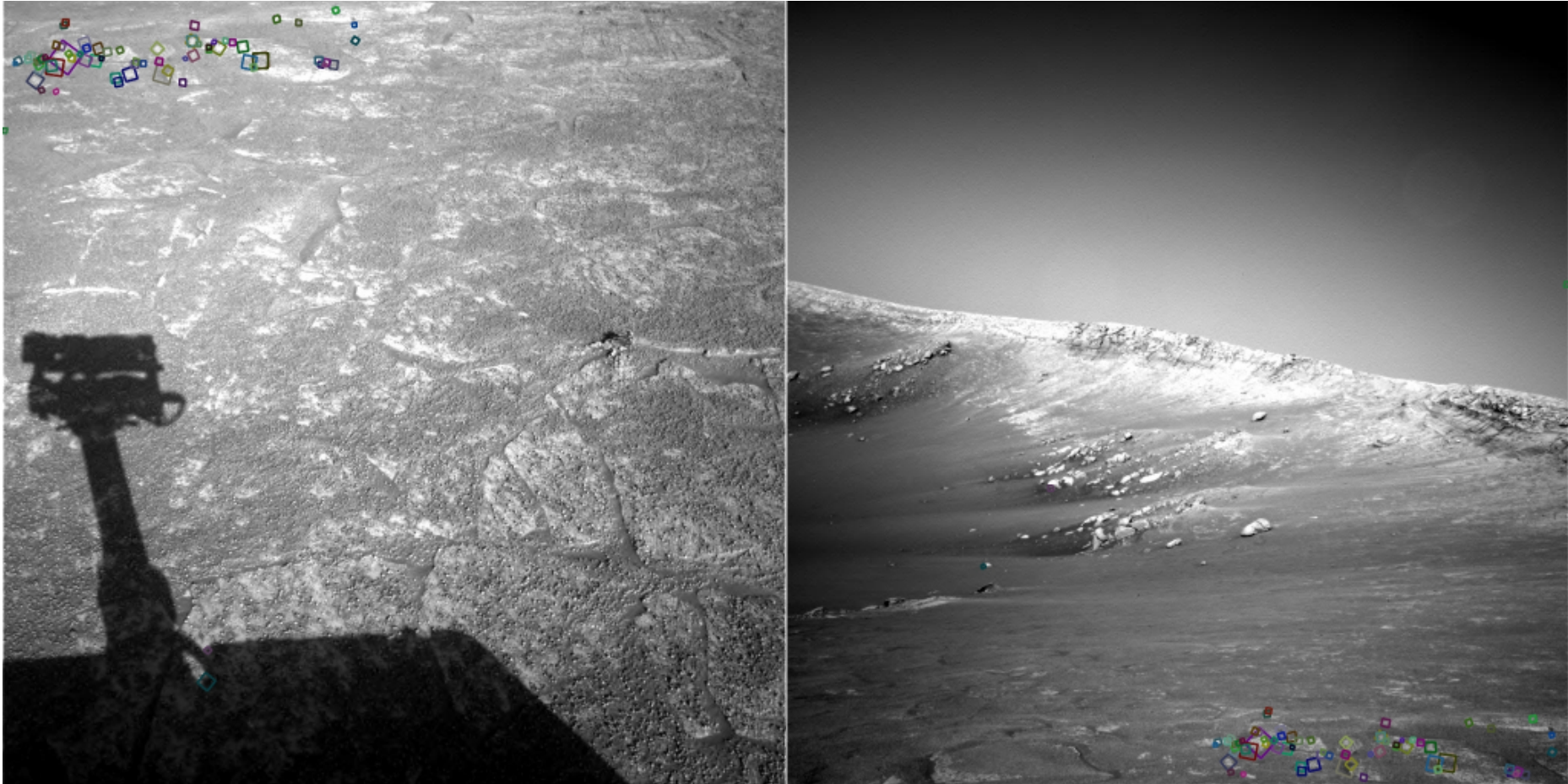
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NASA Mars Rover images

# Answer below (look for tiny colored squares...)

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NASA Mars Rover images  
with SIFT feature matches  
Figure by Noah Snavely

# Human eye movements

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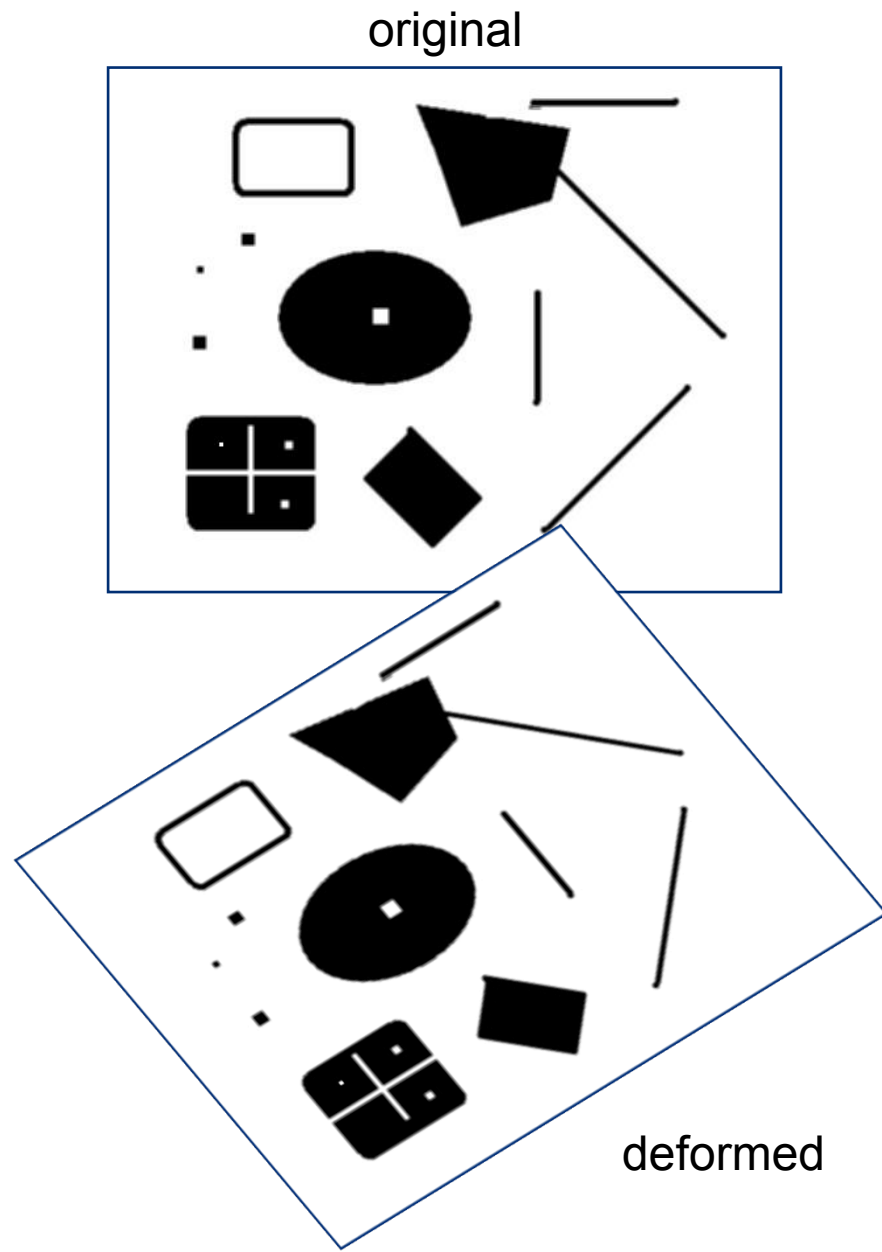


Yarbus eye tracking

# Interest points

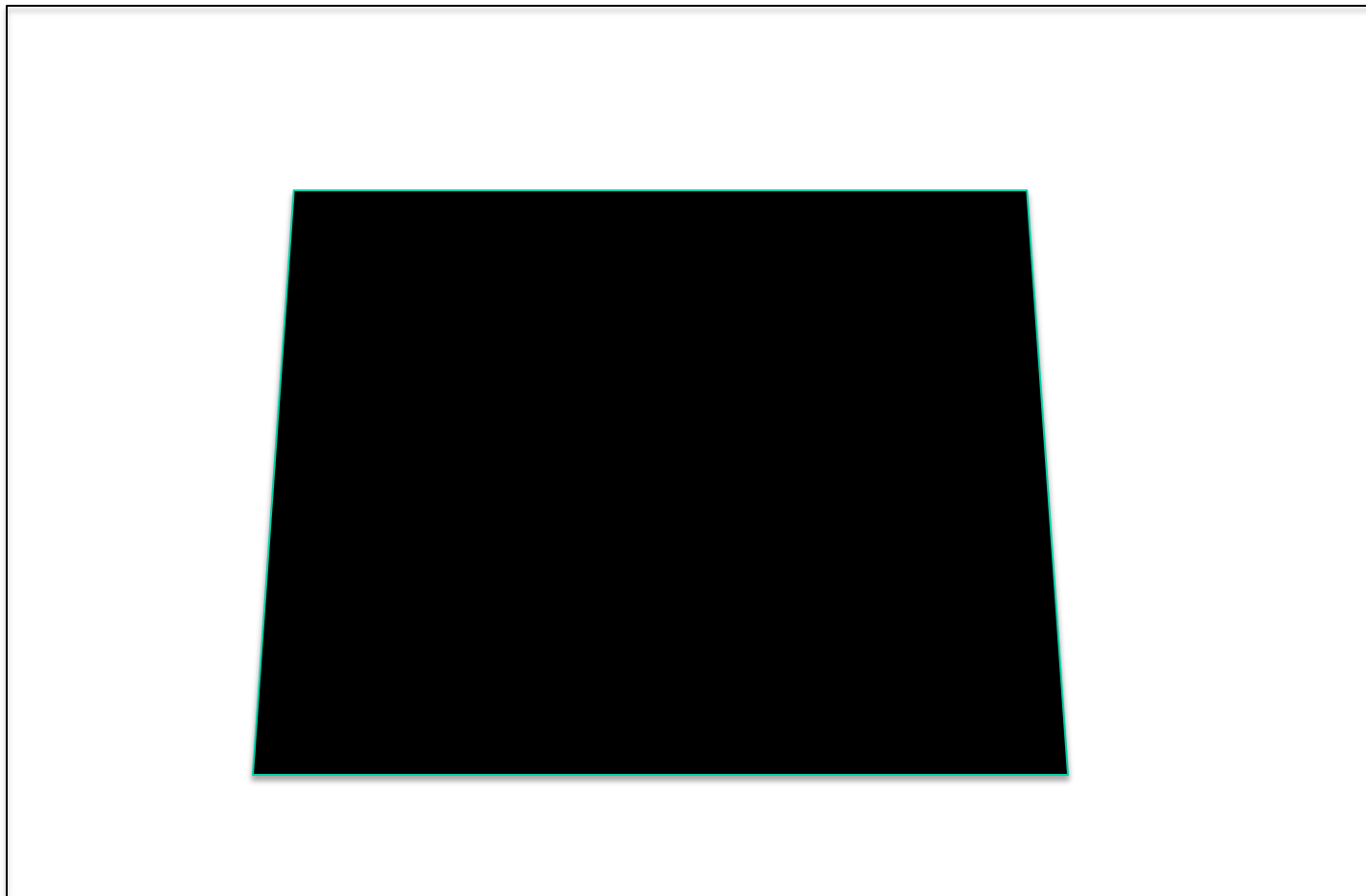
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- Suppose you have to click on some point, go away and come back after I deform the image, and click on the same points again.
  - Which points would you choose?



# Intuition

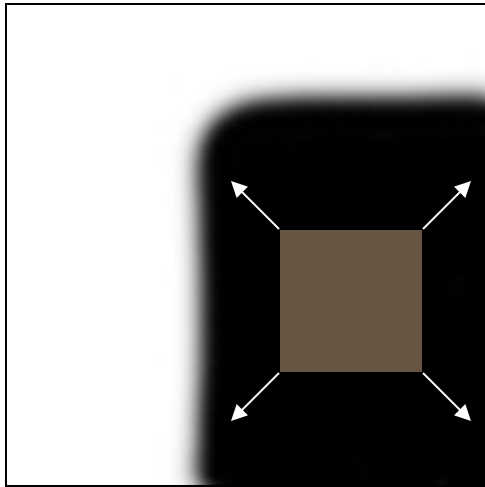
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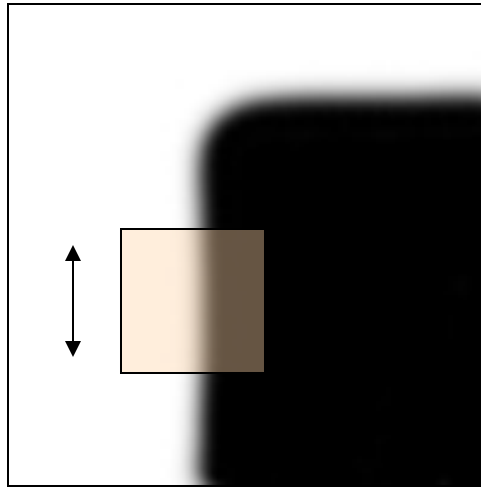
# Corners

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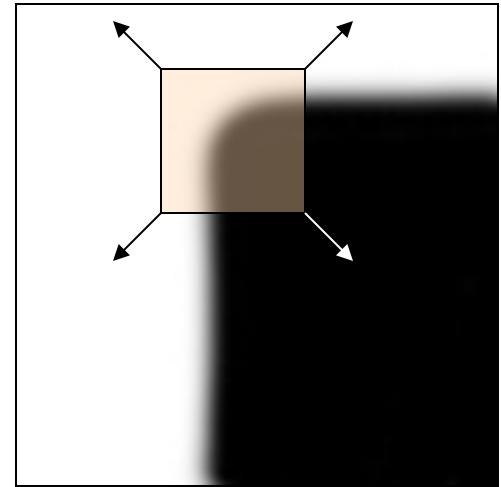
- We should easily recognize the point by looking through a small window
- Shifting a window in *any direction* should give a *large change* in intensity



“flat” region:  
no change in  
all directions



“edge”:  
no change along  
the edge  
direction

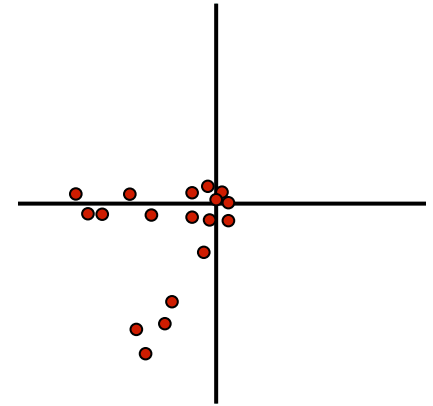
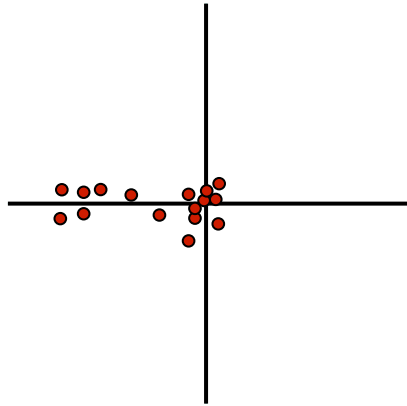
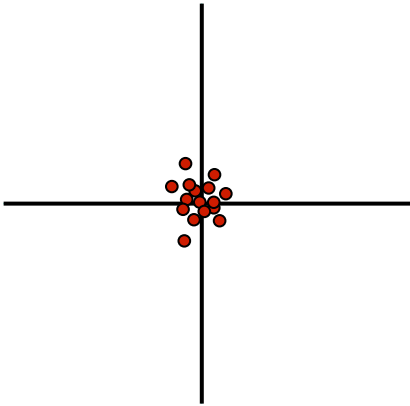
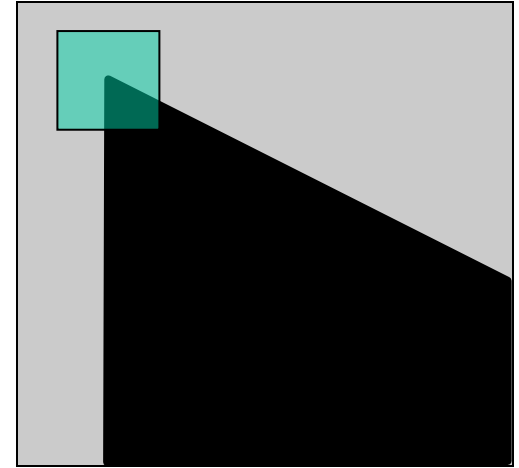
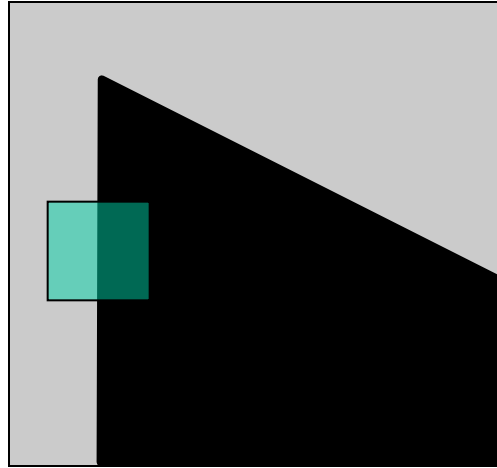
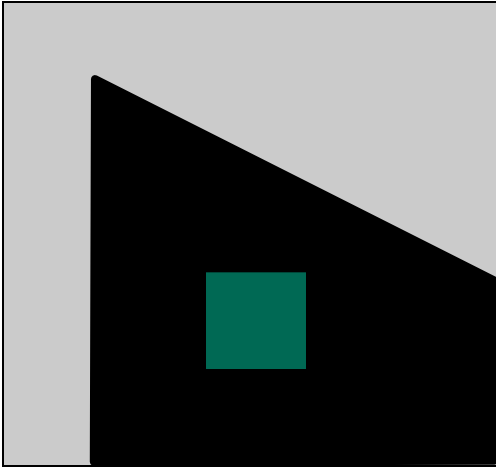


“corner”:  
significant  
change in all  
directions



# Let's look at the gradient distributions

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# Principle Component Analysis

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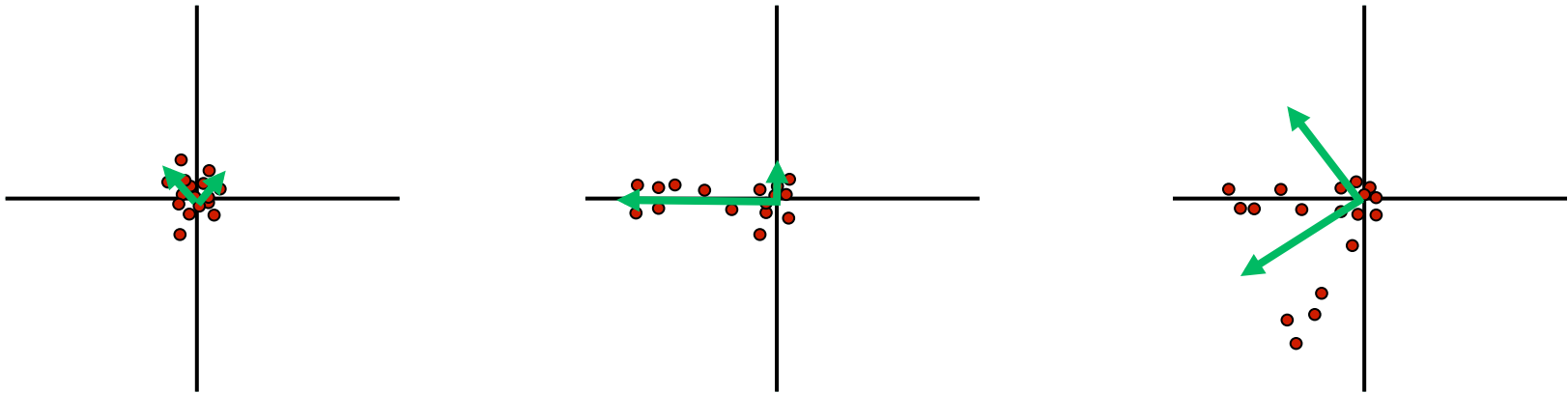
Principal component is the direction of highest variance.

Next, highest component is the direction with highest variance *orthogonal* to the previous components.

How to compute PCA components:

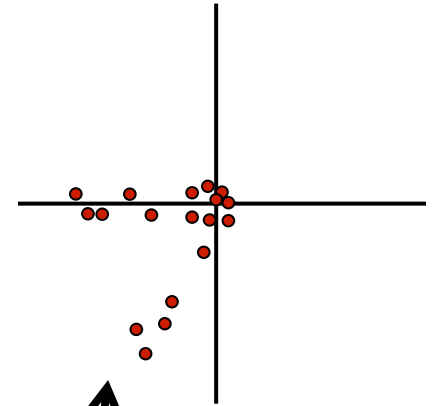
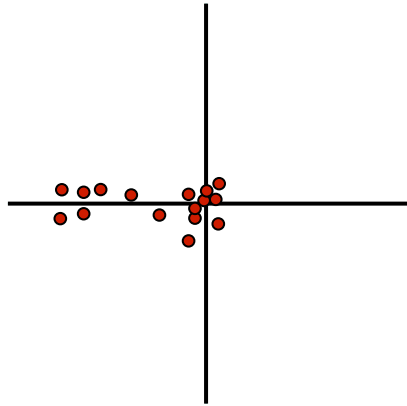
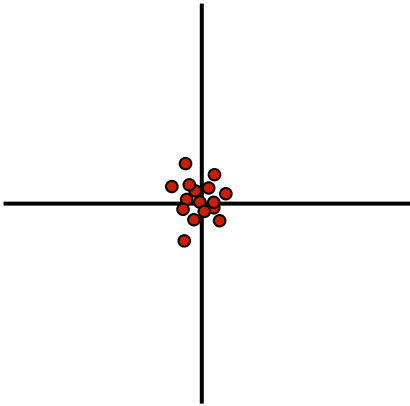
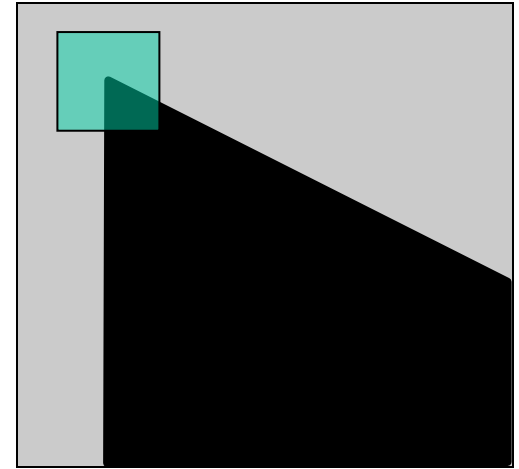
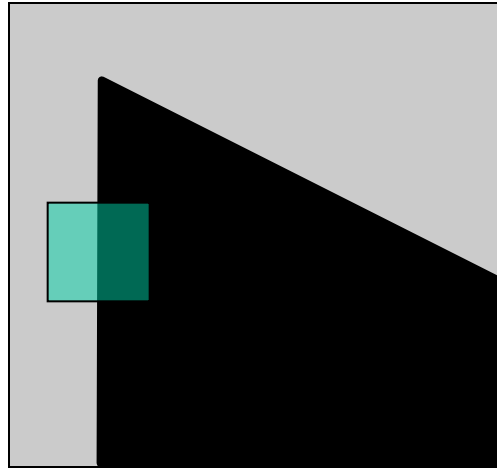
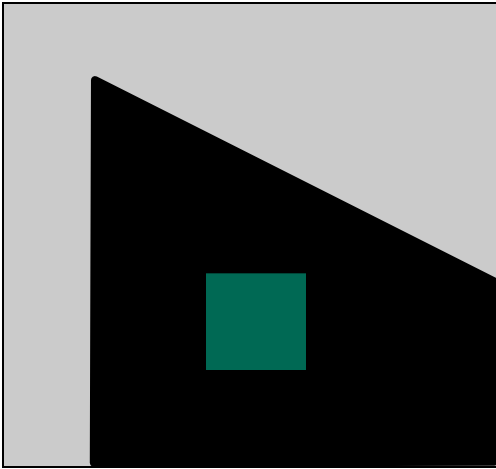
1. Subtract off the mean for each data point.
2. Compute the covariance matrix.
3. Compute eigenvectors and eigenvalues.
4. The components are the eigenvectors ranked by the eigenvalues.

$$Hx = \lambda x$$



# Corners have ...

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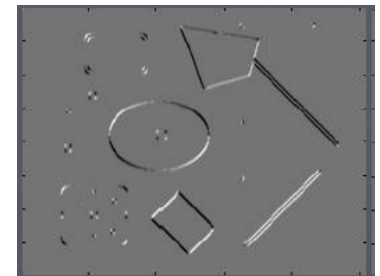
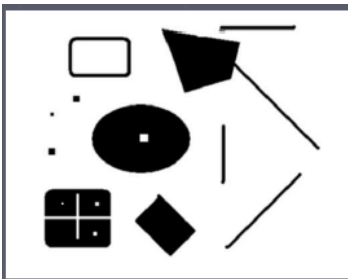
Both eigenvalues are large!

# Second Moment Matrix

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$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x I_x & I_x I_y \\ I_x I_y & I_y I_y \end{bmatrix}$$

2 x 2 matrix of image derivatives (averaged in neighborhood of a point).



Notation:

$$I_x \Leftrightarrow \frac{\partial I}{\partial x}$$

$$I_y \Leftrightarrow \frac{\partial I}{\partial y}$$

$$I_x I_y \Leftrightarrow \frac{\partial I}{\partial x} \frac{\partial I}{\partial y}$$

# The math

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To compute the eigenvalues:

1. Compute the covariance matrix.

$$H = \sum_{(u,v)} w(u,v) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \quad I_x = \frac{\partial f}{\partial x}, I_y = \frac{\partial f}{\partial y}$$

Typically Gaussian weights

2. Compute eigenvalues.

$$H = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \lambda_{\pm} = \frac{1}{2} \left( (a + d) \pm \sqrt{4bc + (a - d)^2} \right)$$

# Corner Response Function

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- Computing eigenvalues are expensive
- Harris corner detector uses the following alternative

$$R = \det(M) - \alpha \cdot \text{trace}(M)^2$$

Reminder:

$$\det \left( \begin{bmatrix} a & b \\ c & d \end{bmatrix} \right) = ad - bc \quad \text{trace} \left( \begin{bmatrix} a & b \\ c & d \end{bmatrix} \right) = a + d$$

# Harris detector: Steps

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1. Compute Gaussian derivatives at each pixel
2. Compute second moment matrix  $M$  in a Gaussian window around each pixel
3. Compute corner response function  $R$
4. Threshold  $R$
5. Find local maxima of response function (nonmaximum suppression)

C.Harris and M.Stephens.

[“A Combined Corner and Edge Detector.”](#) *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

# Harris Detector: Steps

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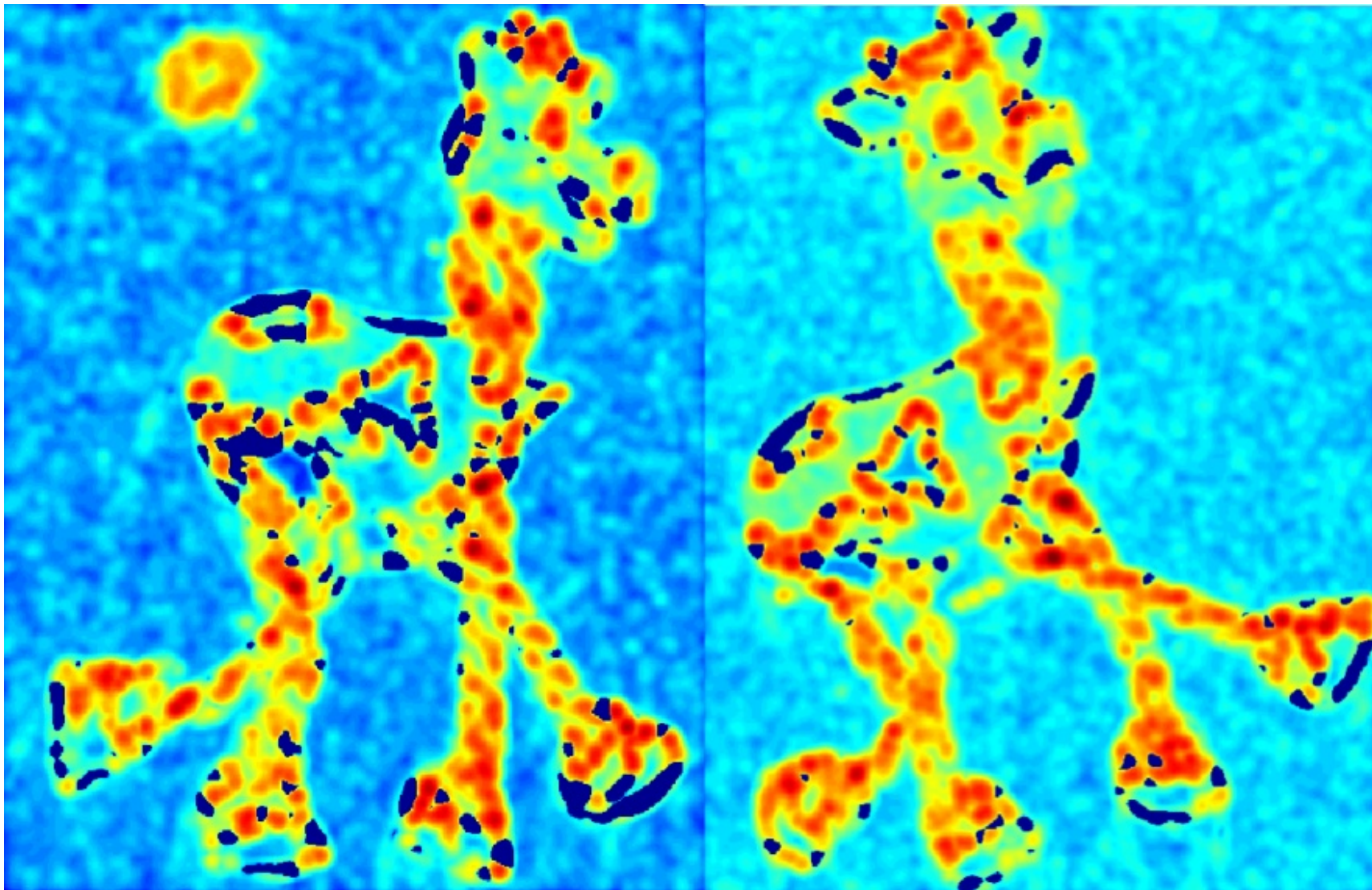




# Harris Detector: Steps

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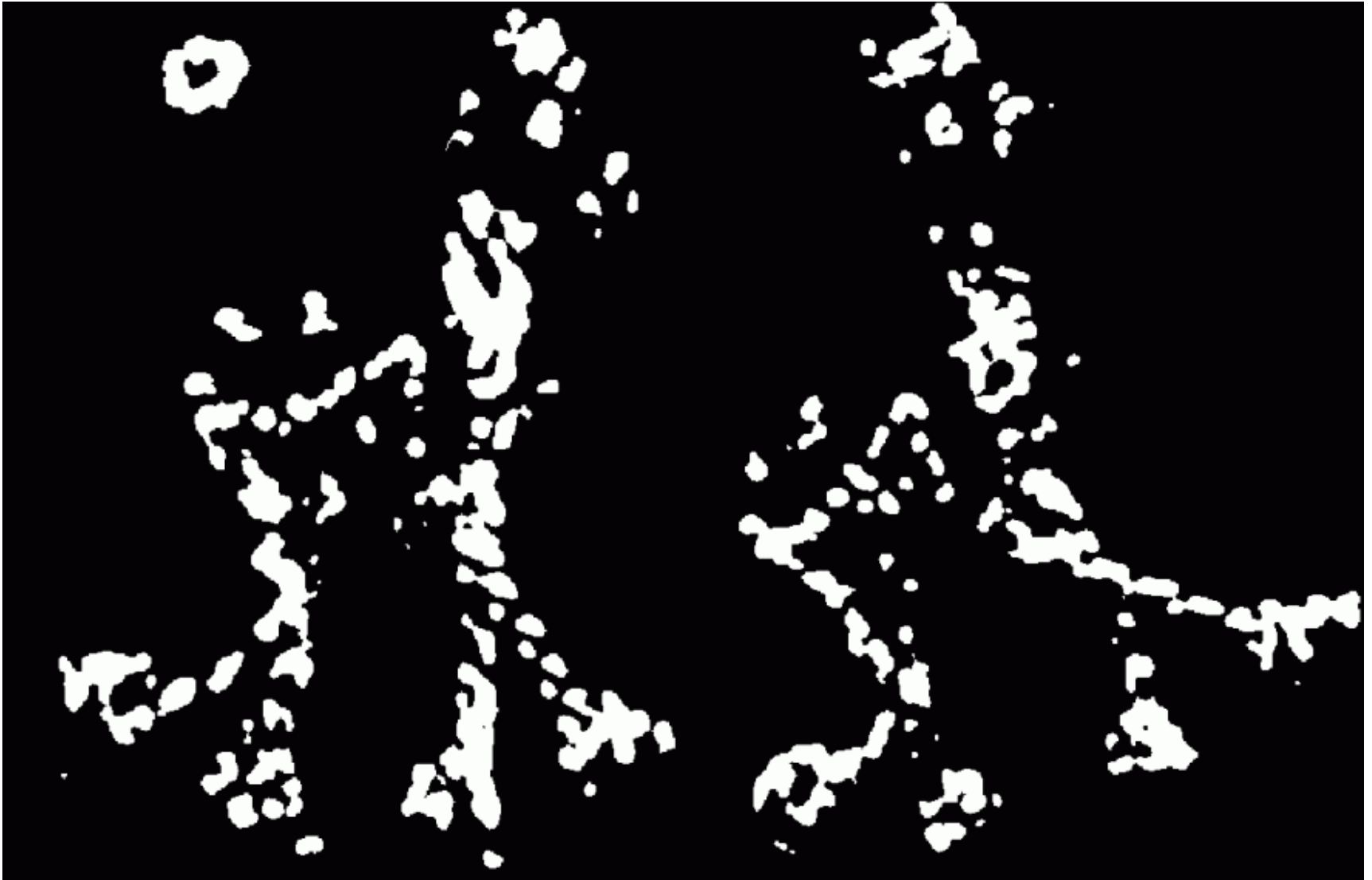
Compute corner response  $R$



# Harris Detector: Steps

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Find points with large corner response:  $R > \text{threshold}$



# Harris Detector: Steps

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Take only the points of local maxima of  $R$



# Harris Detector: Steps

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# Simpler Response Function

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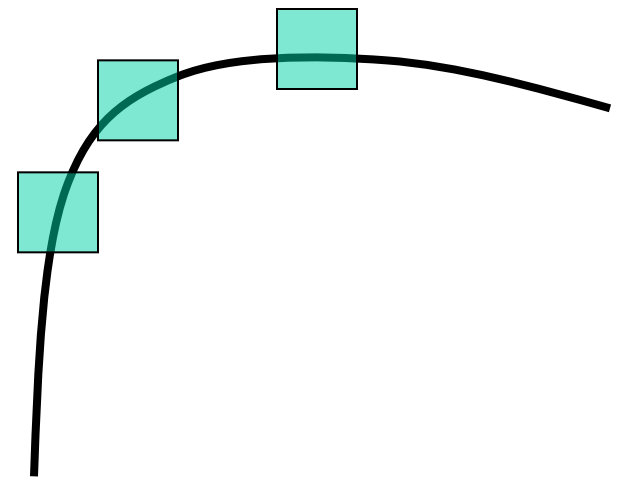
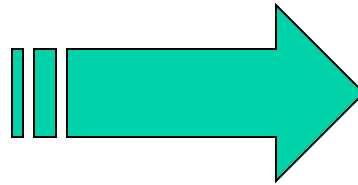
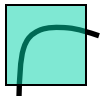
$$R = \det(M) - \alpha \cdot \text{trace}(M)^2$$

$$f = \frac{1}{\frac{1}{\lambda_1} + \frac{1}{\lambda_2}} = \frac{\text{Det}(H)}{\text{Tr}(H)}$$

# Properties of the Harris corner detector

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- Translation invariant? Yes
- Rotation invariant? Yes
- Scale invariant? No



Corner !

All points will be classified as **edges**

# Scale

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Let's look at scale first:



What is the “best” scale?

# Scale Invariance

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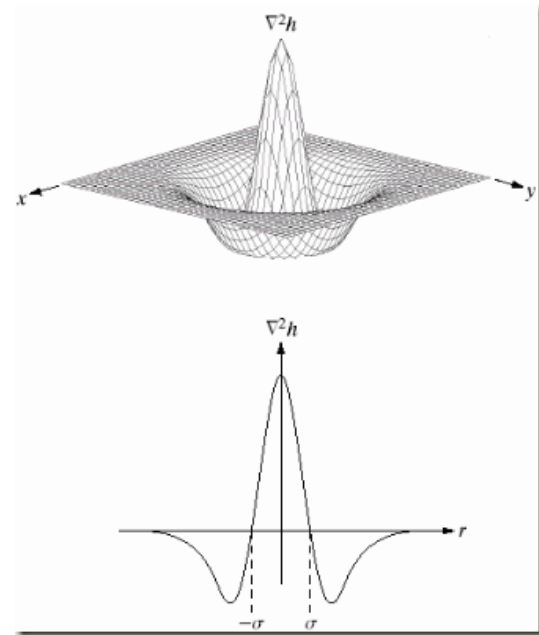
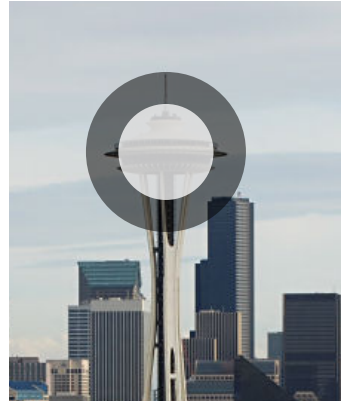
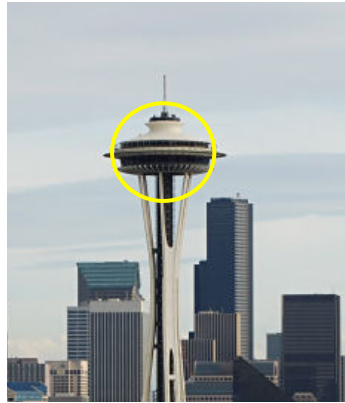


$$f(I_{i_1 \dots i_m}(x, \sigma)) = f(I_{i_1 \dots i_m}(x', \sigma'))$$

How can we independently select interest points in each image, such that the detections are repeatable across different scales?



# Differences between Inside and Outside



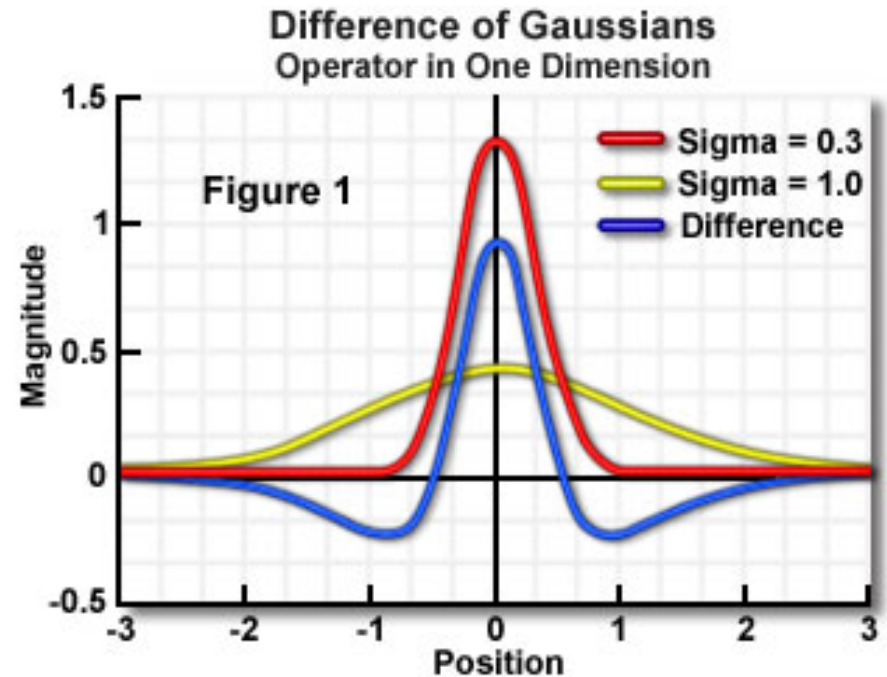
# Scale

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## Why Gaussian?

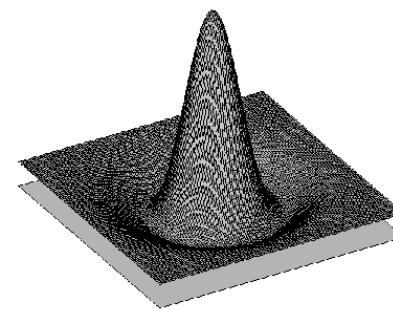
It is invariant to scale change, i.e.,  $f * \mathcal{G}_\sigma * \mathcal{G}_{\sigma'} = f * \mathcal{G}_{\sigma''}$  and has several other nice properties. Lindeberg, 1994

In practice, the Laplacian is approximated using a Difference of Gaussian (DoG).



# Difference-of-Gaussian (DoG)

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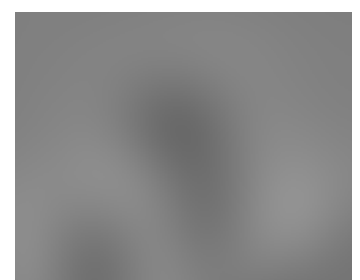
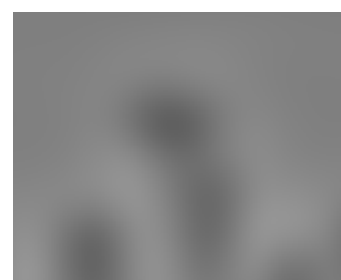
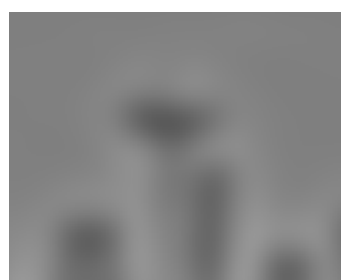
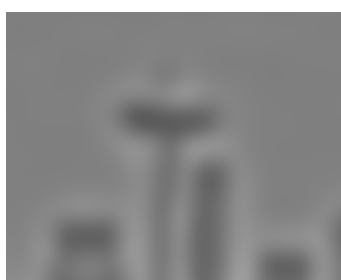
K. Grauman, B. Leibe

# DoG example

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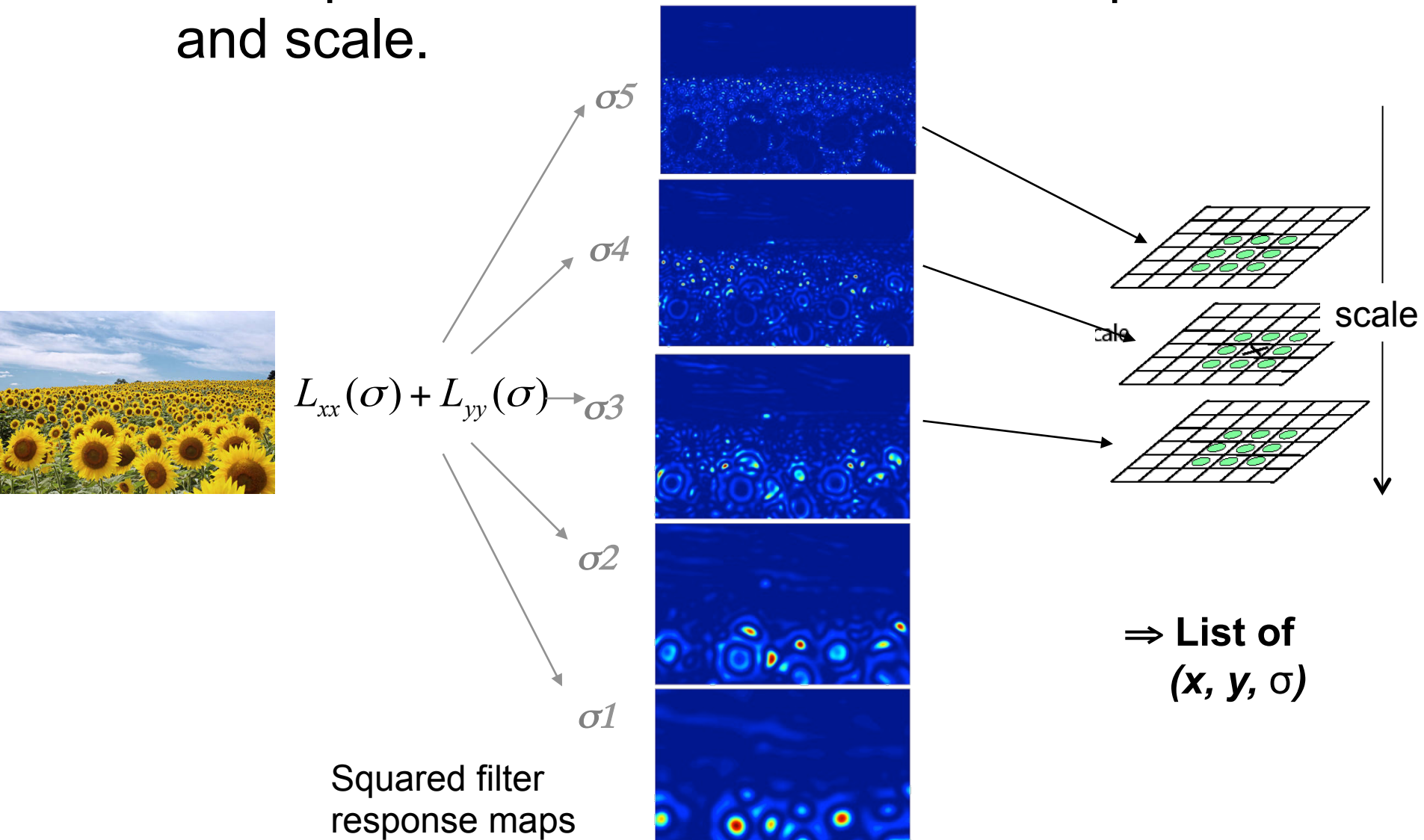
$\sigma = 1$



$\sigma = 66$

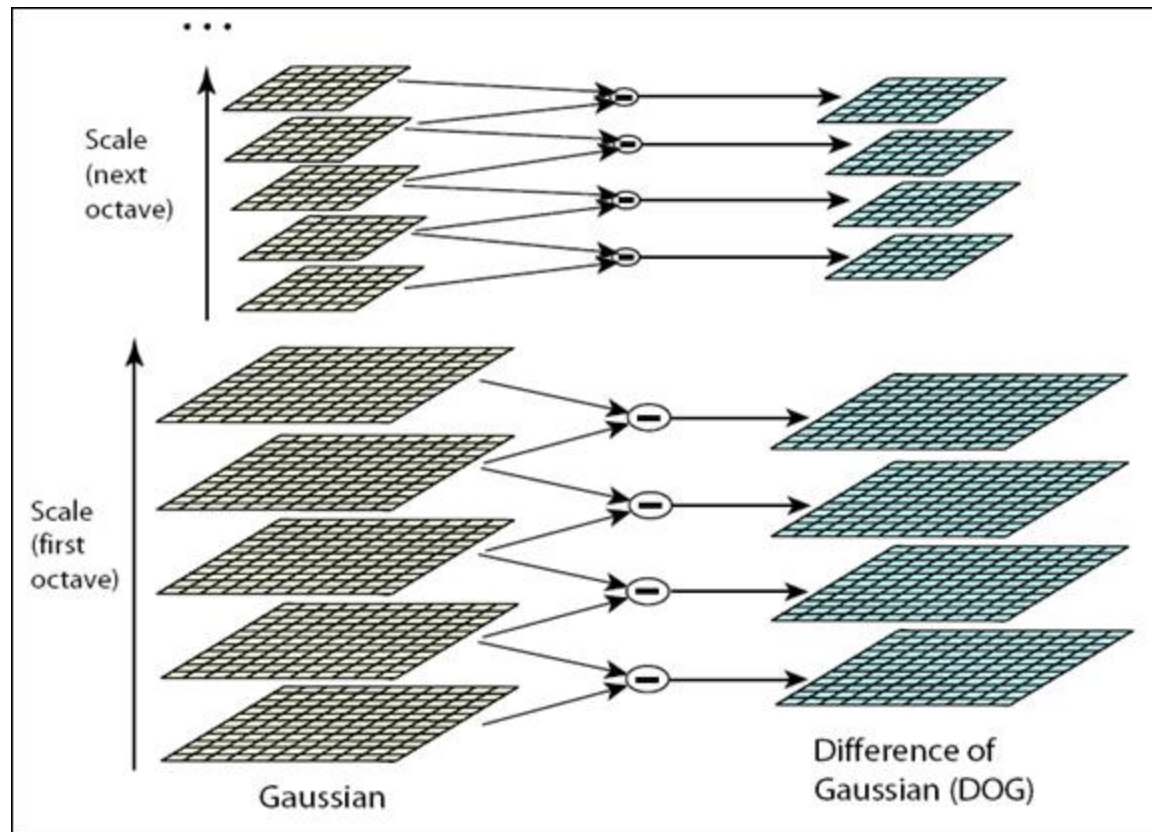
# Scale invariant interest points

Interest points are local maxima in both position and scale.



# Scale

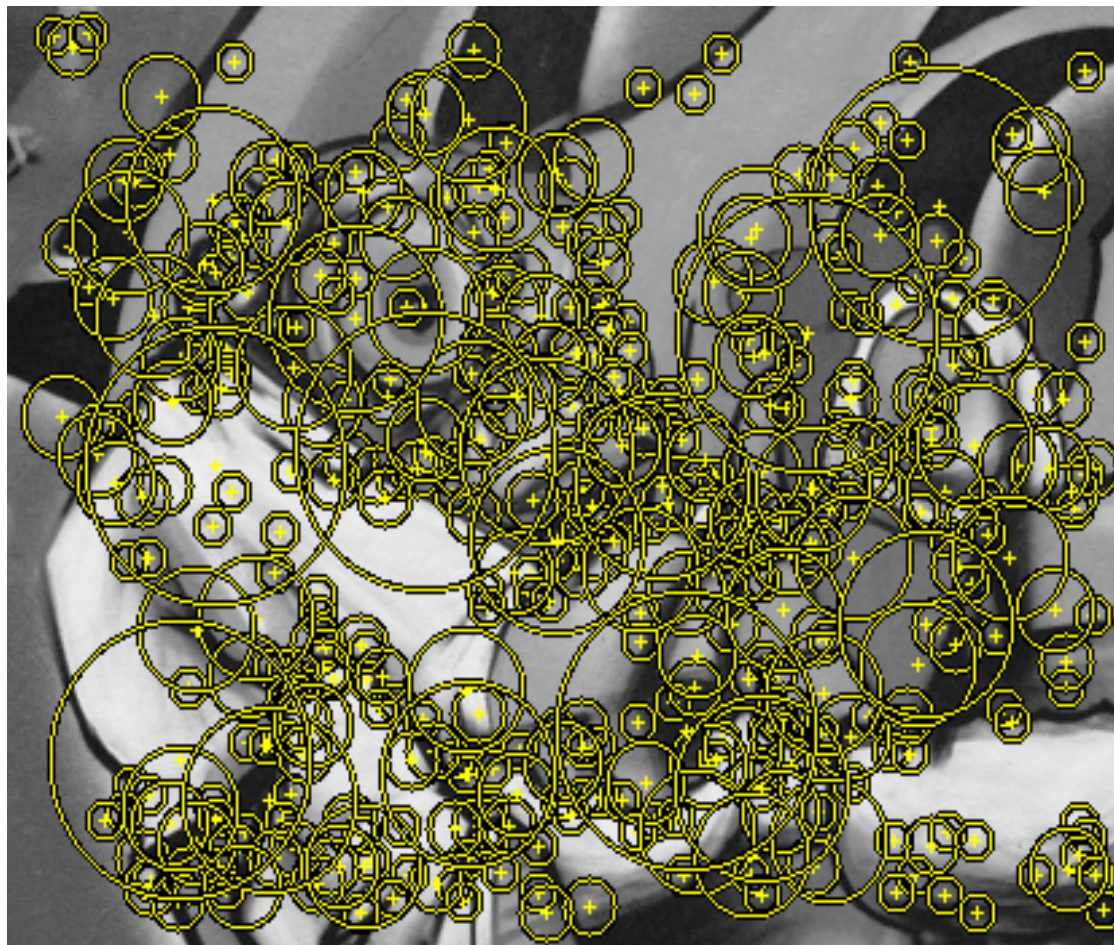
In practice the image is downsampled for larger sigmas.



Lowe, 2004.

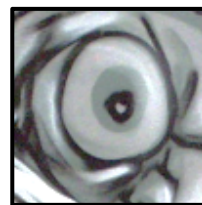
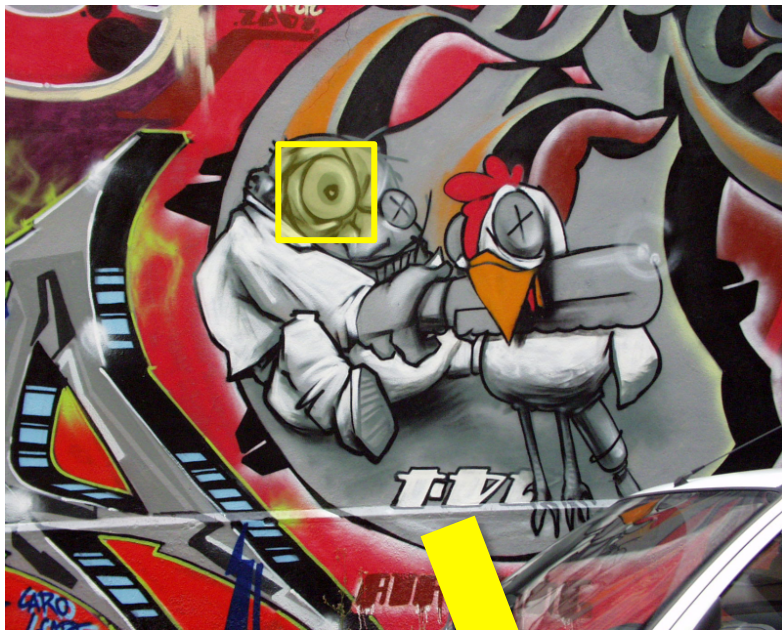
# Results: Difference-of-Gaussian

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K. Grauman, B. Leibe

# How can we find correspondences?

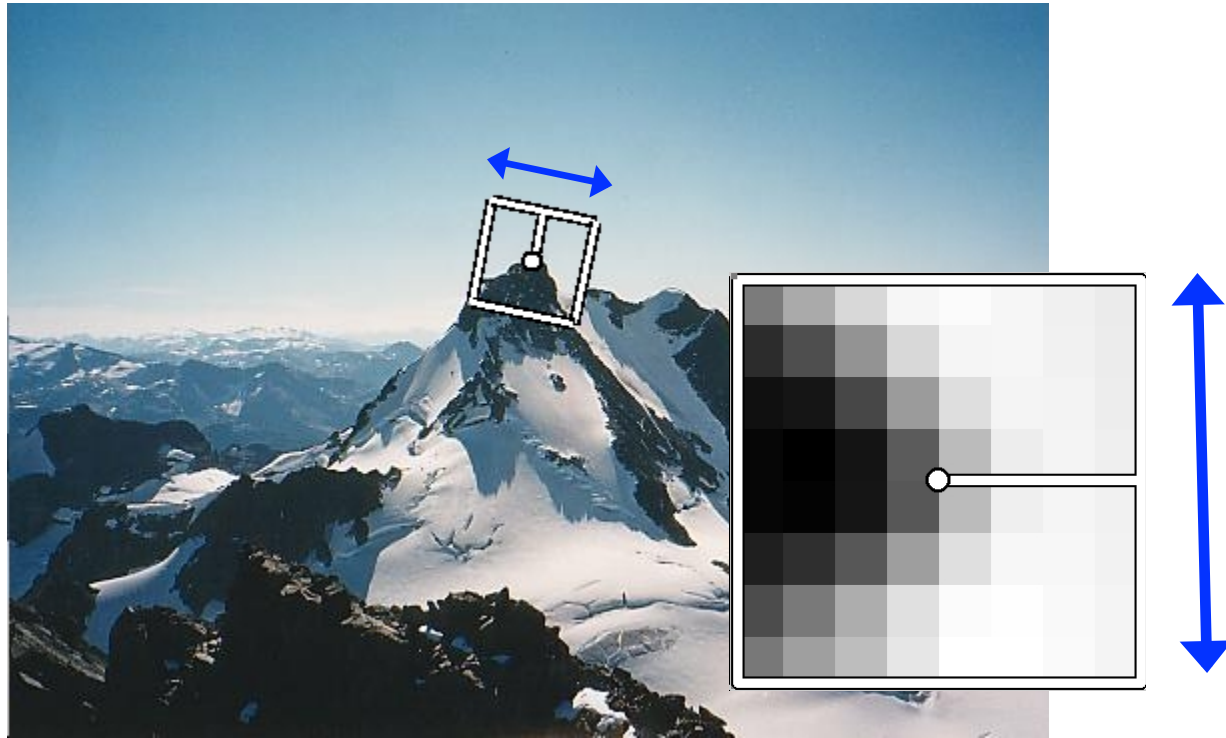


Similarity transform



# Rotation invariance

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- Rotate patch according to its dominant gradient orientation
- This puts the patches into a canonical orientation.

# Orientation Normalization

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- Compute orientation histogram
- Select dominant orientation
- Normalize: rotate to fixed orientation

[Lowe, SIFT, 1999]

