

# Geometric Transformations

EE/CSE 576

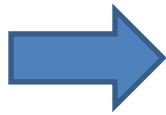
Linda Shapiro

# What are geometric transformations?



Why do we need them?

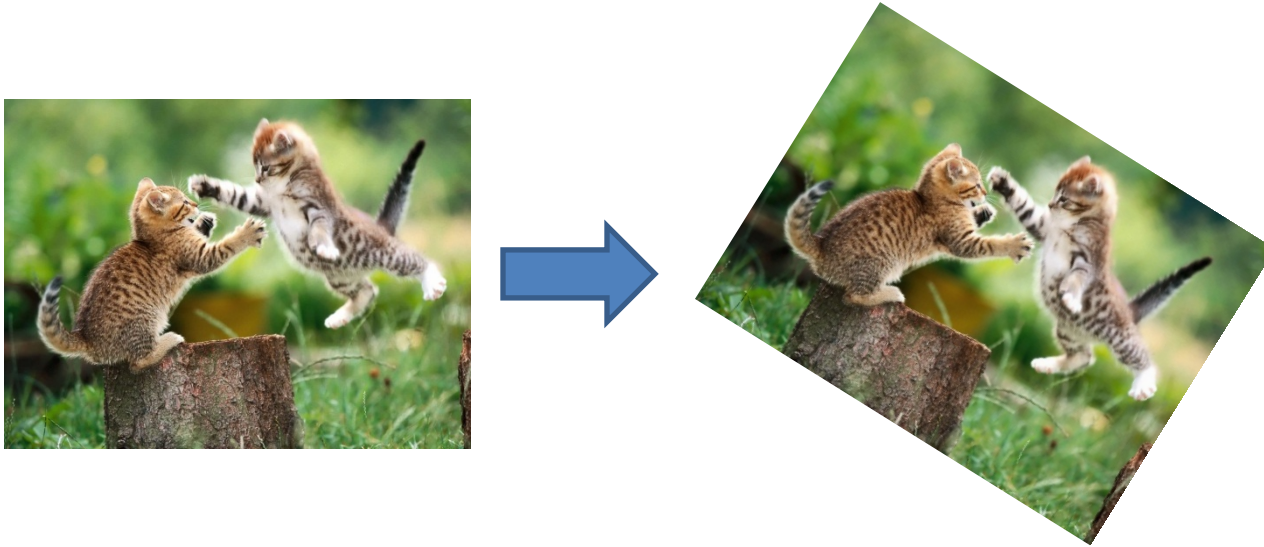
# Translation



$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$

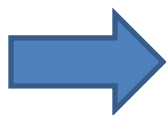
Preserves: Orientation

# Translation and rotation



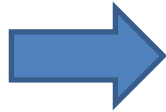
$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) & t_x \\ \sin(\theta) & \cos(\theta) & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

# Scale



$$\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

# Similarity transformations

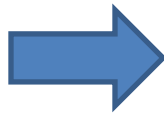


Similarity transform (4 DoF) = translation + rotation + scale

Preserves: Angles

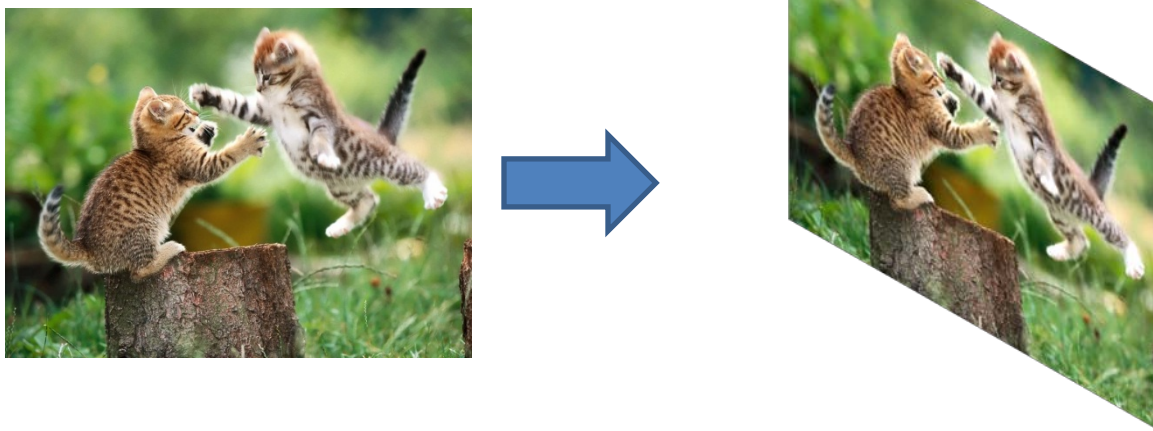


# Aspect ratio



$$\begin{bmatrix} a & 0 & 0 \\ 0 & \frac{1}{a} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

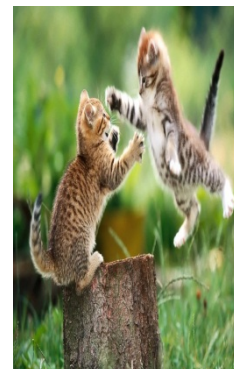
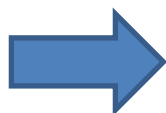
# Shear



$$\begin{bmatrix} 1 & a & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$



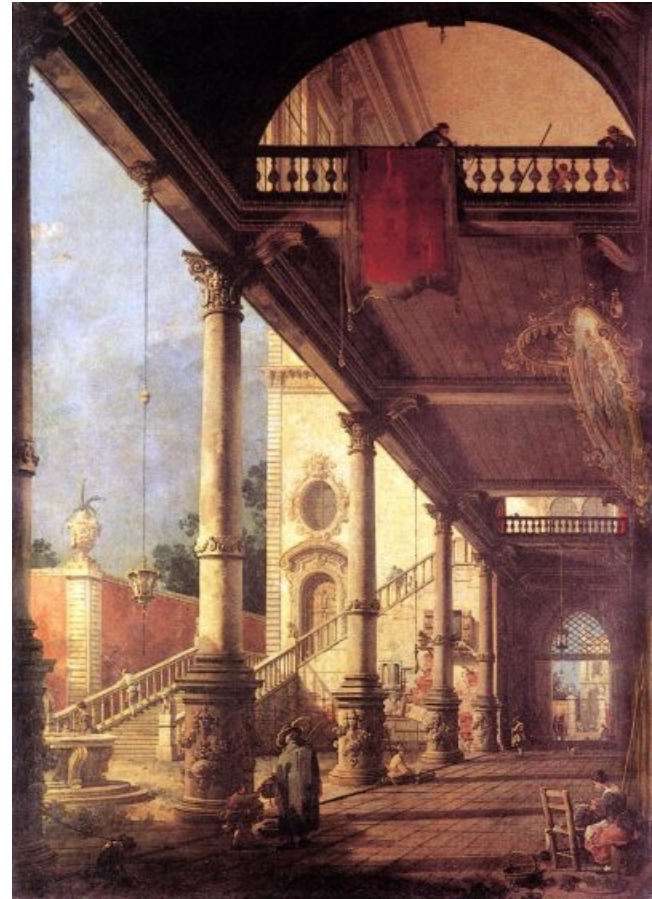
# Affine transformations



Affine transform (6 DoF) = translation + rotation + scale + aspect ratio + shear

Preserves: Parallelism

# What is missing?

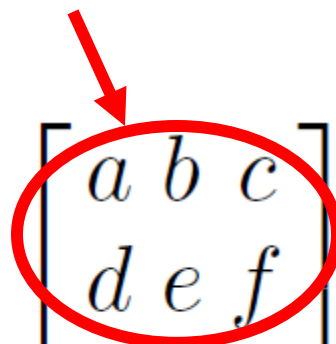


Canaletto

## Are there any other planar transformations?

# General affine

We already used these


$$\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

How do we compute projective transformations?

# Homogeneous coordinates

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

One extra step:

$$x' = u/w$$

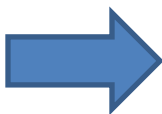
$$y' = v/w$$

# Projective transformations

a.k.a. Homographies

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \quad \begin{aligned} x' &= u/w \\ y' &= v/w \end{aligned}$$

“keystone” distortions



Preserves: Straight Lines

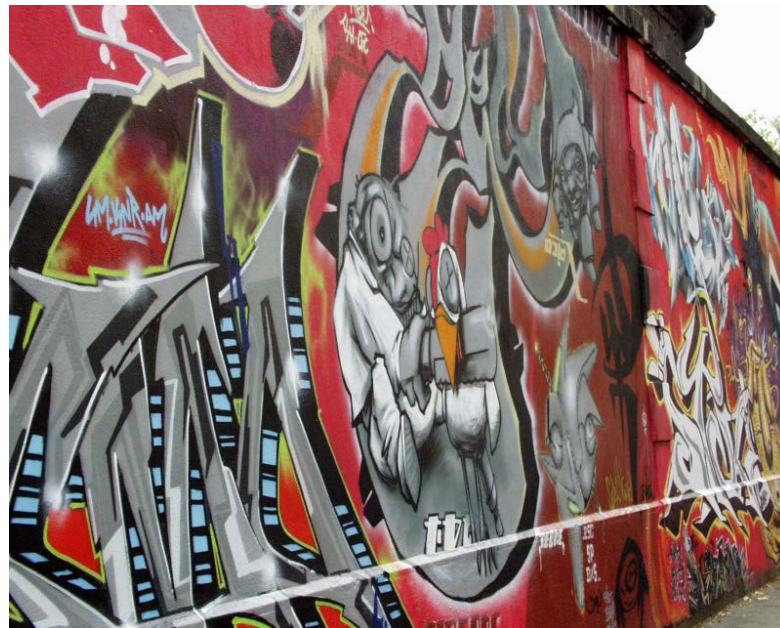
# Finding the transformation

Translation	=	2 degrees of freedom
Similarity	=	4 degrees of freedom
Affine	=	6 degrees of freedom
Homography	=	8 degrees of freedom

How many corresponding points do we need to solve?



# Finding the transformation



- How can we find the transformation between these images?
- How many corresponding points do we need to solve?

# What can I use homographies for?





# For one thing: Panoramas

