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Structure from Motion with Unknown Correspondence

(aka: How to combine EM, MCMC, Nonlinear LS!)

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slides courtesy of Frank Dellaert, adapted by Steve Seitz

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Traditionally: 2 Problems

1. Correspondence
2. Compute 3D positions, camera motion

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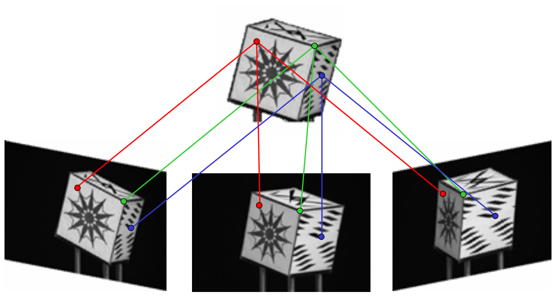
The Correspondence Problem



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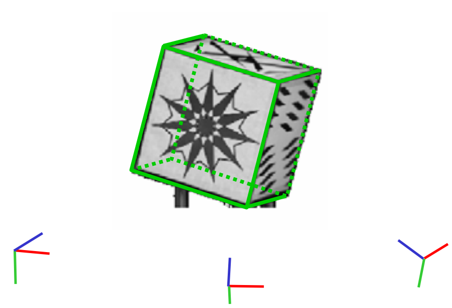
The Structure from Motion Problem

- Find the most likely structure and motion Θ



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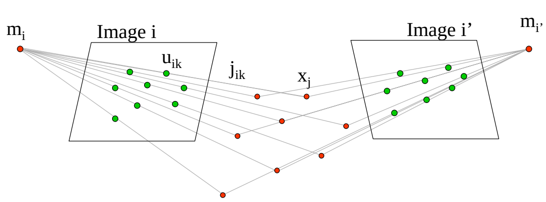
Structure From Motion

Non-linear Least-Squares

(Rick Szeliski's lecture)

$$\sum_{image\ i} \sum_{feature\ k} \|\mathbf{u}_{ik} - \mathbf{h}(\mathbf{m}_i, \mathbf{x}_{j_{ik}})\|^2$$



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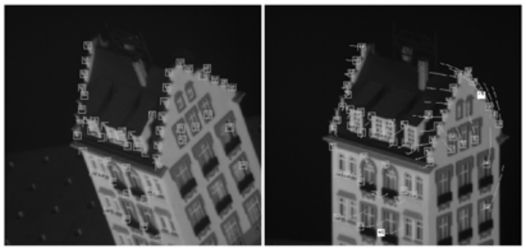
Question

How to recover structure and motion with **unknown correspondence**?

Aside: related subproblems

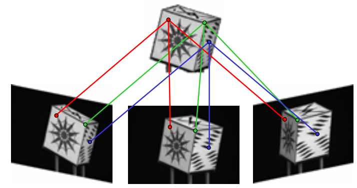
- **known** correspondence, **unknown** structure, motion
  - structure from motion
- **known** motion, **unknown** correspondence, structure
  - stereo
- **known** structure, **unknown** correspondence, motion
  - 3D registration, ICP

Without Correspondence: Try Tracking Features



Lucas-Kanade Tracker (figures from Tommasini et. al. 98)

New Idea: try all correspondences



Likelihood: $P(\Theta; U, J)$

structure + motion — image features — correspondence

Marginalize over J: $P(\Theta; U) = \Sigma_J P(\Theta; U, J)$

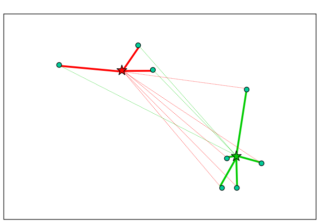
Combinatorial Explosion

- 3 images, 4 features: $4!^3$=13,824
- 5 images, 30 features: $30!^5$=1.3131e+162
- (number of stars:1e+20, atoms: 1e+79)

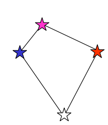
- Computing $P(\Theta ; U, J)$ is intractable !

EM for Correspondence

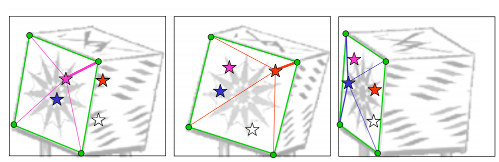
Expectation Maximization



E-Step: Soft Correspondences



$P(J \mid U, \Theta^t)$



View 1 View 2 View 3

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M-Step: SFM

- Compute *expected* SFM solution

$$\sum_{\mathbf{J}} P(\mathbf{J}|\mathbf{U},\Theta^t) \sum_{image\ i} \sum_{feature\ k} \|\mathbf{u}_{ik} - \mathbf{h}(\mathbf{m}_i, \mathbf{x}_{j_{ik}})\|^2$$

- Problem: this requires solving an exponential number of SFM problems
- If we assume correspondences j_{ik} are independent, we can simplify...

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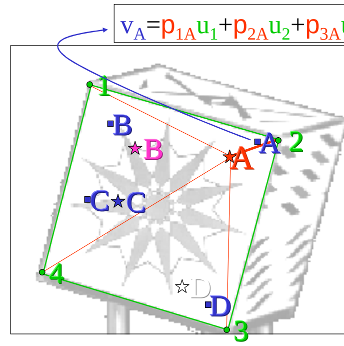
Virtual Measurements

$v_A = p_{1A}u_1 + p_{2A}u_2 + p_{3A}u_3 + p_{4A}u_4$



View 2

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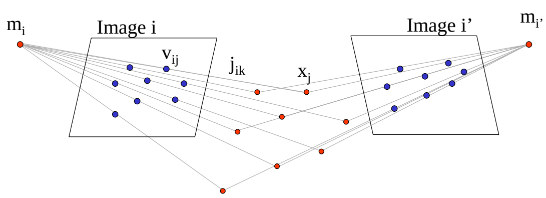
M-Step: SFM

one SFM problem on virtual measurements

$$\sum_{image\ i} \sum_{feature\ j} \|\mathbf{v}_{ij} - \mathbf{h}(\mathbf{m}_i, \mathbf{x}_j)\|^2$$



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Structure from Motion without Correspondence via EM:

1. Generate an initial structure and motion estimate Θ^0.
2. In each image, calculate the n^2 "soft correspondences"
3. Calculate the virtual measurements $\mathbf{v}_{ij}^t$.
4. Find the new estimate Θ^{t+1} for structure and motion using the virtual measurements $\mathbf{v}_{ij}^t$.
5. If not converged, return to step 2.

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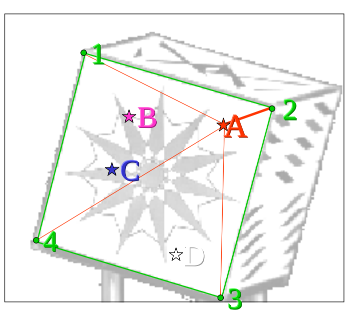
Key Issue: Calculating Weights



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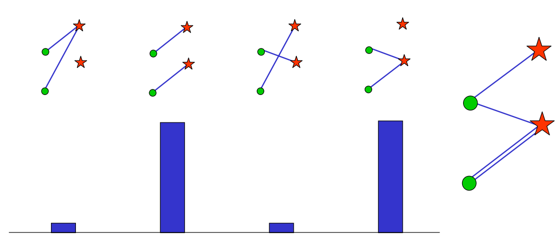
Calculating Weights



Weights = Marginal Probabilities

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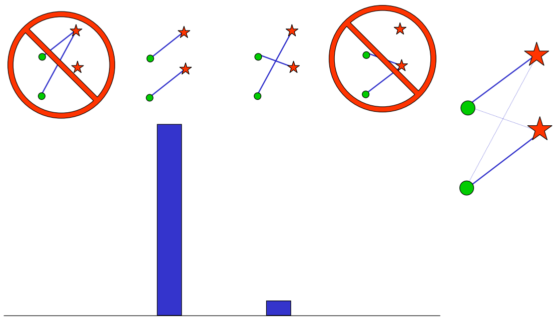
Mutual Exclusion



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Monte Carlo EM

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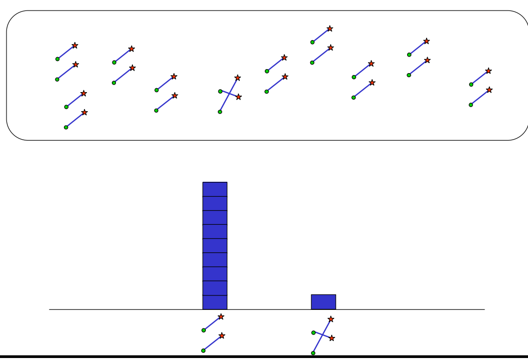
Monte Carlo Approximation



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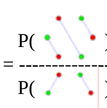
Sampling Assignments using the Metropolis Algorithm (1953)

To sample from $P(\mathbf{J}_i|\mathbf{U}_i,\Theta^t)$ we use the Metropolis algorithm:

1. Start with a valid initial assignment $\mathbf{J}_i^0$. 
2. Propose a new valid assignment $\mathbf{J}_i'$. 
3. Accept with ratio

$$a = \min\left(1, \frac{P(\mathbf{J}_i' \mid \mathbf{U}_i, \Theta^t)}{P(\mathbf{J}_i^r \mid \mathbf{U}_i, \Theta^t)}\right) =$$



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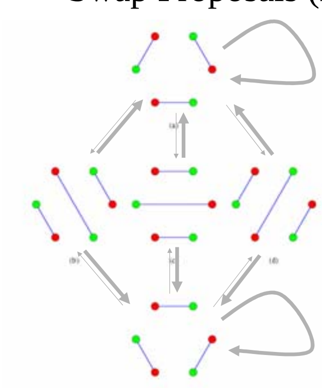
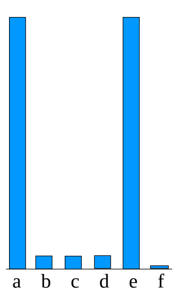
Swap Proposals (inefficient)





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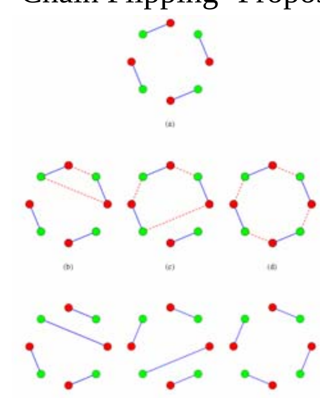
'Chain Flipping' Proposals



Efficient Sampling (Example)

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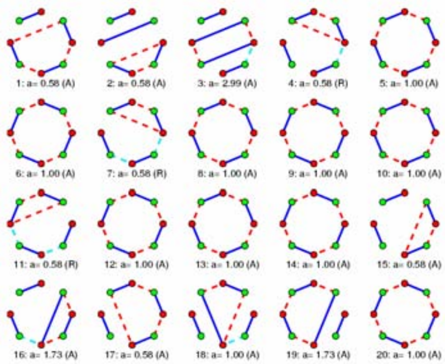

MCMC to deal with mutex

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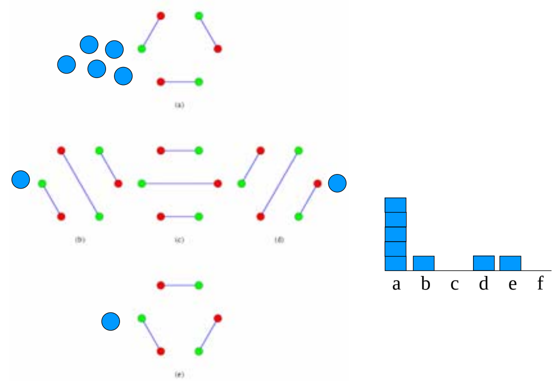

Input Images

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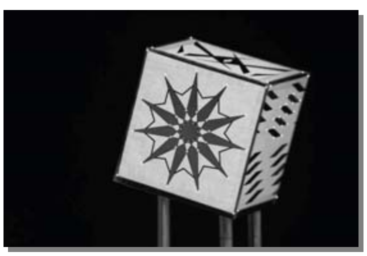
- Manual measurements
- Random order



Results: Cube

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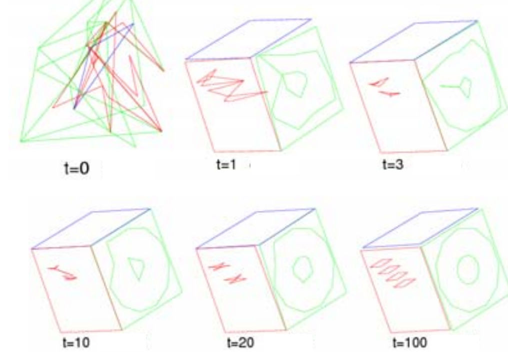

Soft Correspondences

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