CHEMICAL MASTER EQUATION

$$\dot{P}_n = -P_n \Gamma_{n\to m} + P_m \Gamma_{m\to n} \quad m \neq n$$

P_n : Prob. for state n

$\Gamma_{n\to m}$: Transition rate from $n \to m$

EXAMPLE 2 : $\xrightarrow{k_1} m \xrightarrow{k_2} \emptyset$

STATES $m = 0, 1, 2 \ldots$

$$\dot{P}_0 = -k_1 P_0 + k_2 P_1$$

$$\dot{P}_1 = +k_1 P_0 - (k_1 + k_2) P_1 + 2k_2 P_2$$

$$\dot{P}_2 = +k_1 P_1 - (k_1 + 2k_2) P_2 + 3k_2 P_3$$

$$\vdots$$

STEADY STATE

$$P_1 = \frac{k_1}{k_2} P_0$$

Adding the first two eqs.:

$$-k_1 P_1 + 2k_2 P_2 = 0 \Rightarrow P_2 = \frac{k_1}{2k_2} P_1 = \frac{k_1^2}{2k_2^2} P_0$$

Adding the first n eqs.:

$$P_n^* = \frac{\alpha^n}{n!} P_0^* \qquad \alpha = k_1/k_2$$

What is P_0

$$\sum_{n=0}^{\infty} P_n^* = \sum_{n=0}^{\infty} \frac{\alpha_n}{n!} P_0 = 1 \Rightarrow P_0^{-1} = \sum_{n=0}^{\infty} \frac{\alpha_n}{n!} = e^{\alpha}$$

$$P_0 = e^{-\alpha}, \quad P_n = \frac{\alpha^n}{n!} e^{-\alpha}$$

$$\langle n \rangle = \sum_{n=0}^{\infty} n P_n = \alpha = k_1/k_2$$

$$\langle n^2 \rangle = \sum n^2 P_n = \alpha + \alpha^2 \qquad SD: \sqrt{\langle n^2 \rangle - \langle n \rangle^2} = \sqrt{\alpha} = \sqrt{k_1/k_2}$$

EXAMPLE 3:

$$\xrightarrow{k_1} m \xrightarrow{k_2} \emptyset$$

$$m \xrightarrow{k_3} m + P$$

$$P \xrightarrow{k_4} \emptyset$$

m: # OF mRNA

n: # OF PROTEIN

$P_{n,m}$: PROB FOR STATE W/ n, m

$$\dot{P}_{n,m} = -k_1 P_{n,m} - k_2 m P_{n,m} + k_1 P_{n,m-1} + k_2 (m+1) P_{n,m+1}$$
$$- k_3 m P_{n,m} + k_3 m P_{n-1,m} - k_4 n P_{n,m} + k_4 (n+1) P_{n+1,m}$$

$\Rightarrow$ SEE V. SHAREZAEI + P. S. SWAIN PNAS (2008).