

# Finite Model Theory – Homework 2

April 7, 2018

## 1 Ehrenfeucht-Fraïssé Games

1. (0 points)

- (a) Prove that the following properties are not expressible in FO. Hint: reduce them to an inexpressibility problem that you already know well.
  - i. Given a directed graph  $G = (V, E)$ , check if  $|E|$  is even; in other words, check if the number of edges is even.
  - ii. Given a directed graph  $G = (V, E)$ , check if it is a tree.
- (b) Prove that the spoiler has a winning strategy for the EF game with  $k = 4$  pebbles, on the pair of structures  $C_{12}$  and  $C_6 \cup C_6$ . Here  $C_{12}$  represents a directed cycle of length 12:

$$1 \rightarrow 2 \rightarrow \dots \rightarrow 12 \rightarrow 1$$

while  $C_6 \cup C_6$  is the disjoint union of two directed cycles of length 6. Your answer may be informal, but clear enough to allow a human player to win the game (as spoiler).

## 2 Pebble Games

2. (0 points)

When needed, you can use informal arguments to prove that the spoiler, or the duplicator has a strategy in a  $k$ -pebble game.

- (a) Prove that the following query cannot be expressed in  $L_{\infty\omega}^\omega$  (and, hence, neither in datalog): given two unary relations  $A, B$ , check whether  $|A| \leq |B|$ .
- (b) An Eulerian graph is an undirected graph that has a cycle that traverses each edge exactly once. Prove that the property “ $G$  is Eulerian” is not expressible in  $L_{\infty\omega}^\omega$ . Hint: you may want to do a quick search to learn about Eulerian graphs.