

Finite Model Theory – Homework 5

May 13, 2018

1 Algorithmic Finite Model Theory

1. (0 points)

(a) Consider the query:

$$Q(x, y, z, u) = R(x, y) \wedge S(y, z) \wedge T(z, u) \wedge K(u, x)$$

Suppose the four relations have cardinalities N_1, N_2, N_3, N_4 .

Give a formula that represents a tight upper bound on $|Q|$. Your formula should use the cardinalities N_1, N_2, N_3, N_4 and operations like $+, \times, /, \wedge, \max$, for example $\max(N_1/N_2, N_3^{3/2} + N_4)$ (not a real answer).

(b) Consider the same query as above, and repeat your answer for the case when y is a key in S :

$$Q(x, y, z, u) = R(x, y) \wedge S(\underline{y}, z) \wedge T(z, u) \wedge K(u, x)$$

(c) Suppose $|R|, |S|, |T| \leq N$, no upper bound is given for $|A|, |B|$, and the following key constraints hold: $A(x, z, u) : xz \rightarrow u$ and $B(x, y, u) : yu \rightarrow x$. Compute the maximum size of the query:

$$Q(x, y, z, u) = R(x, y) \wedge S(y, z) \wedge T(z, u) \wedge A(\underline{x}, z, u) \wedge B(x, \underline{y}, u)$$

Perhaps a more suggestive notation for the query is:

$$Q(x, y, z, u) = R(x, y) \wedge S(y, z) \wedge T(z, u) \wedge (xz \rightarrow u) \wedge (yu \rightarrow x)$$