Functional Aggregate Queries are FAQs

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Relational AI Inc.

Based on joint work with Mahmoud Abo Khamis and Atri Rudra

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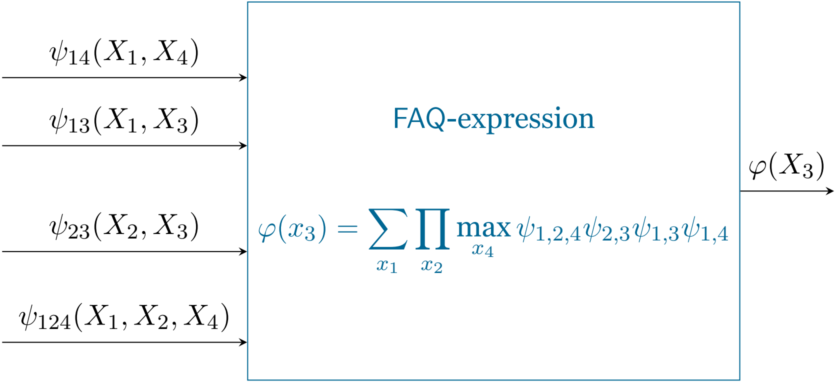
- x_i are values in discrete domain $\mathsf{Dom}(X_i)$
- $\boldsymbol{x} = (x_1, \ldots, x_n) \in \mathsf{Dom}(X_1) \times \cdots \times \mathsf{Dom}(X_n)$
- For any $S \subseteq [n]$,

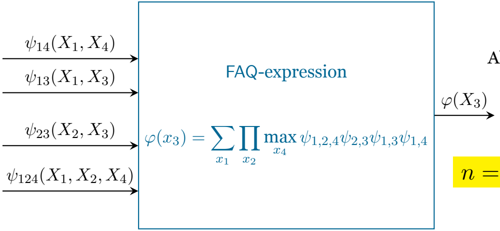
$$
\begin{aligned}
\boldsymbol{x}_S &= (x_i)_{i \in S} \in \prod_{i \in S} \mathsf{Dom}(X_i) \\
\text{e.g. } \boldsymbol{x}_{\{2,5,8\}} &= (x_2, x_5, x_8) \in \mathsf{Dom}(X_2) \times \mathsf{Dom}(X_5) \times \mathsf{Dom}(X_8)
\end{aligned}
$$

Function Aggregate Query: the Problem



All functions have the same range **D**

Function Aggregate Query: the Input



All functions have the same range **D**

$n = 4$

- n *variables* $X_1, \ldots, X_n$

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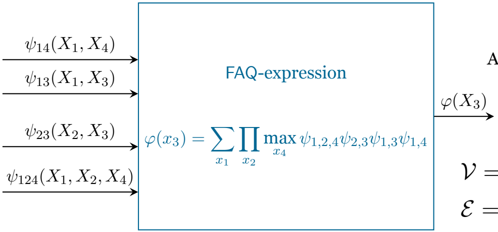

All functions have the same range **D**

$$\mathcal{V} = \{1, 2, 3, 4\}$$
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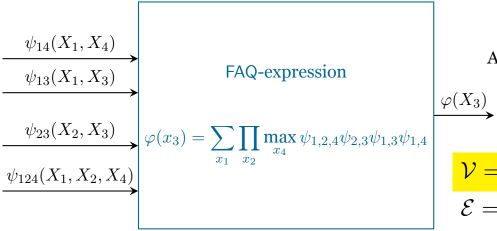

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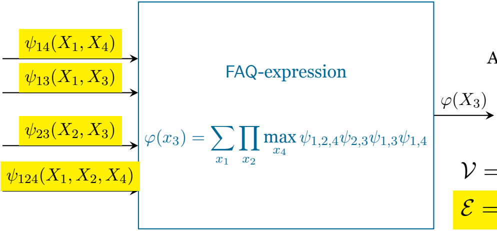

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$$\psi_S : \prod_{i \in S} \mathsf{Dom}(X_i) \to \mathbf{D}$$

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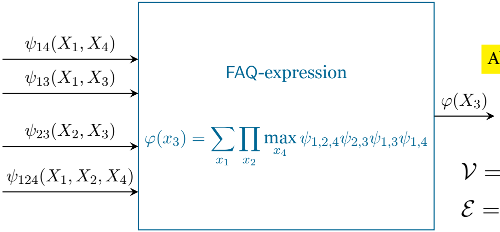

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$\mathbb{R}_+$, $\{\mathsf{true}, \mathsf{false}\}$, $\{0, 1\}$, $2^{\mathcal{U}}$, etc.

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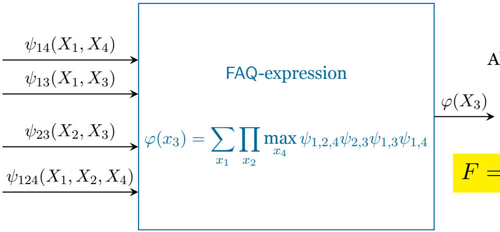

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$F = \{3\}$

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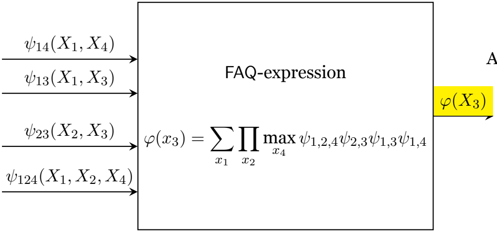
$\uparrow$

$\mathbb{R}_+$, $\{\mathsf{true}, \mathsf{false}\}$, $\{0, 1\}$, $2^{\mathcal{U}}$, etc.

- a set $F \subseteq \mathcal{V}$ of *free variables* (wlog, $F = [f] = \{1, \ldots, f\}$)

Function Aggregate Query: the Output



All functions have the same range $\mathbf{D}$

- Compute the function $\varphi : \prod_{i \in F} \mathsf{Dom}(X_i) \to \mathbf{D}$.

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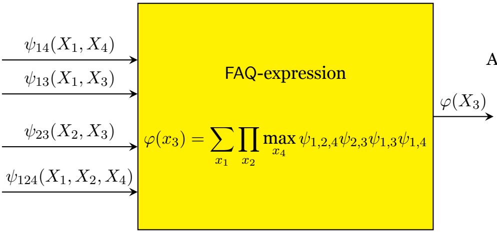

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$$\varphi(\boldsymbol{x}_{[f]}) = \bigoplus^{(f+1)}_{x_{f+1} \in \mathsf{Dom}(X_{f+1})} \cdots \bigoplus^{(n-1)}_{x_{n-1} \in \mathsf{Dom}(X_{n-1})} \bigoplus^{(n)}_{x_n \in \mathsf{Dom}(X_n)} \bigotimes_{S \in \mathcal{E}} \psi_S(\boldsymbol{x}_S)$$

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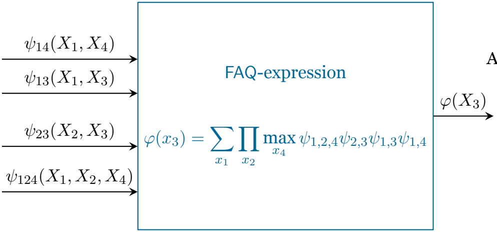

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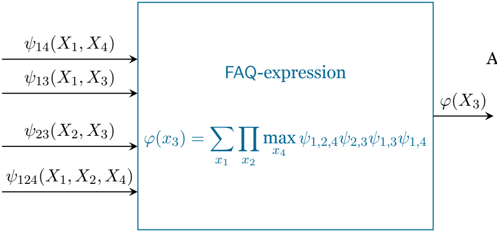

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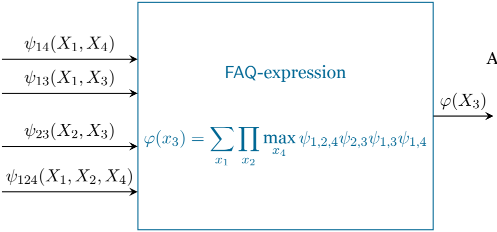

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- Every aggregate $\bigoplus^{(i)}$, either
  - $\left(\mathbf{D}, \bigoplus^{(i)}, \bigotimes\right)$ is a *commutative semiring* (semiring agg)
  - or $\bigoplus^{(i)} = \bigotimes$ (product agg)

SumProd = FAQ-SS *without* free variables

- SumProd: compute the *constant*

$$\varphi = \bigoplus_{x_1} \bigoplus_{x_2} \cdots \bigoplus_{x_n} \bigotimes_{S \in \mathcal{E}} \psi_S(\boldsymbol{x}_S) = \bigoplus_{\boldsymbol{x}} \bigotimes_{S \in \mathcal{E}} \psi_S(\boldsymbol{x}_S)$$

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- Where $(\mathbf{D}, \oplus, \otimes, \mathbf{0}, \mathbf{1})$ is a commutative semiring

Additive identity	$\mathbf{0} \in \mathbf{D} : \mathbf{0} \oplus e = e \oplus \mathbf{0} = e$
Multiplicative identity	$\mathbf{1} \in \mathbf{D} : \mathbf{1} \otimes e = e \otimes \mathbf{1} = e$
Annihilation by $\mathbf{0}$	$\mathbf{0} \otimes e = e \otimes \mathbf{0} = \mathbf{0}$
Distributive law	$a \otimes b \oplus a \otimes c = a \otimes (b \oplus c)$

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- Common examples (there are many more!)

Boolean	$(\{\text{true}, \text{false}\}, \vee, \wedge, \text{false}, \text{true})$
sum-product	$(\mathbb{R}, +, \times, 0, 1)$
max-product	$(\mathbb{R}_+, \max, \times, 0, 1)$
min-plus	$(\mathbb{R} \cup \{\infty\}, \min, +, \infty, 0)$
min-product	$(\mathbb{R}_{>0} \cup \{\infty\}, \min, \times, \infty, 1)$
set	$(2^U, \cup, \cap)$

Problem (CSP = Boolean Join Query)

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- constraint $\psi_S : \prod_{i \in S} \mathsf{Dom}(X_i) \to \{\mathsf{true}, \mathsf{false}\}$ with *support* S
- truth assignment satisfying all constraints
- over all possible truth assignments

Problem (#CSP = Join cardinality query)

count number of satisfying assignments x

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- Sum-product semiring $(\{0, 1\}, +, \times)$

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$$\psi_S(\boldsymbol{x}) = \begin{cases} 1 & \boldsymbol{x}_S \text{ satisfies constraint} \\ 0 & \text{otherwise} \end{cases}$$

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- count 1 only if $\boldsymbol{x}$ satisfies all constraints
- sum over all possible truth assignments

Many examples for FAQ-SS w/o free variables

- Boolean conjunctive query evaluation (Boolean semiring)
- SAT (Boolean semiring)
- Quantifier-free conjunctive query evaluation (set semiring)
- k-colorability (Boolean)
- Permanent (Sum-Product semiring)
- Partition function (Sum-Product semiring)
- etc.

- Sum-Prod = Marginalize a Product Function problem
  - Dechter (Artificial Intelligence 1999)
  - Aji and McEliece (IEEE Trans. Inform. Theory 2000)

Adding Free (i.e. GroupBy) Variables

Problem (FAQ-SS with free variables)

Compute the function

$$\varphi(\boldsymbol{x}_{[f]}) = \bigoplus_{x_{f+1}} \cdots \bigoplus_{x_n} \bigotimes_{S \in \mathcal{E}} \psi_S(\boldsymbol{x}_S) = \bigoplus_{\boldsymbol{x}_{[n]-[f]}} \bigotimes_{S \in \mathcal{E}} \psi_S(\boldsymbol{x}_S)$$

Problem (Conjunctive Query Evaluation – CQE)

$$\Phi(\boldsymbol{x}_{[f]}) = \exists_{x_{f+1}} \cdots \exists_{x_n} \bigwedge_{R \in \mathsf{atoms}(\Phi)} R(\mathsf{vars}(R))$$

- Boolean Semiring $(\{\mathsf{true}, \mathsf{false}\}), \vee, \wedge)$
- FAQ-SS instance:

$$\varphi(\boldsymbol{x}_{[f]}) = \bigvee_{x_{f+1}} \cdots \bigvee_{x_n} \bigwedge_{S \in \mathcal{E}} \varphi_S(\boldsymbol{x}_S).$$

Problem (DB Aggregate Query – count)

```
SELECT count(*)
FROM R, S, T
WHERE R.a=S.a AND R.b=T.b AND S.c=T.c
GROUP BY R.a;
```

- Sum-product semiring $(\mathbb{R}, +, \times, 0, 1)$,

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  - $\psi_T(b,c) = \mathbf{1}_{(b,c)\in T}$
  - Compute the function

$$\varphi(a) = \sum_b \sum_c \psi_R(a,b)\psi_S(a,c)\psi_R(b,c)$$

Problem (DB Aggregate Query – sum)

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SELECT sum(T.c)
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Problem (Matrix Chain Multiplication – MCM)

Let $\boldsymbol{A}_i = (a^{(i)}_{x,y})$, *compute*

$$\underbrace{\boldsymbol{A}}_{p_0 \times p_k} = \underbrace{\boldsymbol{A}_1}_{p_0 \times p_1} \times \underbrace{\boldsymbol{A}_2}_{p_1 \times p_2} \times \cdots \underbrace{\boldsymbol{A}_k}_{p_{k-1} \times p_k} .$$

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  - Compute new function $\varphi : [p_0] \times [p_k] \to \mathbf{D}$

$$\varphi(x_0, x_k) = \sum_{x_1} \cdots \sum_{x_{k-1}} \prod_{i=0}^{k-1} \psi_{i,i+1}(x_i, x_{i+1}).$$

Inference in Probabilistic Graphical Models

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  - Compute some marginal distribution
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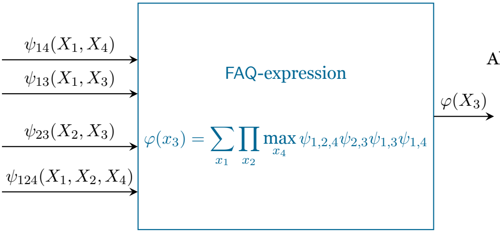
- These problems are *precisely* FAQ-SS on
  - $(\mathbb{R}_+, +, \times)$
  - $(\mathbb{R}_+, \max, \times)$

Many more examples

- Discrete Fourier Transform
- Hollant Problem (as in Holographic algorithms)
- Graph Homomorphism Problem
- Weighted CSP
- List recoverable codes
- LDPC codes
- With a squint: also called Aggregate over Factorized DB
  - Bakibayev et al. (VLDB 2014), Olteanu-Zavodny (TODS 2015)
- etc.

Why the generality of FAQ again?



All functions have the same range $\mathbf{D}$

- Compute the function $\varphi : \prod_{i \in F} \mathsf{Dom}(X_i) \to \mathbf{D}$.
- φ defined by the FAQ-*expression*

$$\varphi(\boldsymbol{x}_{[f]}) = \bigoplus_{x_{f+1} \in \mathsf{Dom}(X_{f+1})}^{(f+1)} \cdots \bigoplus_{x_{n-1} \in \mathsf{Dom}(X_{n-1})}^{(n-1)} \bigoplus_{x_n \in \mathsf{Dom}(X_n)}^{(n)} \bigotimes_{S \in \mathcal{E}} \psi_S(\boldsymbol{x}_S)$$

- For each $\bigoplus^{(i)}$
  - Either $\left(\mathbf{D}, \bigoplus^{(i)}, \bigotimes\right)$ is a *commutative semiring*
  - Or $\bigoplus^{(i)} = \bigotimes$

Quantified Conjunctive Queries

Problem (QCQ with free variables)

Given $Q_i \in \{\exists, \forall\}$, *for* $i > f$.

$$\Phi(X_1, \ldots, X_f) = Q_{f+1} X_{f+1} \cdots Q_n X_n \left(\bigwedge_{R \in \mathsf{atoms}(\Phi)} R \right),$$

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- FAQ instance:
  - $(\{0,1\}, \{\max, \times\}, \times)$
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- FAQ instance:
  - $(\{0,1\} \cup \mathbb{R}_+, \{\max, \times, +\}, \times)$
  - Compute the constant

$$\varphi = \sum_{x_1} \cdots \sum_{x_f} \bigoplus_{x_{f+1} \in \{0,1\}}^{(f+1)} \cdots \bigoplus_{x_n \in \{0,1\}}^{(n)} \prod_{S \in \mathcal{E}} \psi_S(\boldsymbol{x}_S)$$

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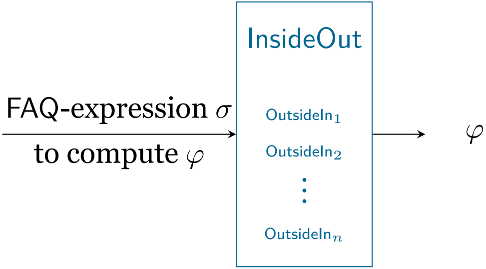

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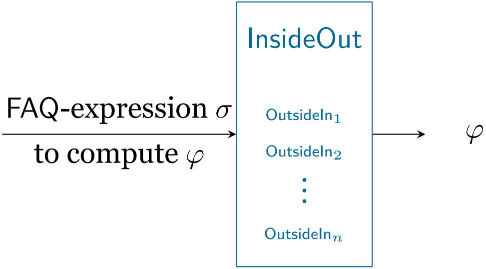

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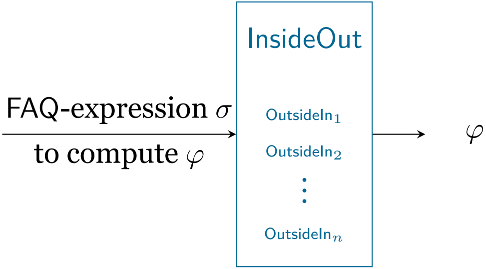

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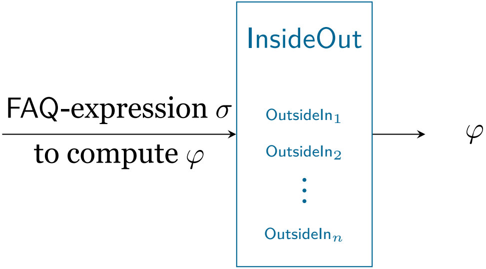

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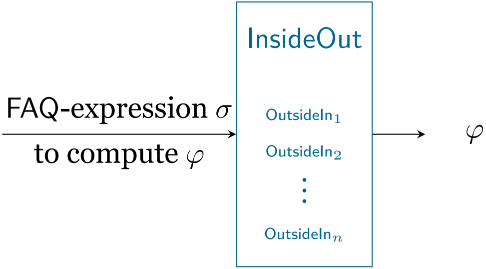

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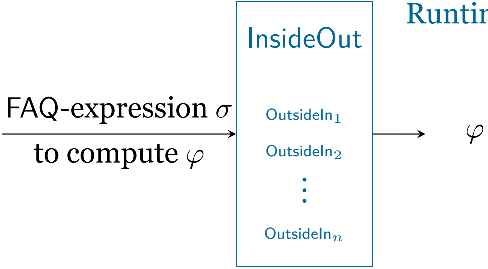
Runtime = $\tilde{O}(N^{\mathsf{faqw}(\sigma)} + |\varphi|)$



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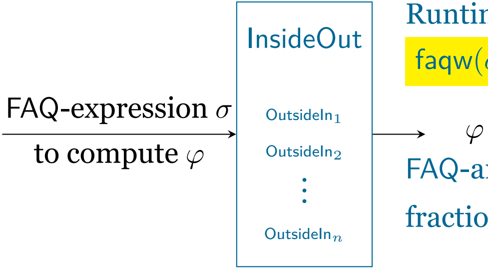

Runtime = $\tilde{O}(N^{\mathsf{faqw}(\sigma)} + |\varphi|)$

$\mathsf{faqw}(\sigma)$ is the "width" of σ

FAQ-analog of fractional hypertree width

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Roadmap

FAQ-expr. σ
for φ, hypergraph $\mathcal{H}$

Roadmap

$\mathsf{EVO}(\varphi)$ = set of expressions
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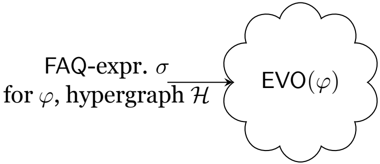

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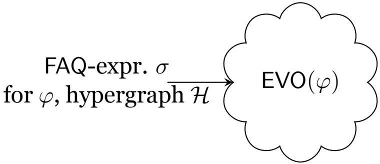

$$\varphi(x_3) = \sum_{x_2} \sum_{x_1} \sum_{x_5} \max_{x_4} \psi_{1,2,4}\psi_{2,3}\psi_{1,3}\psi_{1,4}\psi_{2,5}$$

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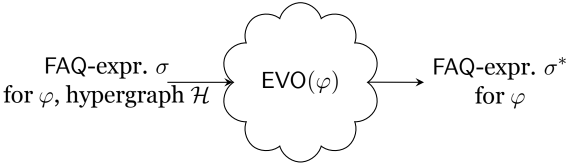
$$\sigma^* = \arg\min_{\tau \in \mathsf{EVO}(\varphi)} \mathsf{faqw}(\tau)$$



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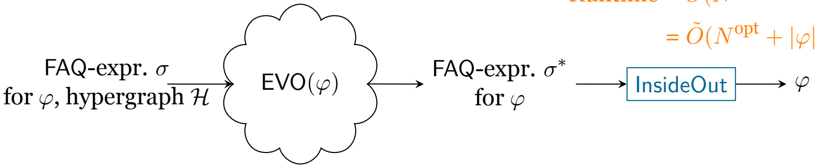

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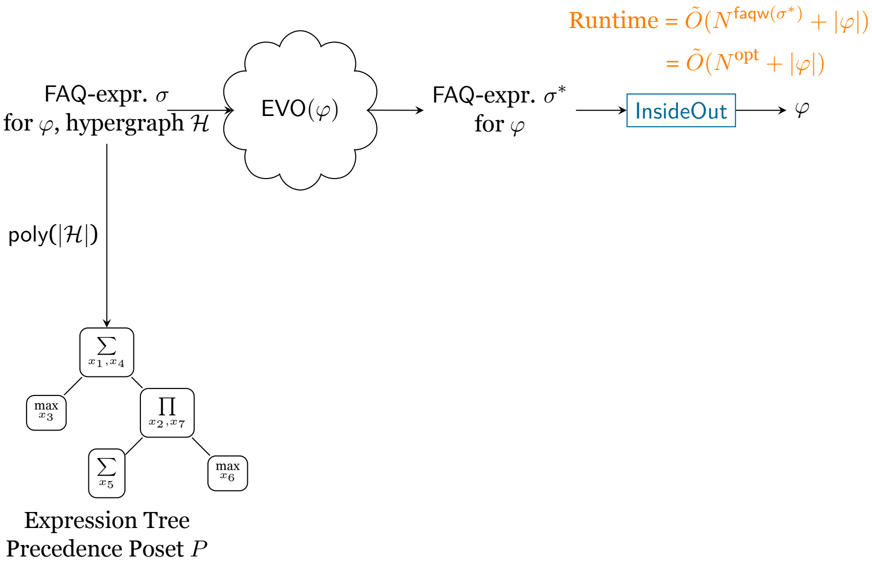
Runtime = $\tilde{O}(N^{\mathsf{faqw}(\sigma^*)} + |\varphi|)$
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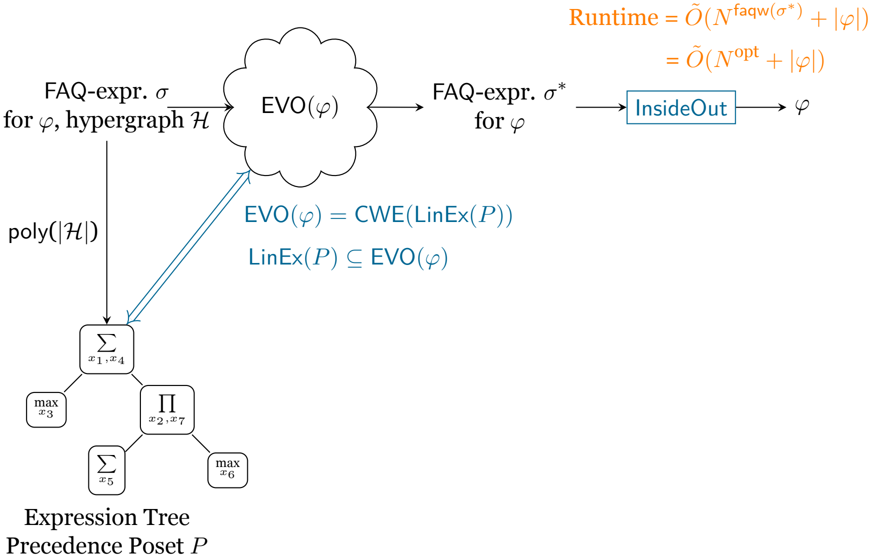

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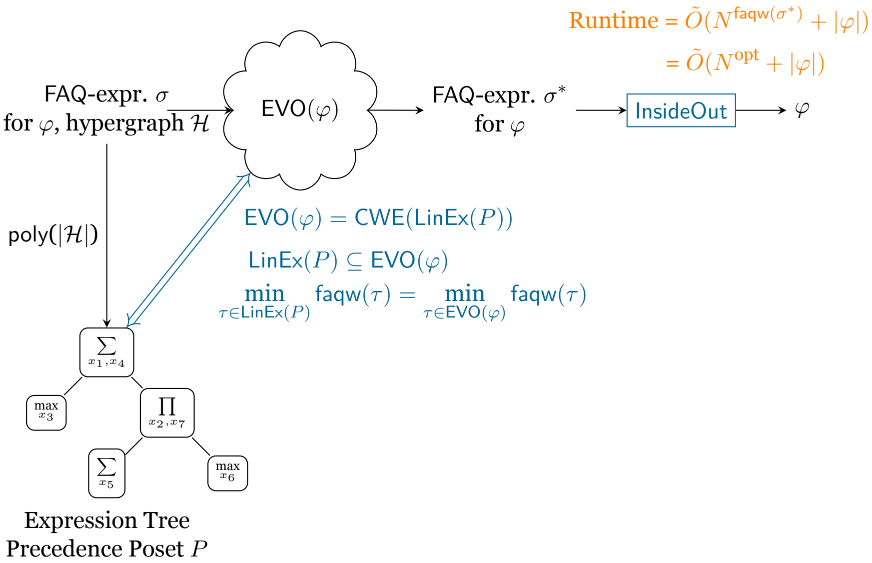

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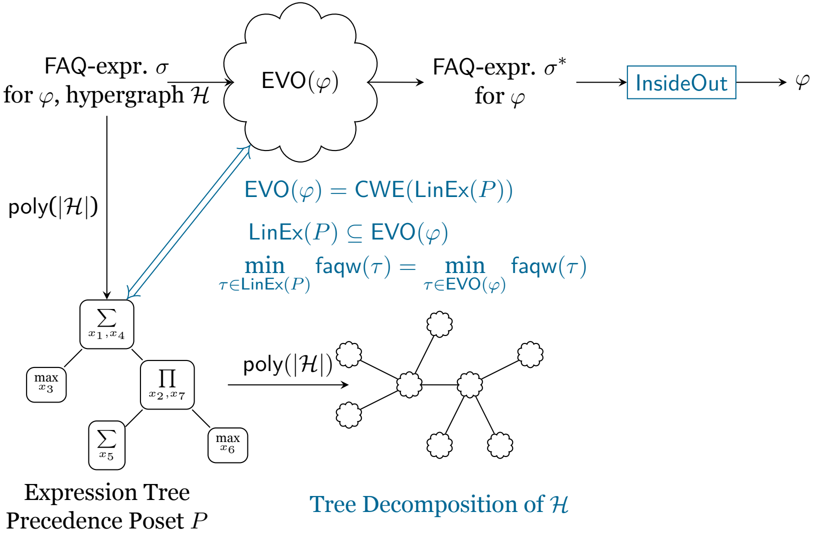
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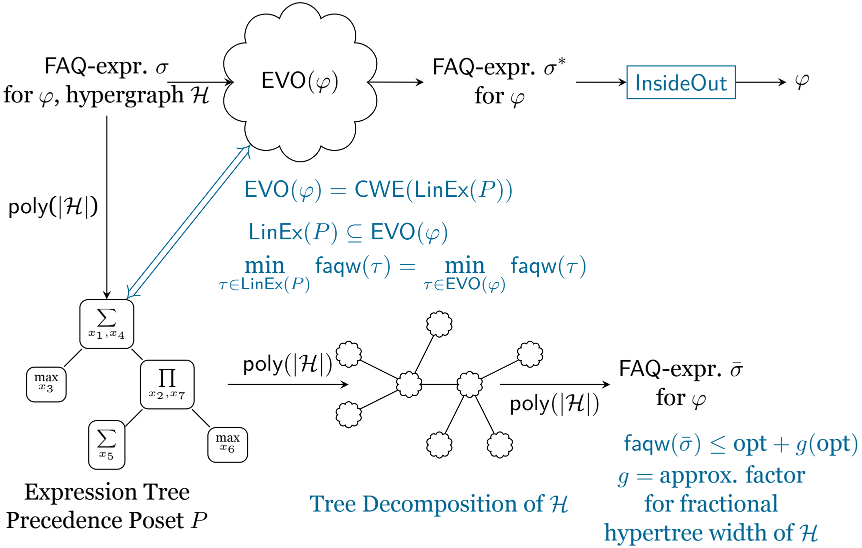

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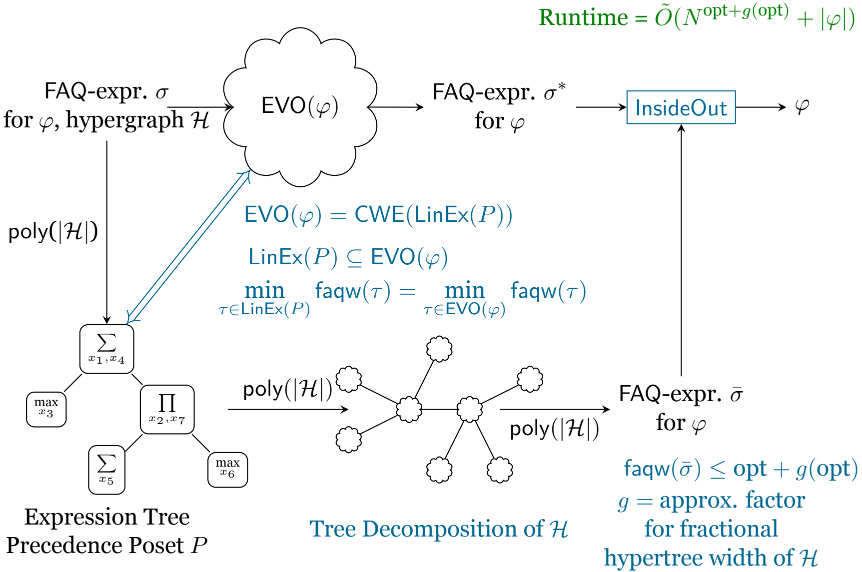

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Some Corollaries

Problem	Previous Algo.	InsideOut
#QCQ	No non-trivial algo	$\tilde{O}(N^{\mathsf{faqw}(\varphi)} + \lvert\varphi\rvert)$
QCQ	$\tilde{O}(N^{\mathsf{PW}(\mathcal{H})} + \lvert\varphi\rvert)$ Chen-Dalmau (LICS 2012)	$\tilde{O}(N^{\mathsf{faqw}(\varphi)} + \lvert\varphi\rvert)$ $\mathsf{faqw}(\varphi) \lneqq \mathsf{PW}(\varphi)$
#CQ	$\tilde{O}(N^{\mathsf{DM}(\mathcal{H})} + \lvert\varphi\rvert)$ Durand-Mengel (ICDT 2013)	$\tilde{O}(N^{\mathsf{faqw}(\varphi)} + \lvert\varphi\rvert)$ $\mathsf{DM}(\mathcal{H}) = \mathsf{faqw}(\varphi)$
Joins	$\tilde{O}\left(N^{\mathsf{fhtw}(\mathcal{H})} + \lvert\varphi\rvert\right)$ Grohe-Marx (SODA'06)	$\tilde{O}\left(N^{\mathsf{faqw}(\varphi)} + \lvert\varphi\rvert\right)$ $\mathsf{fhtw}(\mathcal{H}) = \mathsf{faqw}(\varphi)$
Marginal Distrib. MAP query	$\tilde{O}(N^{\mathsf{htw}(\varphi)} + \lvert\varphi\rvert)$ $\tilde{O}(N^{\mathsf{htw}(\varphi)} + \lvert\varphi\rvert)$ Kask et al. (Artif. Intel. 2005)	$\tilde{O}(N^{\mathsf{faqw}(\varphi)} + \lvert\varphi\rvert)$ $\tilde{O}(N^{\mathsf{faqw}(\varphi)} + \lvert\varphi\rvert)$ $\mathsf{faqw}(\varphi) \lneqq \mathsf{htw}(\varphi)$
Matrix Chain Mult.	DP bound	DP Bound
DFT	$O(N \log_p N)$	$O(N \log_p N)$
	Aji-McEliece (IEEE Trans. IT 2000) Dechter (Artif. Intell. 1999) Textbook	

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  - SAT $\in$ **P** for β-acyclic queries

How are factors accessed?

- A factor oracle for each input factor ψ_S
  - Oracle = (complexity people's) fancy name for an `API`
- Output factor: a table of tuples $\langle \boldsymbol{x}_F, \varphi[\boldsymbol{x}_F] \rangle$ but only store tuples for which $\varphi[\boldsymbol{x}_F] \neq \mathbf{0}$
- Called the listing representation

Example (Salary : $\{\text{ID}, \text{Name}\} \to \mathbb{R}_+$)

ID	Name	Salary
1234	Dan Suciu	$400,000
5678	Current Hung Ngo	$100,000
9999	Future Hung Ngo	$1,000,000

- Other representations $\Rightarrow$ different sets of results
  - SAT $\in$ **P** for β-acyclic queries
  - Minesweeper and Tetris join algorithms

Notation: conditional factor

- Let ψ_S be a factor
- Let $K \subseteq [n]$, $\boldsymbol{y}_K \in \prod_{i \in K} \mathsf{Dom}(X_i)$
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$$\psi_S(\cdot \mid \boldsymbol{y}_K) : \prod_{i \in S} \mathsf{Dom}(X_i) \to \mathbf{D}$$

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$$I_1 = \left\{ x_1 \mid \psi_S(\cdot \mid x_1) \not\equiv \mathbf{0}, \forall S \in \mathcal{E} \right\}$$

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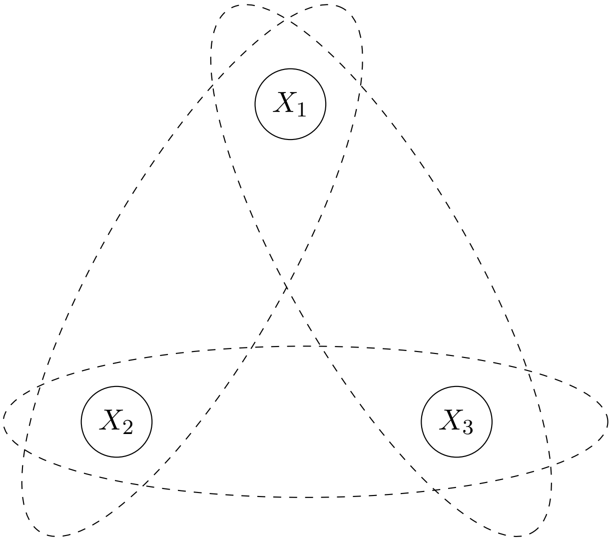
Background: fractional edge cover of $\mathcal{H}$



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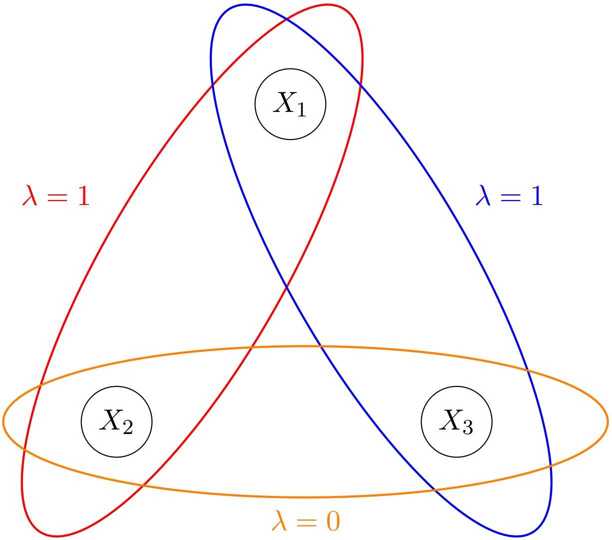

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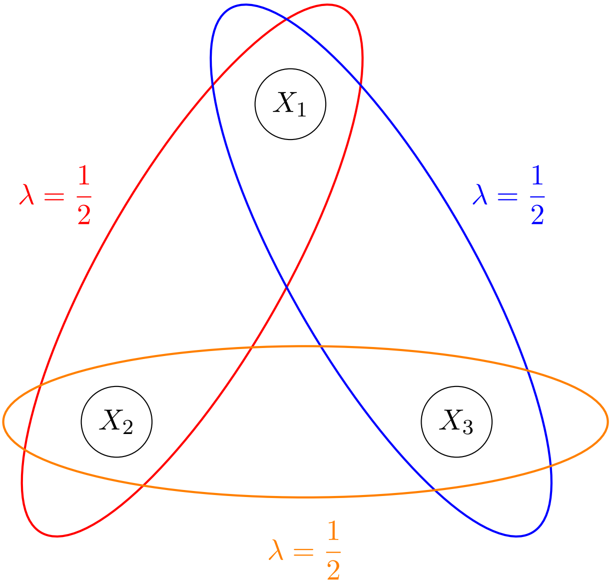

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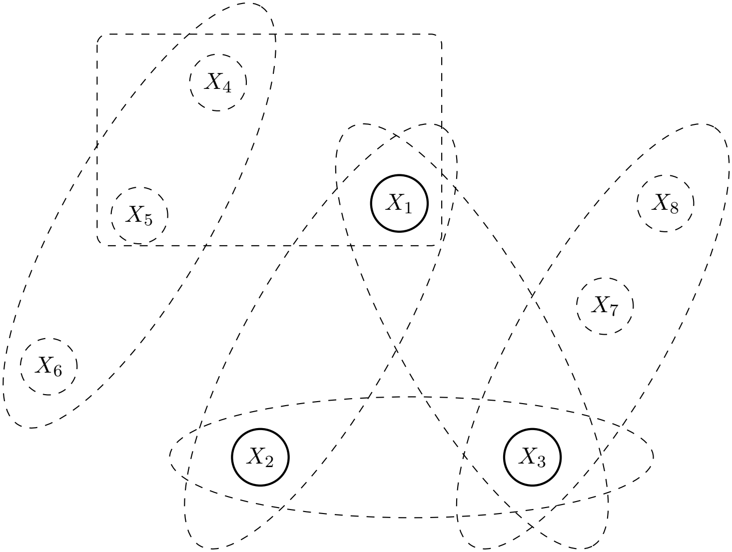

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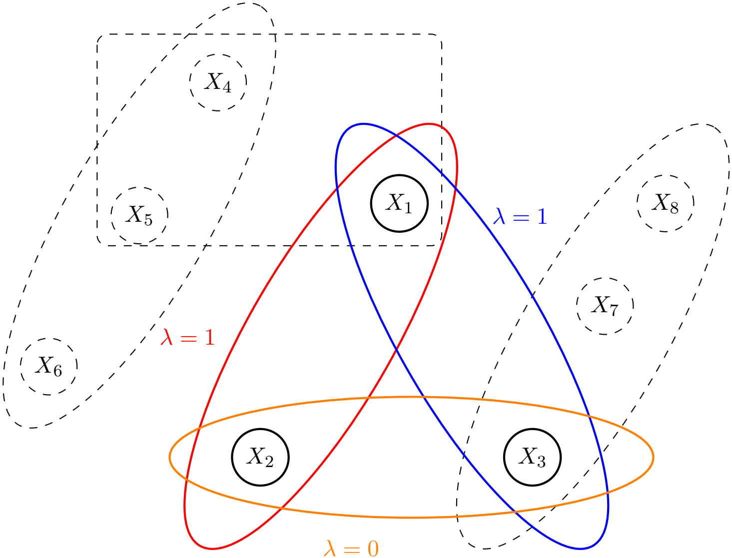

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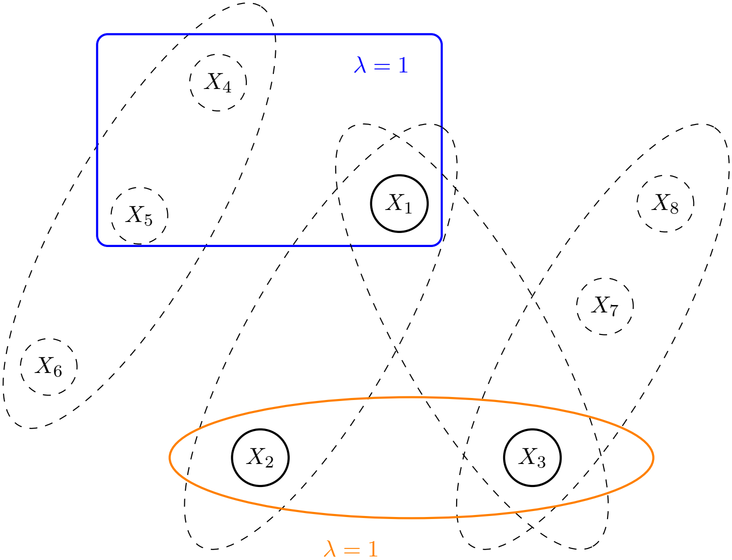

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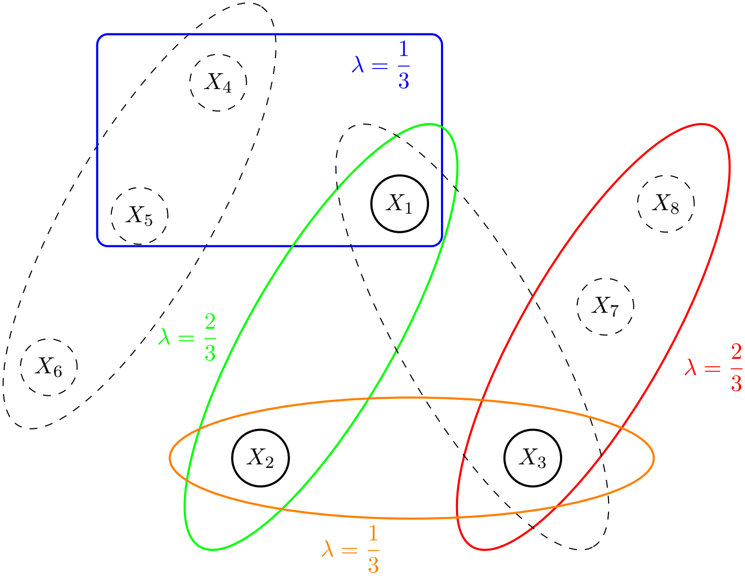

Background: fractional edge cover

- $\mathcal{H} = (\mathcal{V}, \mathcal{E})$ a multi hypergraph
- $B \subseteq \mathcal{V}$
- a fractional edge cover of B is a vector $(\lambda_S)_{S \in \mathcal{E}}$, s.t.

$$\begin{aligned} \sum_{S : v \in S} \lambda_S &\geq 1, \ \forall v \in B \\ \lambda_S &\geq 0, \ \forall S \in \mathcal{E}. \end{aligned}$$

- called fractional edge cover of $\mathcal{H}$ when $B = \mathcal{V}$

Pros of OutsideIn

Theorem (Runtime of OutsideIn)

Let $(\lambda_S)_{S\in\mathcal{E}}$ *be* ***any*** fractional edge cover *of* $\mathcal{H}$*, then* OutsideIn *runs in time*

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- Virtually zero memory footprint (backtracking search!).

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- $Q = R(A, B) \bowtie S(B, C) \bowtie T(A, C)$, $|R| = |S| = |T| = N$

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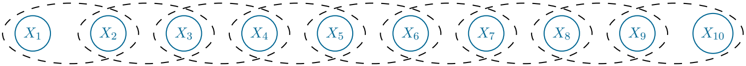

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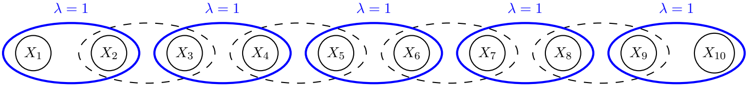

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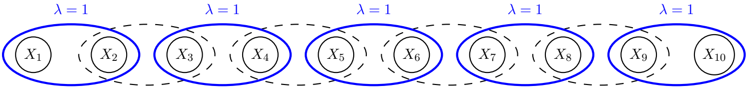

- More generally, horrible whenever fhtw < AGM-bound.

Table of Contents

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&= \sum_a \sum_b \sum_c b \cdot R(a,b)S(b,c) \sum_d T(c,d) \\
&= \sum_a \sum_b \sum_c b \cdot R(a,b)S(b,c)F(c) \\
&= \sum_a \sum_b b \cdot R(a,b) \sum_c S(b,c)F(c) \\
&= \sum_a \sum_b b \cdot R(a,b)G(b) \\
&= \sum_a \sum_b b \cdot R(a,b)G(b) \\
&= \sum_a H(a).
\end{aligned}
$$

Example: $\varphi() = \sum_a \sum_b \sum_c \sum_d R(a,b)S(b,c)T(c,d) \times b$

$$
\begin{aligned}
\varphi &= \sum_a \sum_b \sum_c \sum_d R(a,b)S(b,c)T(c,d)b \\
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&= \sum_a \sum_b b \cdot R(a,b) \sum_c S(b,c)F(c) \\
&= \sum_a \sum_b b \cdot R(a,b)G(b) \\
&= \sum_a \sum_b b \cdot R(a,b)G(b) \\
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\end{aligned}
$$

Runtime: $\tilde{O}(|R| + |S| + |T|)$.

FAQ-SS *without* free variables

$$\varphi = \bigoplus_{x_1} \cdots \bigoplus_{x_n} \bigotimes_{S \in \mathcal{E}} \psi_S(\boldsymbol{x}_S)$$

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↑

$\otimes$ distributes over $\oplus$

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$$\psi_{U_n - \{n\}}(\boldsymbol{x}_{U_n - \{n\}})$$

FAQ-SS *without* free variables

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&= \bigoplus_{x_1} \cdots \bigoplus_{x_{n-1}} \bigotimes_{n \notin S} \psi_S(\boldsymbol{x}_S) \otimes \psi_{U_n - \{n\}}(\boldsymbol{x}_{U_n - \{n\}})
\end{aligned}$$

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\end{aligned}
$$

$$U_n = \bigcup_{S : n \in S} S$$

New instance of FAQ-SS

$$
\begin{aligned}
\mathcal{H}_{n-1} &= ([n-1], \mathcal{E}_{n-1}) \\
\mathcal{E}_{n-1} &= \mathcal{E} + (U_n - \{n\}) - \{S \mid n \in S\}
\end{aligned}
$$

Naïve InsideOut = Variable Elimination + OutsideIn

$$\varphi = \bigoplus_{x_1} \cdots \bigoplus_{x_{n-1}} \bigotimes_{n \notin S} \psi_S(\boldsymbol{x}_S) \left(\bigoplus_{x_n} \bigotimes_{S \in \partial(n)} \psi_S(\boldsymbol{x}_S) \right)$$

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$\uparrow$

But how do we compute this? $\psi_{U_n - \{n\}}(\boldsymbol{x}_{U_n - \{n\}})$

Naïve InsideOut = Variable Elimination + OutsideIn

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Run the OutsideIn algorithm!

Naïve InsideOut = Variable Elimination + OutsideIn

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$\psi_{U_n - \{n\}}(\boldsymbol{x}_{U_n - \{n\}})$

Run the OutsideIn algorithm!

$$\text{Runtime} \approx |U_n| \cdot |\partial(n)| \cdot \prod_{S \in \partial(n)} |\psi_S|^{\lambda_S^{(n)}}$$

Naïve InsideOut = Variable Elimination + OutsideIn

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Theorem (Runtime of Naïve InsideOut)

For a given variable ordering $v_1, \dots, v_n$, the runtime of Naïve InsideOut *makes is $\tilde{O}$ of*

$$\sum_{k=1}^{n} |U_k| \cdot |\partial(v_k)| \cdot \prod_{S \in \partial(v_k)} |\psi_S|^{\lambda_S^{(k)}} \leq mn \sum_{k=1}^{n} \prod_{S \in \partial(v_k)} |\psi_S|^{\lambda_S^{(k)}}$$

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Corollary (Textbook result – Zhang-Poole'95, Dechter'96)

For a "good" variable ordering, PGM inference can be done in time $O(mn^2 D^{w+1})$ where w = tree-width$(\mathcal{H})$ = $\mathrm{tw}(\mathcal{H})$.

Pros and Cons of Naïve InsideOut

- The good: $O(kN)$-time for k-path query!

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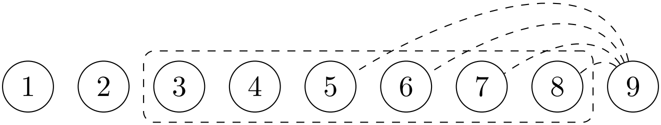
- The bad: consider the following (part of a) graph



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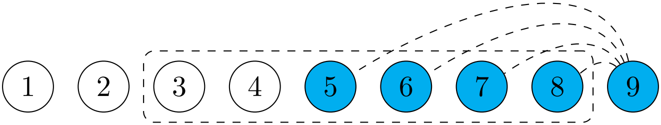

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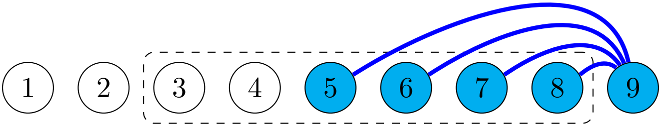

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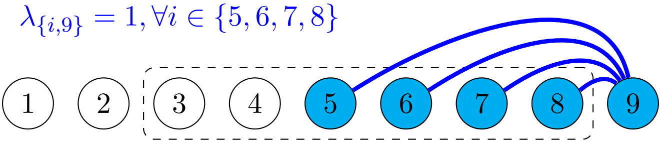
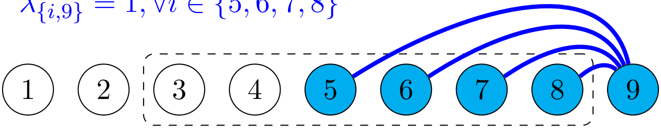


Pros and Cons of Naïve InsideOut

- The good: $O(kN)$-time for k-path query!
- The bad: consider the following (part of a) graph

$\lambda_{\{i,9\}} = 1, \forall i \in \{5,6,7,8\}$



- Say $|\psi_S| = N$, then runtime $O(N^4)$.

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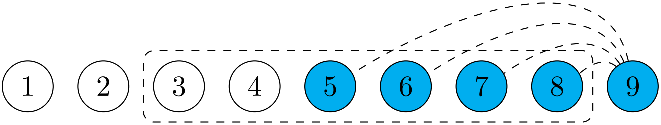

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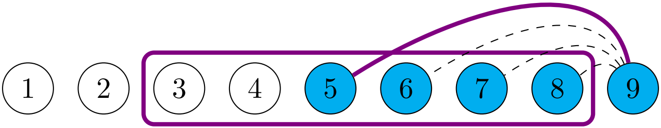

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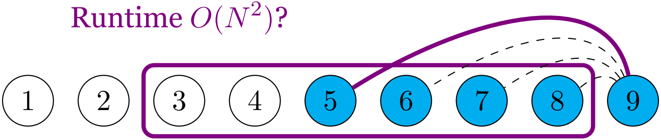

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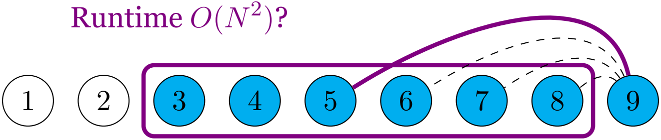
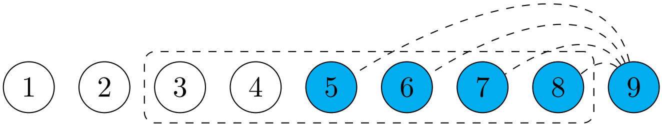


- Say $|\psi_S| = N$, then runtime $O(N^4)$.
- Good idea, but naïve application increases U_9

Indicator projection

- Projection onto U_9:



Indicator projection

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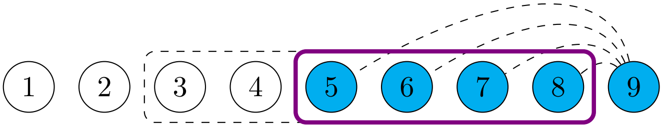

Indicator projection

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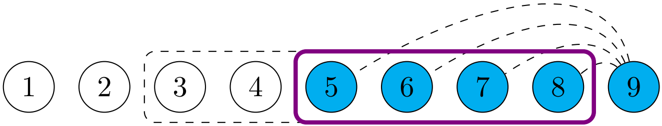

- Formally, $S \in \mathcal{E}, U \subseteq \mathcal{V}$ s.t. $S \cap U \neq \emptyset$,

$$\pi_U \psi_S : \prod_{i \in S \cap U} \mathsf{Dom}(X_i) \to \{\mathbf{0}, \mathbf{1}\},$$

where

$$\pi_U \psi_S(\boldsymbol{x}_{S \cap U}) = \begin{cases} \mathbf{1} & \text{if } \exists \boldsymbol{y}_S \text{ s.t. } \psi_S(\boldsymbol{y}_S) \neq \mathbf{0} \text{ and } \boldsymbol{y}_{S \cap U} = \boldsymbol{x}_{S \cap U} \\ \mathbf{0} & \text{otherwise} \end{cases}$$

Not so naïve InsideOut Example

$$\varphi = \sum_{a,b,c,d,e,f} R(a,b)S(a,c)T(b,c,d,e)W(e,f)V(d,f)$$

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Not so naïve InsideOut Example

$$
\begin{aligned}
\varphi &= \sum_{a,b,c,d,e,f} R(a,b)S(a,c)T(b,c,d,e)W(e,f)V(d,f) \\
&= \sum_{a,b,c,d,e} R(a,b)S(a,c)T(b,c,d,e) \underbrace{\sum_{f} \pi_{de}T(d,e)\, W(e,f)V(d,f)}_{\tilde{O}(N^{3/2})} \\
&= \sum_{a,b,c,d,e} R(a,b)S(a,c)T(b,c,d,e)M_1(d,e)
\end{aligned}
$$

Not so naïve InsideOut Example

$$\varphi = \sum_{a,b,c,d,e,f} R(a,b)S(a,c)T(b,c,d,e)W(e,f)V(d,f)$$

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$$= \sum_{a,b,c,d,e} R(a,b)S(a,c)T(b,c,d,e)M_1(d,e)$$

$$= \sum_{a,b,c} R(a,b)S(a,c) \sum_{d,e} \pi_b R(b) \pi_S(c)\, T(b,c,d,e)M_1(d,e)$$

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$$
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&= \sum_{a,b,c} R(a,b)S(a,c)M_2(b,c)
\end{aligned}
$$

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&= \sum_{a,b,c,d,e} R(a,b)S(a,c)T(b,c,d,e)M_1(d,e) \\
&= \sum_{a,b,c} R(a,b)S(a,c) \sum_{d,e} \pi_b R(b)\pi_S(c)\, T(b,c,d,e)M_1(d,e) \\
&= \sum_{a,b,c} R(a,b)S(a,c)M_2(b,c) \\
&= \underbrace{\sum_{a,b,c} R(a,b)S(a,c)M_2(b,c)}_{\tilde{O}(N^{3/2})}
\end{aligned}$$

InsideOut = VE + indicator-projections + OutsideIn

$$\varphi = \bigoplus_{x_1} \cdots \bigoplus_{x_{n-1}} \bigotimes_{n \notin S} \psi_S(\mathbf{x}_S) \left(\bigoplus_{x_n} \bigotimes_{S \in \partial(n)} \psi_S(\mathbf{x}_S) \right)$$

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InsideOut = VE + indicator-projections + OutsideIn

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Time $\leq mn\mathsf{AGM}(U_n)$

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Theorem (Runtime of InsideOut)

For FAQ-SS, InsideOut *runs in time* $\tilde{O}$ *of*

$$mn \sum_{k=1}^{n} \mathsf{AGM}(U_k) \leq mn \sum_{k=1}^{n} N^{\rho^*_{\mathcal{H}}(U_k)} \leq mn^2 N^{\max_k \rho^*_{\mathcal{H}}(U_k)}$$

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Corollary (New (textbook?) result)

For a "good" variable ordering $v_1, \ldots, v_n$*, PGM inference can be done in time* $O(mn^2 N^w)$ *where* $w = \mathsf{fhtw}(\mathcal{H})$.

InsideOut = VE + indicator-projections + OutsideIn

$$\varphi = \bigoplus_{x_1} \cdots \bigoplus_{x_{n-1}} \bigotimes_{n \notin S} \psi_S(\mathbf{x}_S) \bigoplus_{x_n} \bigotimes_{S \in \partial(n)} \psi_S(\mathbf{x}_S) \otimes \bigotimes_{\substack{S \notin \partial(n) \\ S \cap U_n \neq \emptyset}} \pi_{U_n} \psi_S(\mathbf{x}_{S \cap U_n})$$

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Corollary (Grohe-Marx (SODA'06))

Join can be computed in time $\tilde{O}(N^{\mathsf{fhtw}(\mathcal{H})} + |output|)$.

InsideOut = VE + indicator-projections + OutsideIn

$$\varphi = \bigoplus_{x_1} \cdots \bigoplus_{x_{n-1}} \bigotimes_{n \notin S} \psi_S(\mathbf{x}_S) \bigoplus_{x_n} \bigotimes_{S \in \partial(n)} \psi_S(\mathbf{x}_S) \otimes \bigotimes_{\substack{S \notin \partial(n) \\ S \cap U_n \neq \emptyset}} \pi_{U_n} \psi_S(\mathbf{x}_{S \cap U_n})$$

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Corollary (Grohe-Marx (SODA'06))

CSP *on instances with bounded* fhtw *are fixed-parameter tractable.*

InsideOut = VE + indicator-projections + OutsideIn

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Corollary (New?)

Join cardinality and any aggregate query (over a conjunction) can be computed in time $\tilde{O}(N^{\mathsf{fhtw}(\mathcal{H})})$

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Corollary (Pichler-Skritek 2011, Durand-Mengel (ICDT'2013))

For query graphs $\mathcal{H}$ *with bounded* fhtw, *quantifier-free* #CQ *is polynomial-time solvable.*

InsideOut for general FAQ

$$\varphi = \cdots \bigoplus_{x_n}^{(n)} \bigotimes_{S \in \mathcal{E}} \psi_S(\mathbf{x}_S)$$

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InsideOut for general FAQ

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InsideOut for general FAQ

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If $\oplus^{(n)} = \otimes$ and $\oplus^{(n)}$ is *not-idempotent*

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Table of Contents

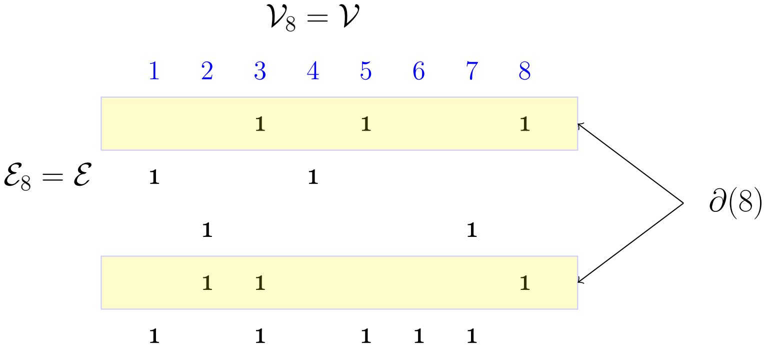
Variable ordering and support set sequence

$$\mathcal{V}_8 = \mathcal{V}$$

$$\mathcal{E}_8 = \mathcal{E}$$

1	2	3	4	5	6	7	8
		1		**1**			**1**
1			**1**				
	1					**1**	
	1	**1**					**1**
1		**1**		**1**	**1**	**1**	

Variable ordering and support set sequence



Variable ordering and support set sequence

$\mathcal{V}_8 = \mathcal{V}$

	1	2	3	4	5	6	7	8
			1		1			1
$\mathcal{E}_8 = \mathcal{E}$	1			1				
		1					1	
		1	1					1
	1		1		1	1	1	
U_8		1	1		1			1

$\partial(8)$

Variable ordering and support set sequence

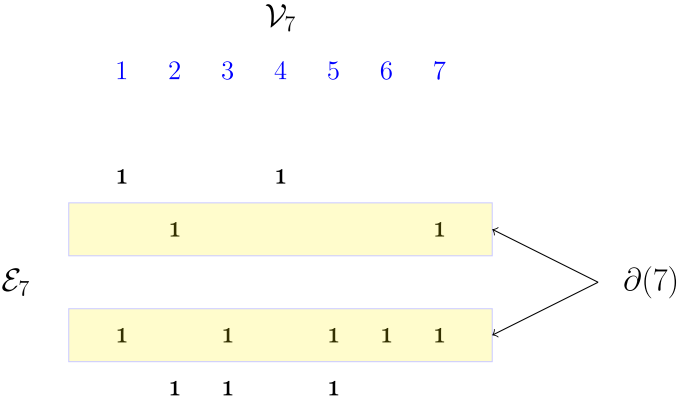
	$\mathcal{V}_7$						
	1	2	3	4	5	6	7
	1			**1**			
		1					**1**
$\mathcal{E}_7$							
	1		**1**		**1**	**1**	**1**
$U_8 - \{8\}$		**1**	**1**		**1**		

Variable ordering and support set sequence

$\mathcal{V}_7$

	1	2	3	4	5	6	7
	1			**1**			
		1					**1**
$\mathcal{E}_7$							
	1		**1**		**1**	**1**	**1**
		1	**1**		**1**		

Variable ordering and support set sequence



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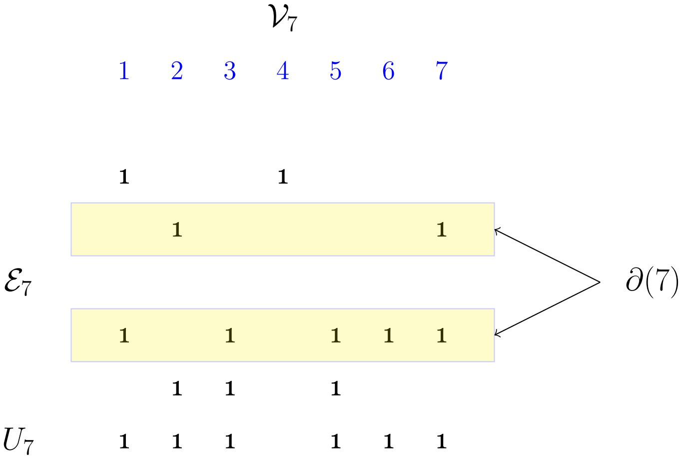

Variable ordering and support set sequence

$\mathcal{V}_6$

	1	2	3	4	5	6
	1			**1**		
$\mathcal{E}_6$						
		1	**1**		**1**	
$U_7 - \{7\}$	**1**	**1**	**1**		**1**	**1**

Variable ordering and support set sequence

$\mathcal{V}_6$

	1	2	3	4	5	6
	1			**1**		
$\mathcal{E}_6$						
		1	**1**		**1**	
	1	**1**	**1**		**1**	**1**

Variable ordering and support set sequence

$\mathcal{V}_6$

	1	2	3	4	5	6	
	1			**1**			
$\mathcal{E}_6$							
		1	**1**		**1**		
	1	**1**	**1**		**1**	**1**	$\leftarrow \partial(6)$

Variable ordering and support set sequence

$\mathcal{V}_6$

	1	2	3	4	5	6	
	1			**1**			
$\mathcal{E}_6$							
		1	**1**		**1**		
	1	**1**	**1**		**1**	**1**	$\longleftarrow \partial(6)$
U_6	**1**	**1**	**1**		**1**	**1**	

Variable ordering and support set sequence

$\mathcal{V}_5$

	1	2	3	4	5
	1			**1**	
$\mathcal{E}_5$					
		1	**1**		**1**
$U_6 - \{6\}$	**1**	**1**	**1**		**1**

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\end{aligned}
$$

- $(U_n^\sigma, U_{n-1}^\sigma, \cdots, U_1^\sigma)$ is called the suport set sequence of σ.

FAQ-width of a variable ordering σ

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Dynamic Programming for

for Matrix Chain Multiplication

and The FFT

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$$\leq \quad \tilde{O}(mn)\left[nN^{\mathsf{faqw}(\sigma)} + |\varphi|\right].$$

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Let φ be an FAQ-query. A *φ-equivalent variable ordering* is a vertex ordering $\sigma = (v_1, \ldots, v_n)$ that satisfies:

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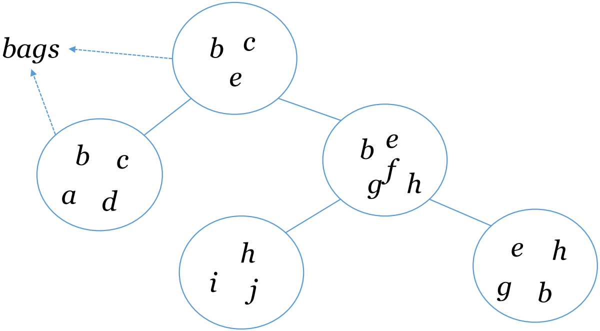
Definition (FAQ-width of an FAQ-query φ)

$$\mathsf{faqw}(\varphi) := \min_{\sigma \in \mathsf{EVO}(\varphi)} \mathsf{faqw}(\sigma).$$

Detour: Tree decomposition of $\mathcal{H} = (\mathcal{V}, \mathcal{E})$

{a,b,d}, {c, d}, {b, c, d), {b, e}, {c, e}, {b, e, f},
{b, e, g}, {g, f, h}, {i, j, h}, {e, g}, {e, g, b}, {e, b, h}.



Every hyperedge is covered by some bag

{a,b,d}, {c, d}, {b, c, d), {b, e}, {c, e}, {b, e, f},
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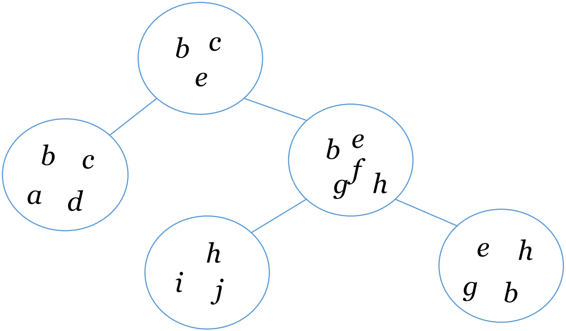

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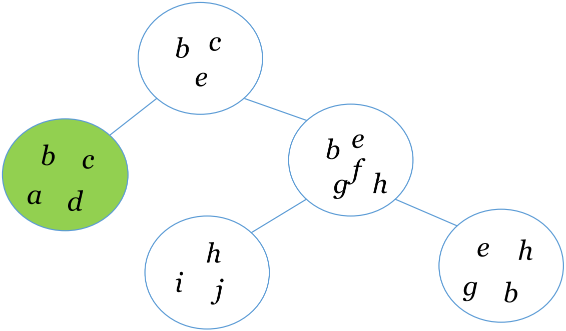

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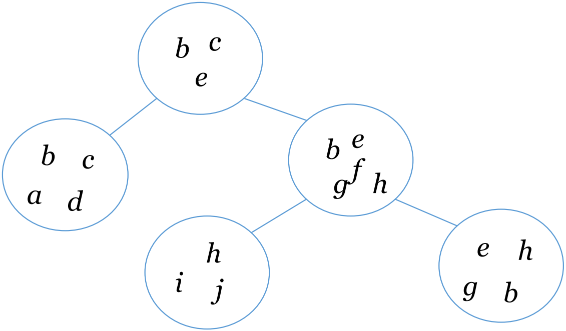

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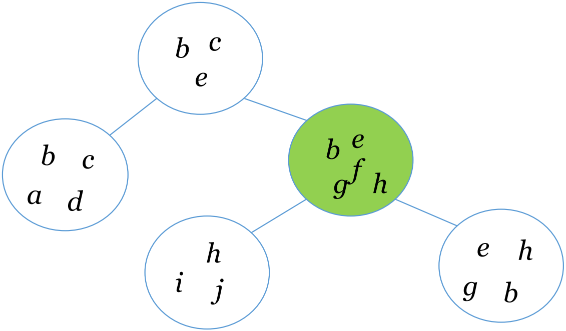

Running intersection property (RIP)

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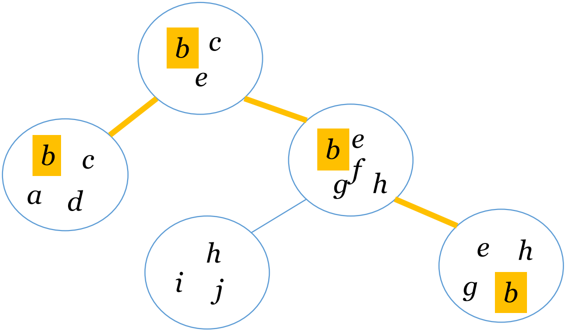

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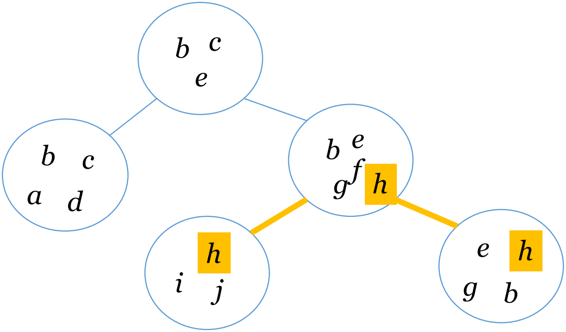

Tree decomposition, formally

- Hypergraph $\mathcal{H} = (\mathcal{V}, \mathcal{E})$
- A tree decomposition is a pair (T, χ)
  - $T = (V(T), E(T))$ is a tree
  - $\chi : V(T) \to 2^{\mathcal{V}}$, $\chi(t)$ called *bag*
  - Coverage property :

$$\forall F \in \mathcal{E}, \qquad \exists t \in V(T) \text{ s.t. } F \subseteq \chi(t).$$

  - RIP :

$$\forall v \in \mathcal{V}, \qquad \{t \mid t \in V(T), v \in \chi(t)\} \text{ is a connected sub-tree}$$

  - Non-redundant if no bag is contained in another.

Variable Ordering and Tree decomposition

- Given $\mathcal{H} = (\mathcal{V}, \mathcal{E})$, and a variable ordering $\sigma = (v_1, \ldots, v_n)$
- Corresponding support sets $U^\sigma_n, U^\sigma_{n-1}, \ldots, U^\sigma_1$

Proposition (Folklorish)

- *For every variable ordering σ, there is a tree decomposition whose bags are precisely the sets U^σ_k*
- *For every tree decomposition (T, χ), there is a variable ordering σ such that every U^σ_k is covered by some bag.*

faqw in terms of tree decompositions

Definition (FAQ-width of a variable ordering σ)

$$\mathsf{faqw}(\sigma) := \max_{k \in K} \rho^*_{\mathcal{H}}(U_k^\sigma).$$

Definition (FAQ-width of an FAQ-query φ)

$$\mathsf{faqw}(\varphi) := \min_{\sigma \in \mathsf{EVO}(\varphi)} \mathsf{faqw}(\sigma).$$

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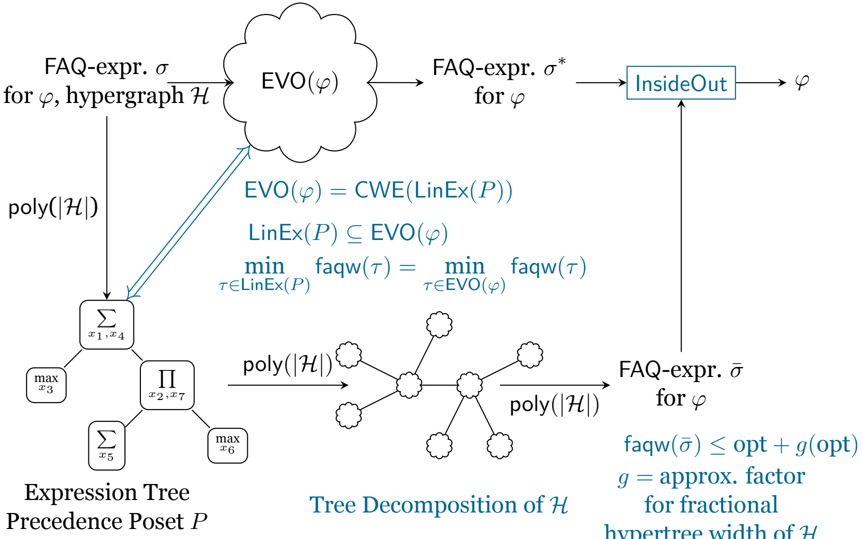
Roadmap Reminder



Characterizing EVO(φ)

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FAQ with two blocks of semiring aggregates

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Precedence Poset:

$$u \prec v \text{ whenever } u \in L \text{ and } v \notin L.$$

Approximating $\mathsf{faqw}(\varphi)$

Theorem

Given an approximation algorithm for $\mathsf{fhtw}(\mathcal{H})$ *within a bound*

$$g(\mathsf{fhtw}(\mathcal{H})),$$

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Best known $g(x) = O(x^3)$ [Marx'12]

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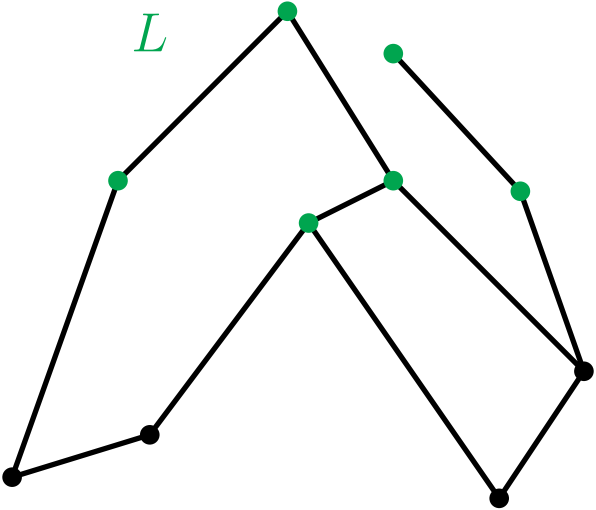

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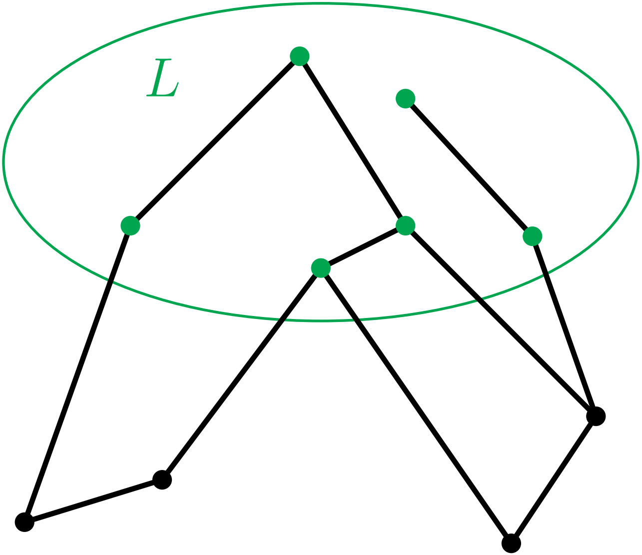

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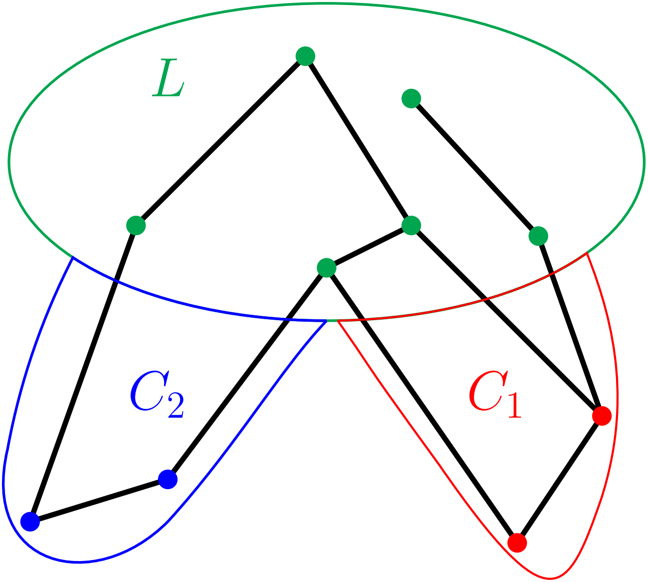

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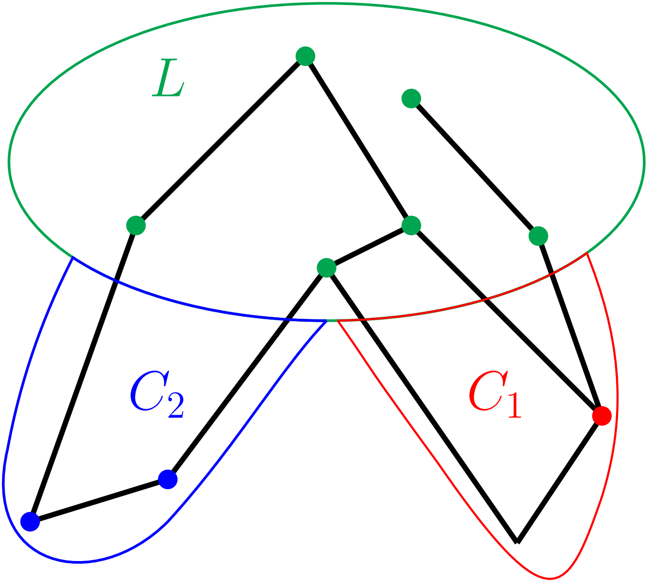

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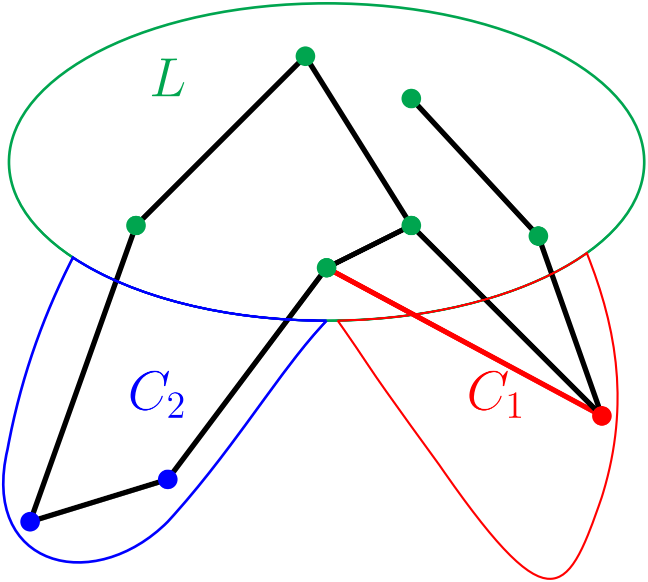

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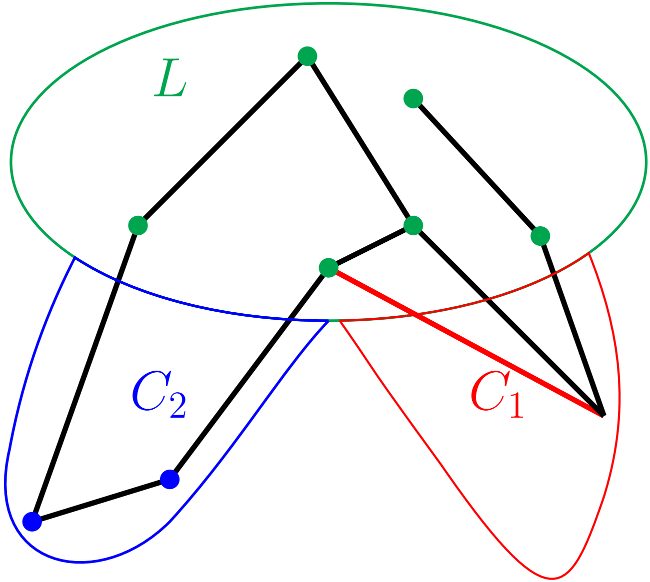

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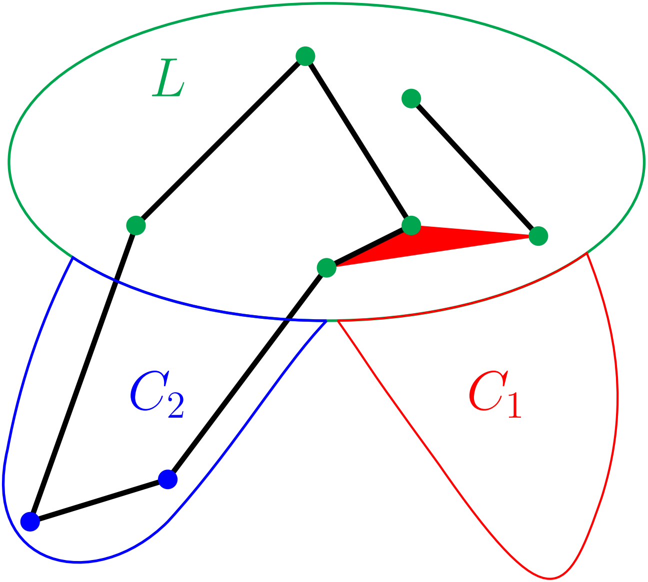

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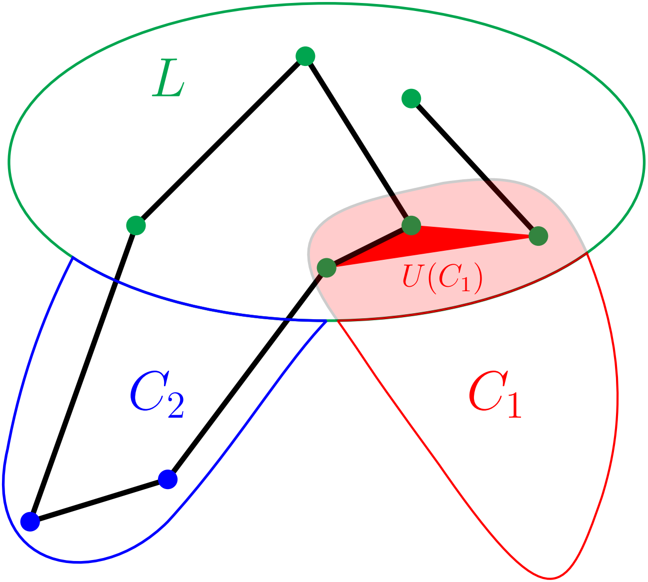

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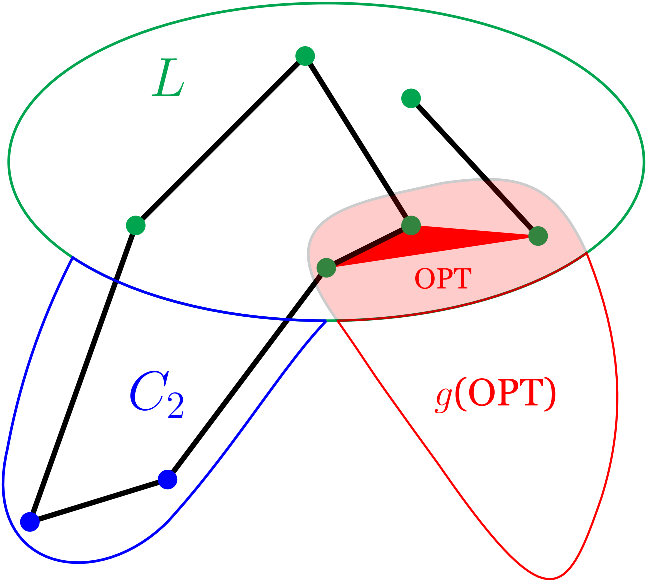

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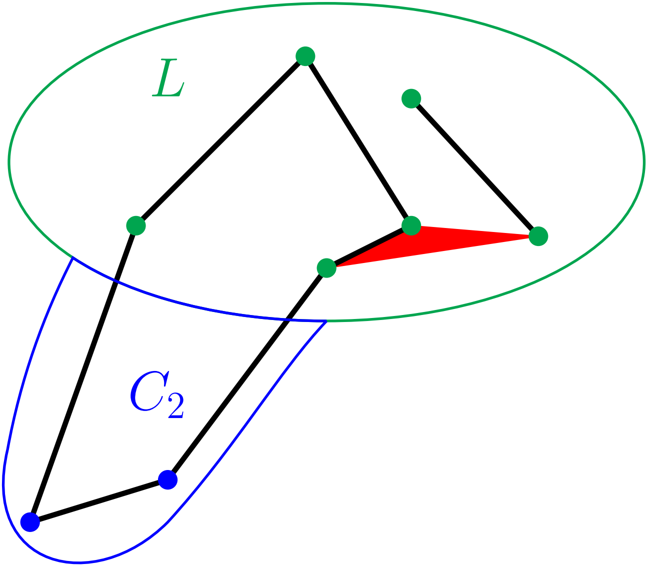

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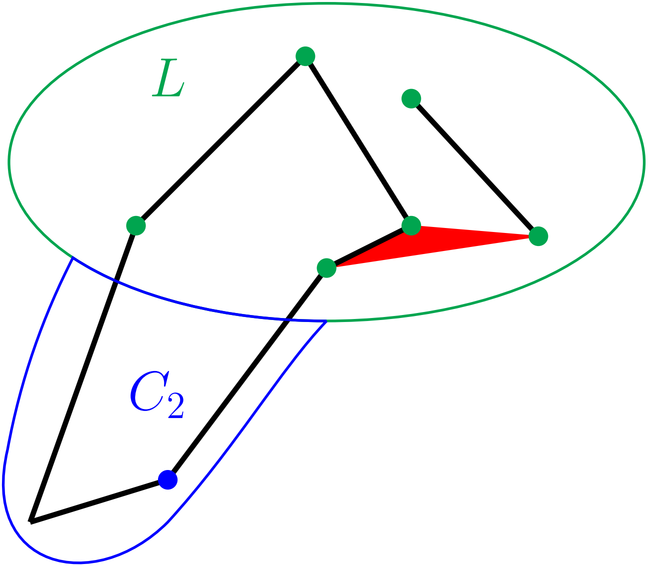

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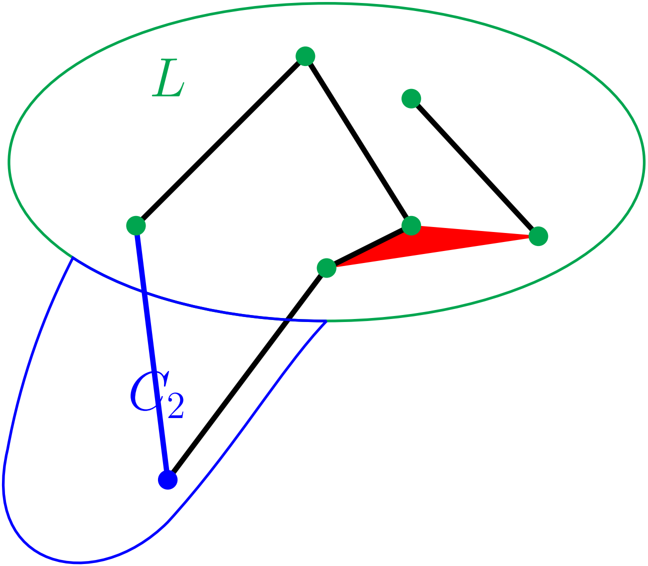

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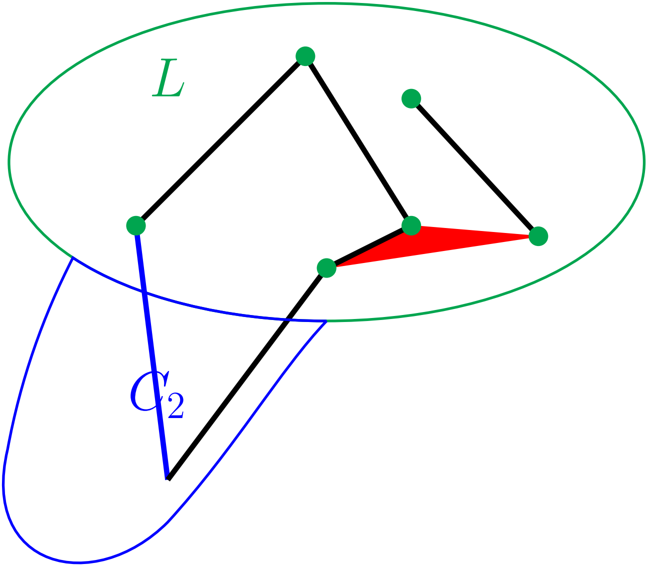

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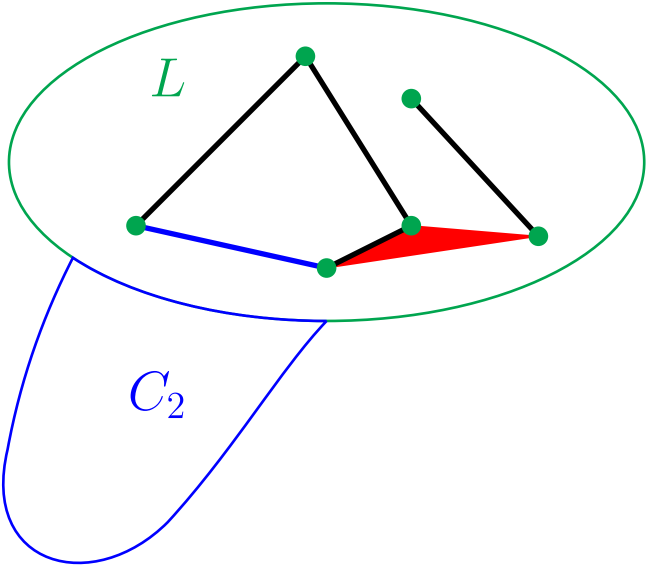

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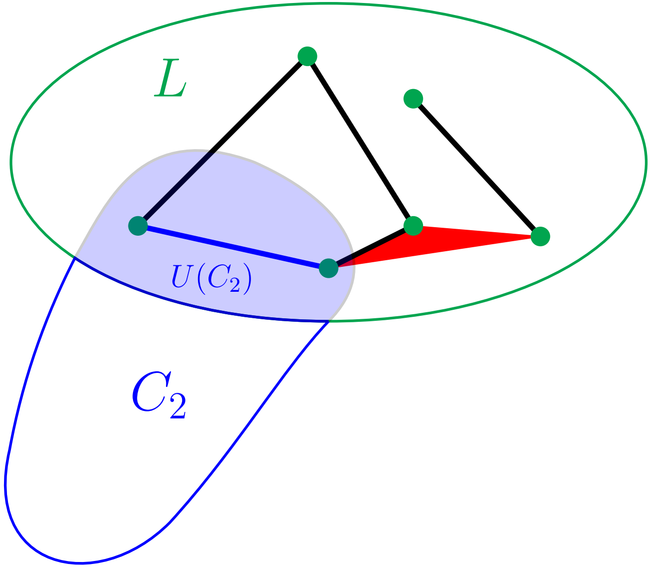

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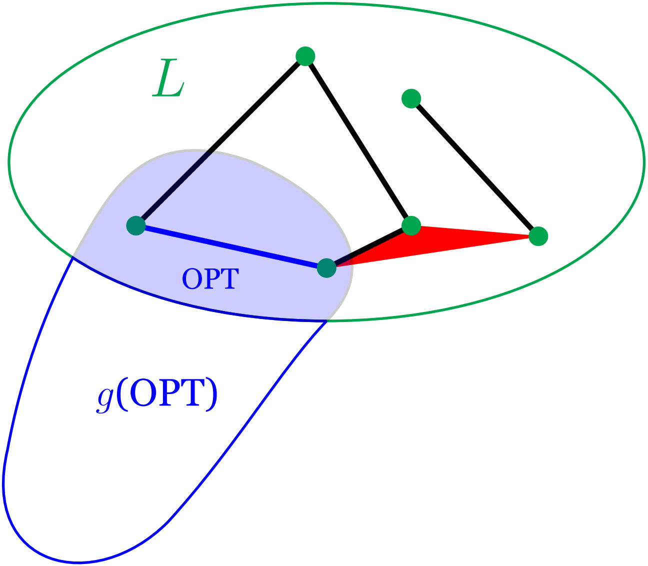

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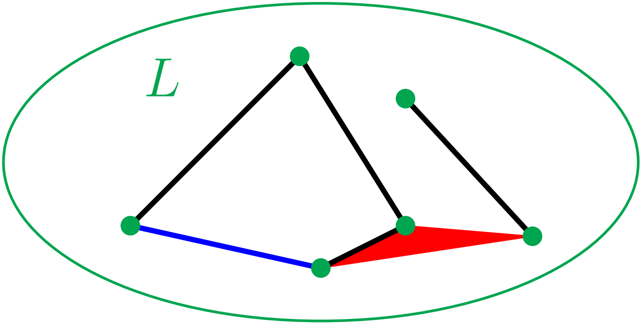

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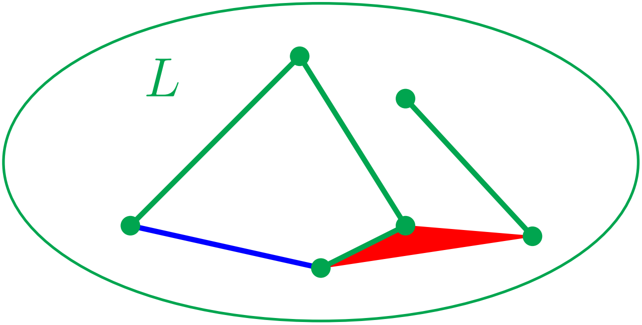

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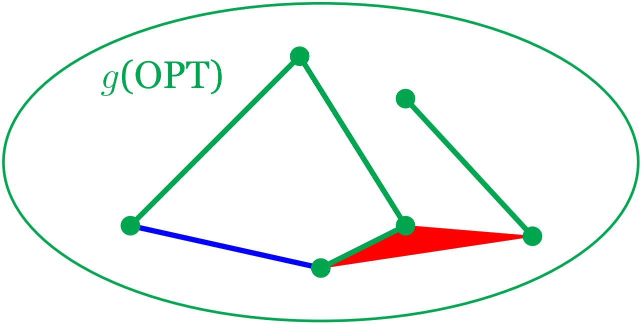

Approximating $\mathsf{faqw}(\varphi)$

Corollary

FAQ *with two block of semiring aggregates is solvable in time*

$$O\left(\text{poly}(m,n)\cdot N^{O\left(\mathsf{faqw}^3(\varphi)\right)}\right).$$

Approximating faqw(φ)

Corollary (Durand-Mengel [ICDT'13])

#CQ is solvable in time

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FAQ with only semiring aggregates

$$\varphi(\boldsymbol{x}_{[f]}) = \bigoplus_{x_{f+1}}^{(f+1)} \cdots \bigoplus_{x_n}^{(n)} \bigotimes_{S \in \mathcal{E}} \psi_S(\boldsymbol{x}_S)$$

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- Expression Tree defines the precedence poset

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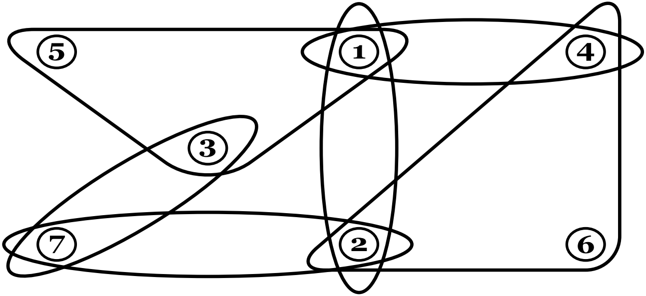
  - Compartmentalization Step.
  - Compression Step.

$$\varphi = \sum_{x_1} \sum_{x_2} \max_{x_3} \sum_{x_4} \sum_{x_5} \max_{x_6} \max_{x_7} \psi_{12}\, \psi_{135}\, \psi_{14}\, \psi_{246}\, \psi_{27}\, \psi_{37}$$



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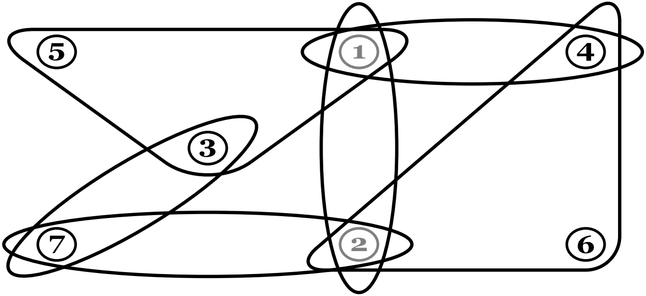

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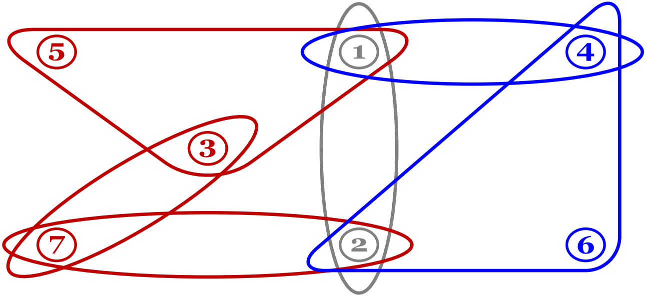

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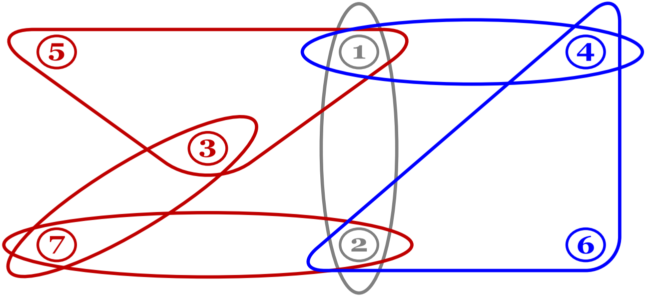

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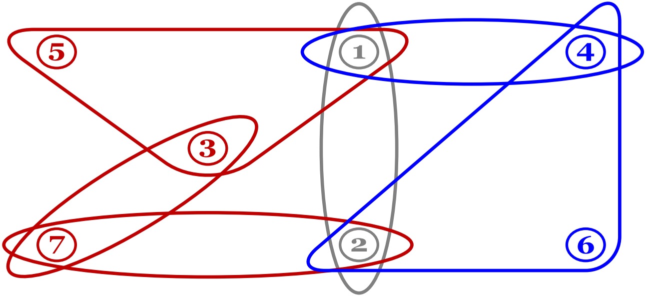
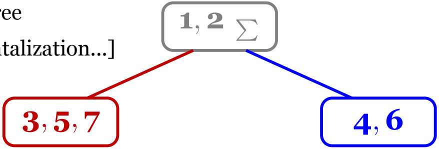


Expression Tree

[Compartmentalization...]



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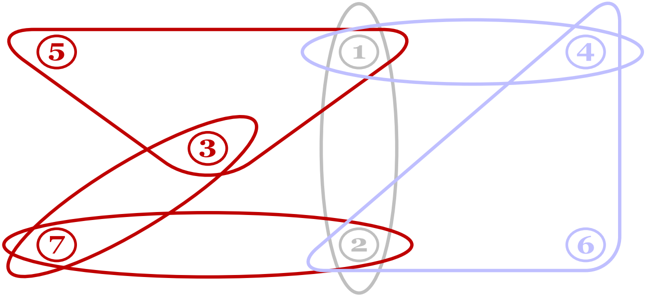

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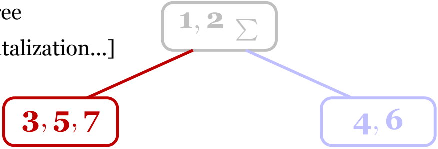

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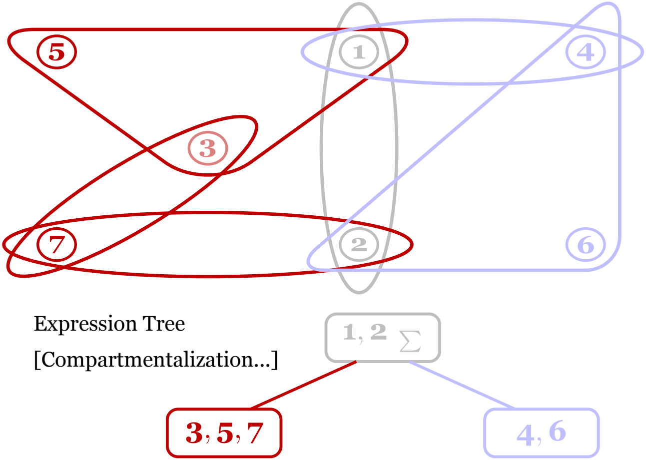

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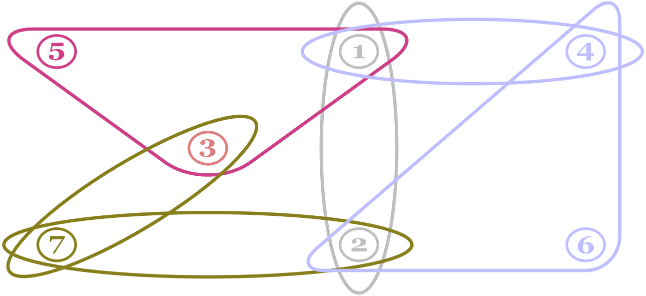

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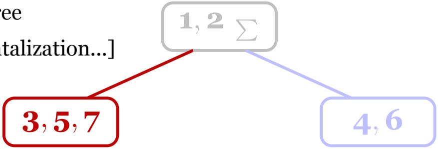

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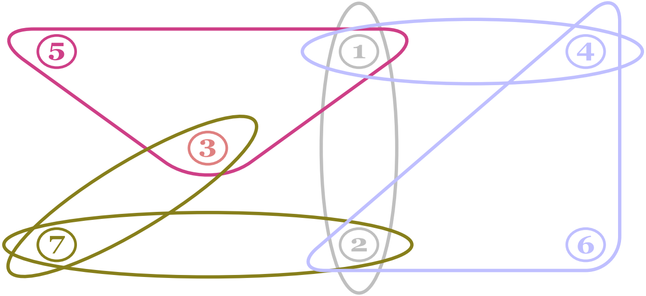
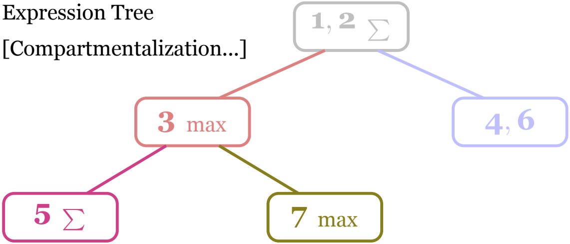


Expression Tree

[Compartmentalization...]



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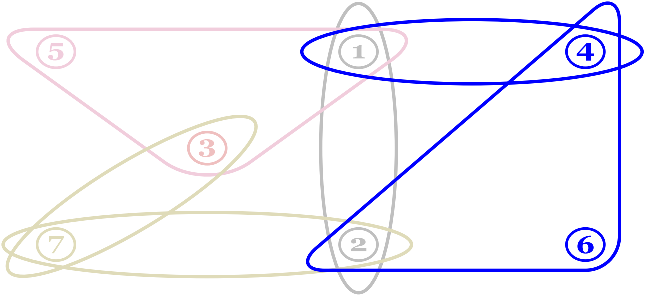

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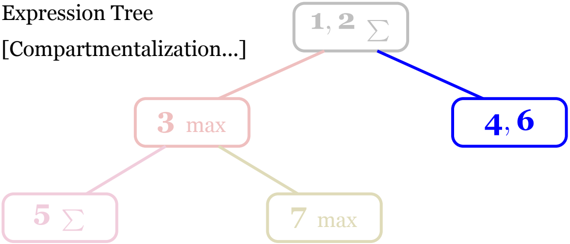

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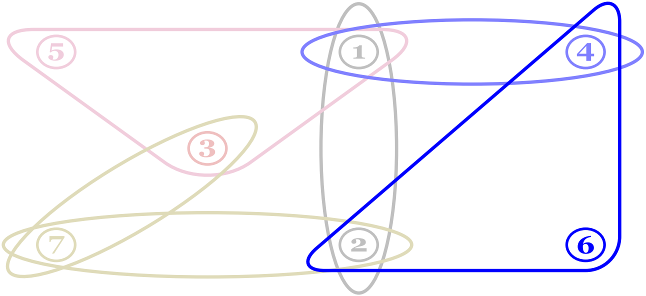

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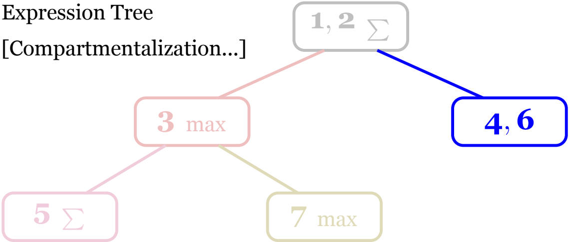

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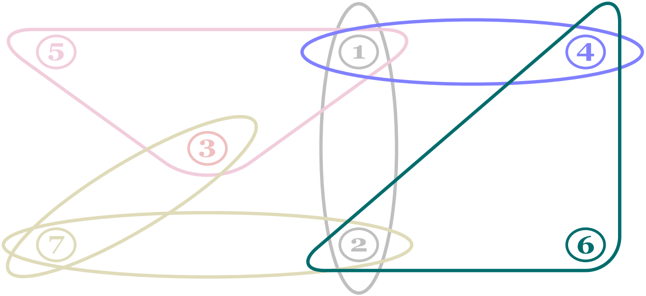

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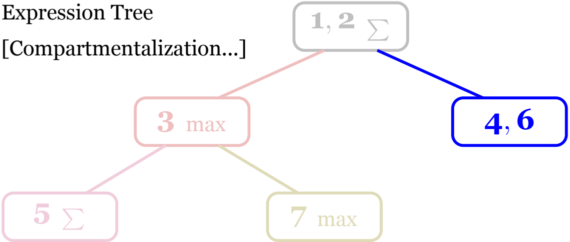

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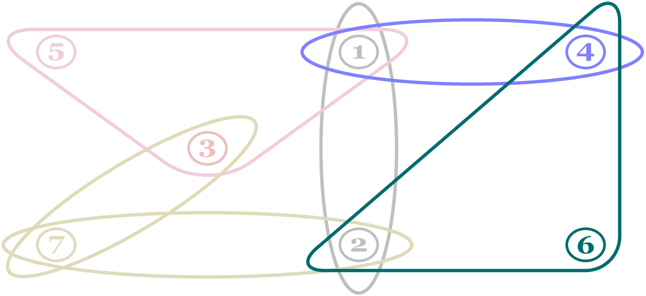

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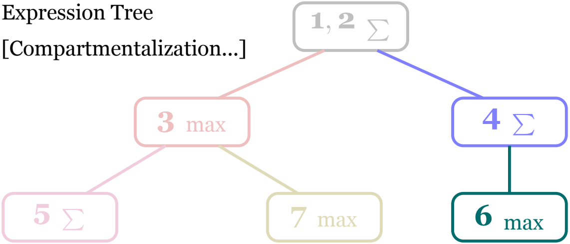

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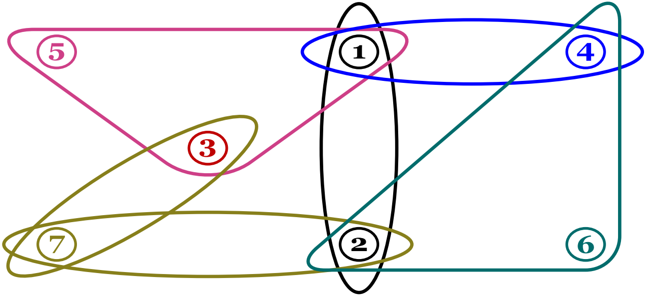

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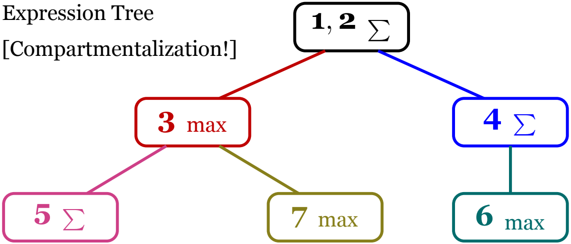

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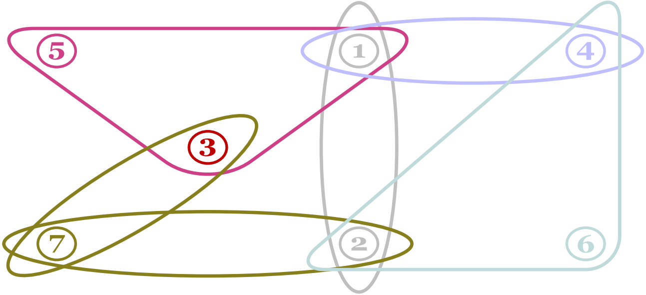

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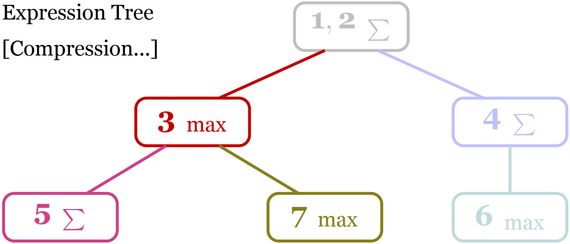

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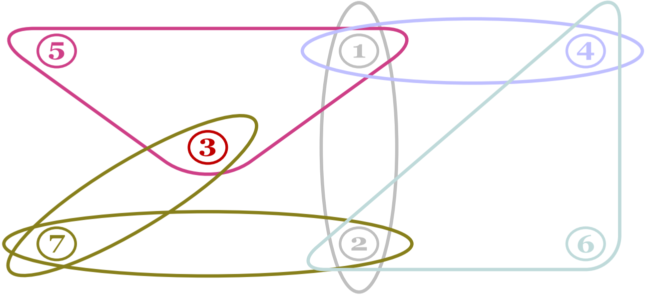

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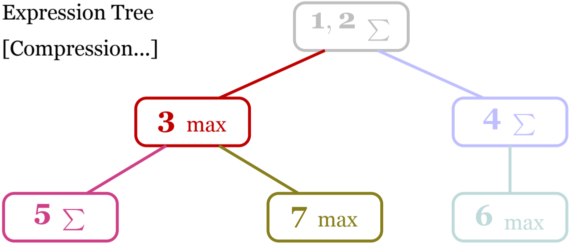

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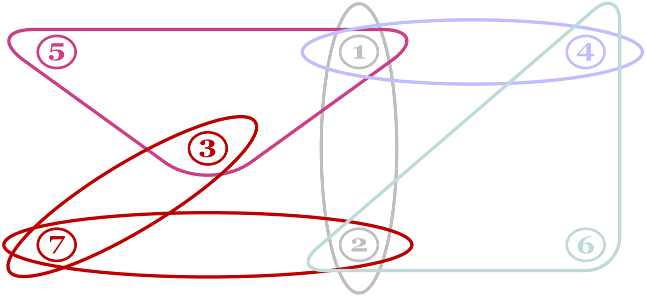

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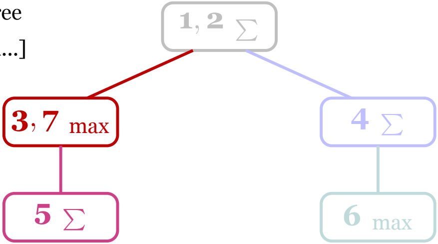

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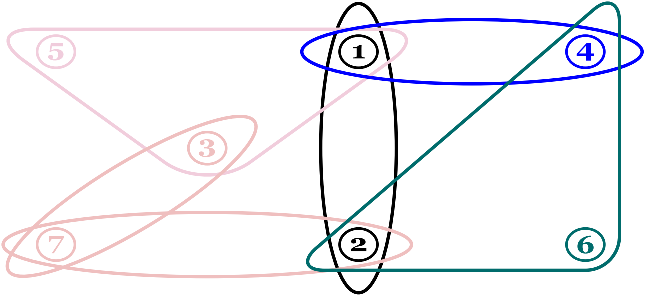

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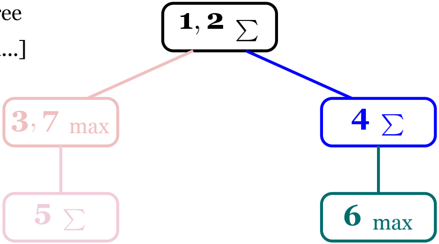

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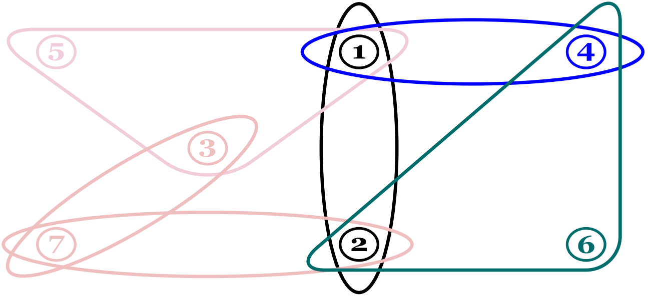

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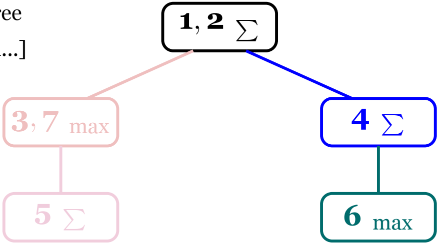

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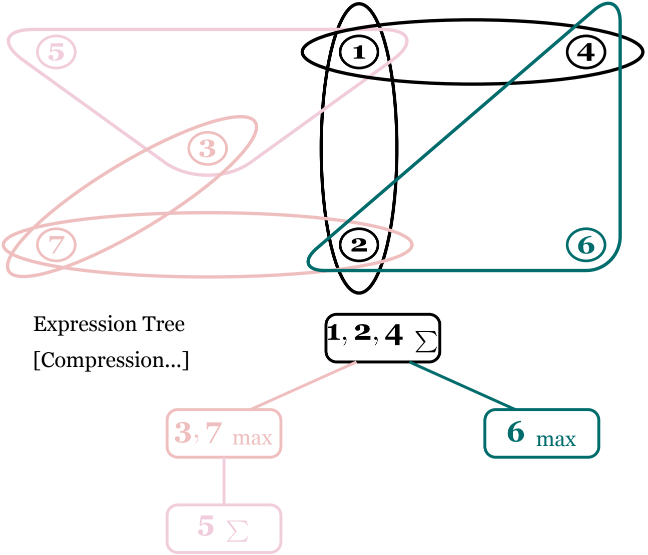

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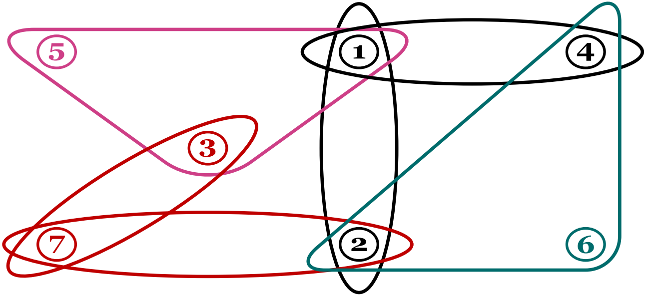

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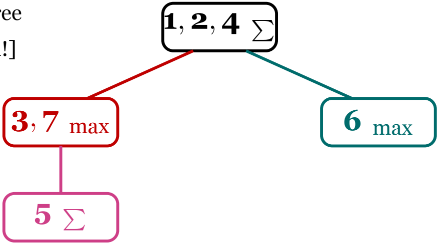

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$$\mathsf{LinEx}(P) \subseteq \mathsf{EVO}(\varphi).$$

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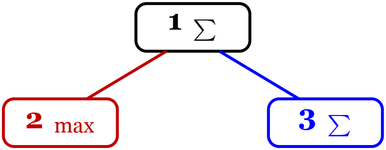

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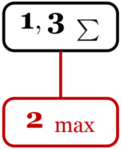

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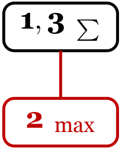

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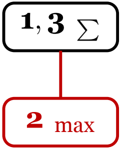

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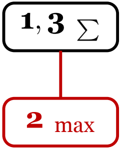

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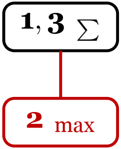

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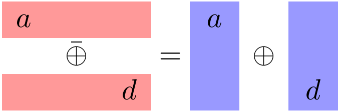

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Hence, $\oplus$ and $\bar{\oplus}$ are identical!

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Two semantically different aggregates may be identical by accident, e.g. min and $\times$ under $\{0, 1\}$

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Given an approximation algorithm for $\mathsf{fhtw}(\mathcal{H})$ *within a bound*

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Examples: QCQ,

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FAQ with *idempotent* product aggregates

Let $\mathbf{D}_I \supseteq \{\mathbf{0}, \mathbf{1}\}$ be a set of idempotent elements of $\otimes$.

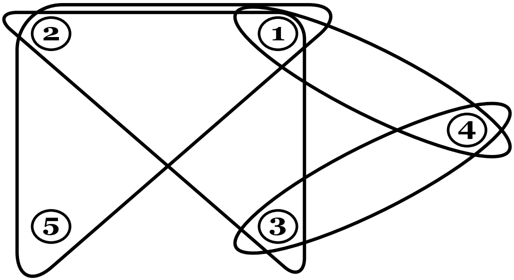
$$\varphi(\boldsymbol{x}_{[f]}) = \underbrace{\bigoplus_{x_{f+1}}^{(f+1)} \cdots \bigoplus_{x_{f+\ell}}^{(f+\ell)}}_{\text{Semiring aggregates}} \underbrace{\bigoplus_{x_{f+\ell+1}}^{(f+\ell+1)} \cdots \bigoplus_{x_n}^{(n)}}_{\text{Products or semirings closed under } \mathbf{D}_I} \underbrace{\bigotimes_{S\in\mathcal{E}}}_{\text{Closed under } \mathbf{D}_I} \underbrace{\psi_S(\boldsymbol{x}_S)}_{\in \mathbf{D}_I}$$

Examples: QCQ, #QCQ

$$\varphi(\boldsymbol{x}_{[f]}) = \bigoplus_{x_{f+1}}^{(f+1)} \cdots \bigoplus_{x_n}^{(n)} \prod_{S\in\mathcal{E}} \psi_S(\boldsymbol{x}_S).$$

$$\varphi = \sum_{x_1} \cdots \sum_{x_l} \bigoplus_{x_{l+1}}^{(l+1)} \cdots \bigoplus_{x_n}^{(n)} \prod_{S\in\mathcal{E}} \psi_S(\boldsymbol{x}_S).$$

$$\varphi = \sum_{x_1} \max_{x_2} \prod_{x_3} \max_{x_4} \max_{x_5} \psi_{123}\, \psi_{125}\, \psi_{14}\, \psi_{34}$$



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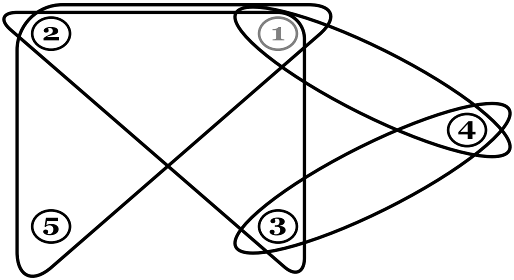

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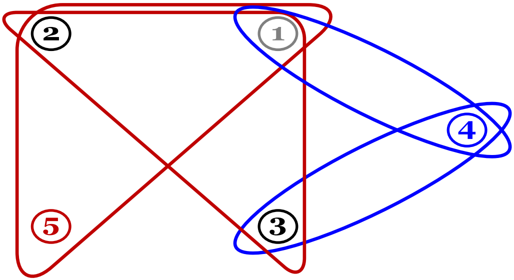

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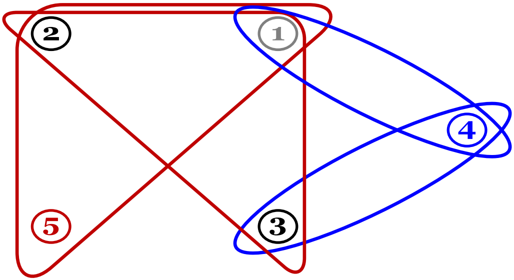

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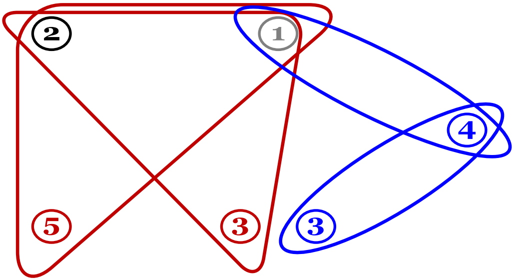

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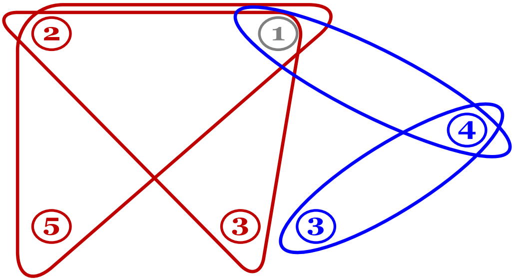

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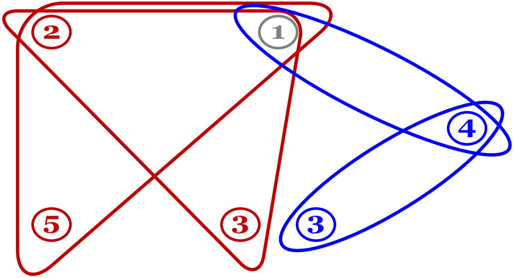
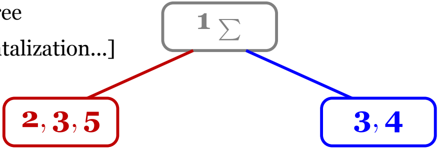


Expression Tree

[Compartmentalization...]



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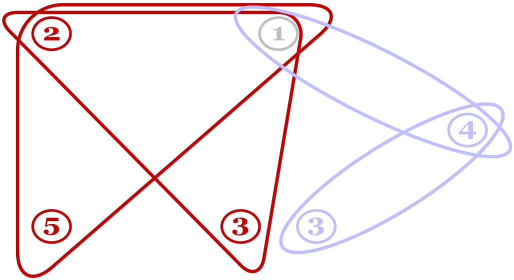

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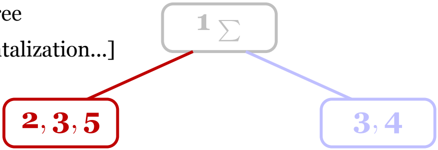

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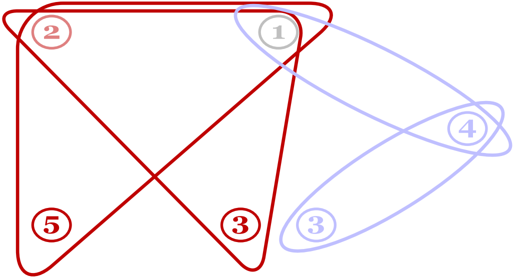

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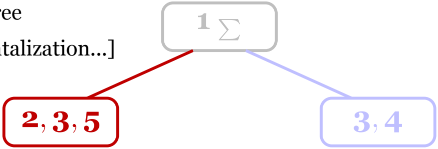

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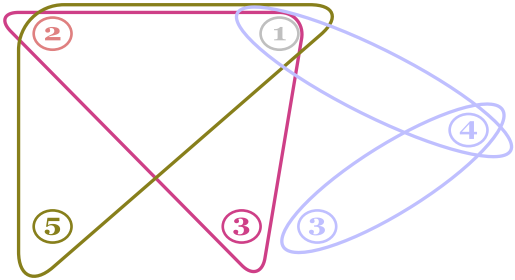

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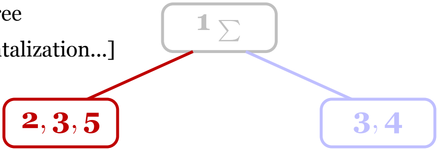

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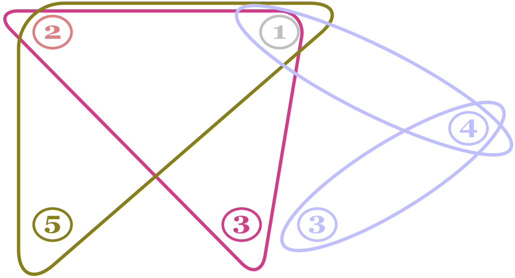

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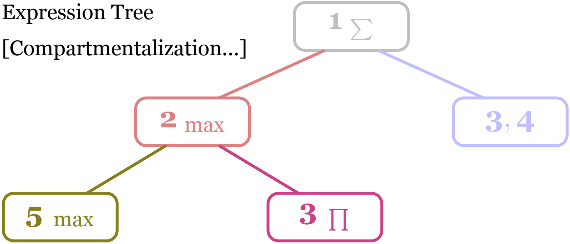

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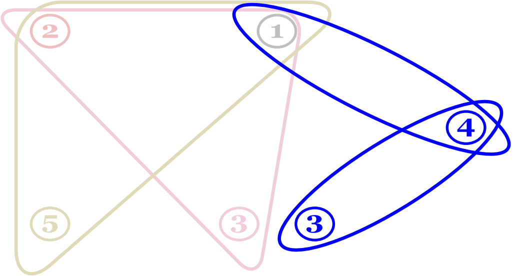

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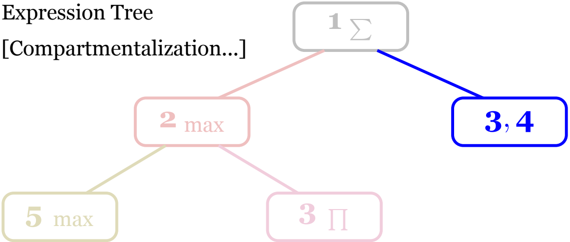

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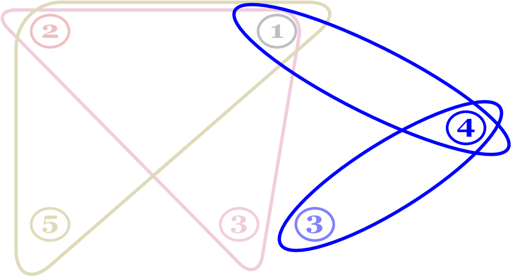

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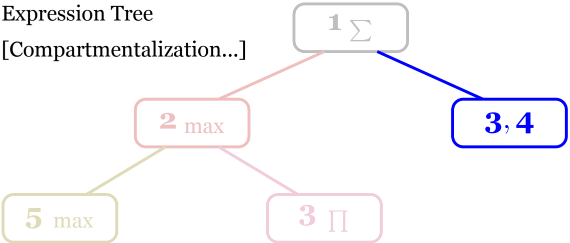

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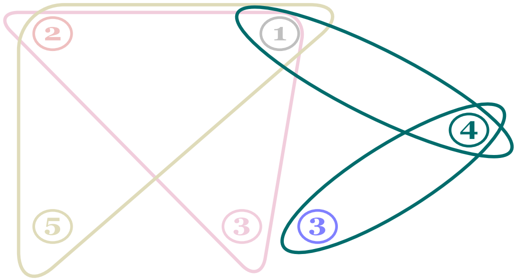

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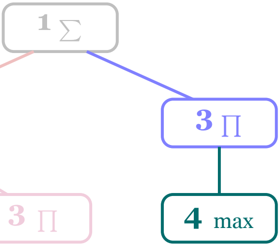

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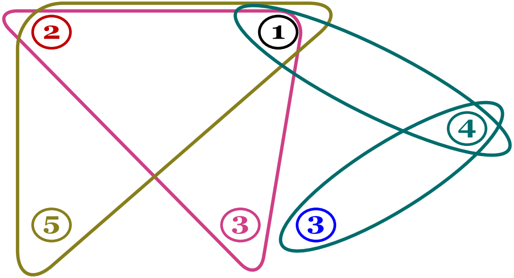

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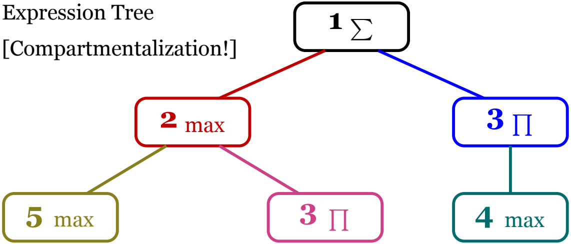

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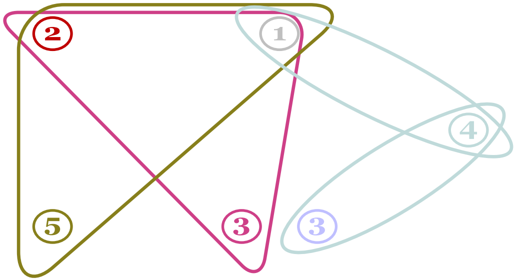

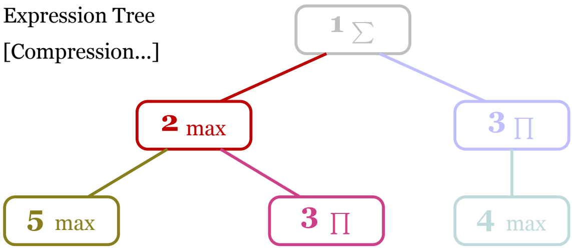

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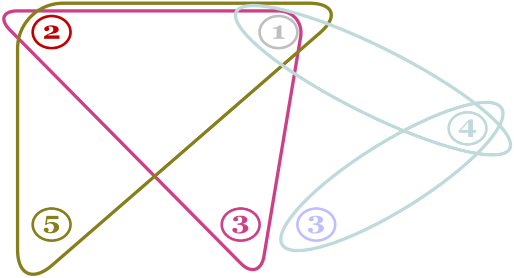

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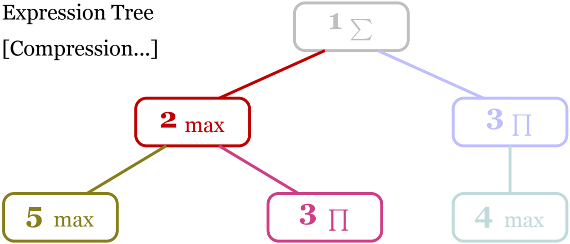
[Compression...]



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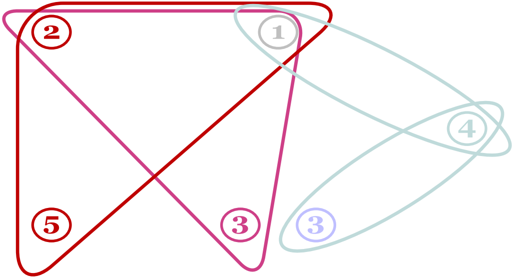

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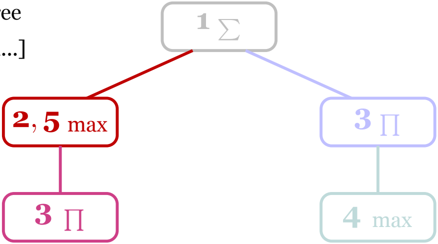

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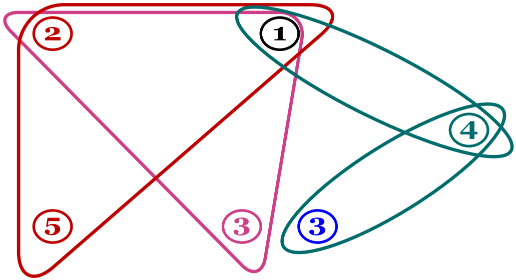

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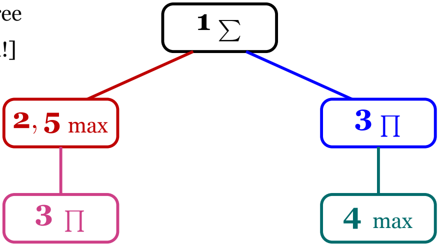

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Many Thanks!
Any FAQ**?**