Binary Search:
A Fundamental Algorithm

FIT 100

Binary search is a simple, common sense way to search through an *ordered* set of items.

Questions, often referred to as queries or probes, are asked to find if the desired item is smaller or larger.

If the question, hereafter called probe, is chosen from the middle of the sequence, ½ the possibilities are eliminated with each answer.

It's like 20 questions, but MUCH more specific.

FIT 100 Algorithm vs. Program

❖ Remember that an algorithm is an abstraction.

❖ A program incorporates all the details needed for implementation of an instance of the algorithm

❖ The description of binary search on the first slide was an algorithm – the idea of binary search given abstractly

❖ A program for binary search– which is your goal– will encode the algorithm for a specific situation, in a specific language, with specific assumptions

❖ Today we will look at:
  □ Binary search in general
  □ Review the Day Finder application – the specific implementation of binary search

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FIT 100 Pick a Card, Any Card

❖ The simplicity of binary search is this:
  □ As long as there are things in some kind of order, each question allows you to throw out half

  □ Ask another question that will divide the remaining items into two halves, throwing one out

  □ The question that allows you to cut down the search space is often called a probe

  □ Continue until there is only one thing left, and that is the item you are seeking

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FIT 100 Another Example of the Algorithm at Work

❖ Use binary search to locate a letter in the alphabet
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

❖ Use binary search to locate a letter in the alphabet
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
After M?

❖ Use binary search to locate a letter in the alphabet
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
After M? No

A B C D E F G H I J K L M ~~N O P Q R S T U V W X Y Z~~
After G?

❖ Use binary search to locate a letter in the alphabet
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
After M? No

A B C D E F G H I J K L M ~~N O P Q R S T U V W X Y Z~~
After G? Yes

~~A B C D E F G~~ H I J K L M ~~N O P Q R S T U V W X Y Z~~
After J?

❖ Use binary search to locate a letter in the alphabet
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
After M? No

A B C D E F G H I J K L M ~~N O P Q R S T U V W X Y Z~~
After G? Yes

~~A B C D E F G~~ H I J K L M ~~N O P Q R S T U V W X Y Z~~
After J? Yes

~~A B C D E F G H I J~~ K L M ~~N O P Q R S T U V W X Y Z~~
After L?

FIT 100 — Another Example of the Algorithm at Work

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
After M? No

A B C D E F G H I J K L M ~~N O P Q R S T U V W X Y Z~~
After G? Yes

~~A B C D E F G~~ H I J K L M ~~N O P Q R S T U V W X Y Z~~
After J? Yes

~~A B C D E F G~~ H I J K L M ~~N O P Q R S T U V W X Y Z~~
After L? No

~~A B C D E F G~~ H I J K L ~~M N O P Q R S T U V W X Y Z~~
After K?

FIT 100 — Another Example of the Algorithm at Work

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
After M? No

A B C D E F G H I J K L M ~~N O P Q R S T U V W X Y Z~~
After G? Yes

~~A B C D E F G~~ H I J K L M ~~N O P Q R S T U V W X Y Z~~
After J? Yes

~~A B C D E F G H I J~~ K L M ~~N O P Q R S T U V W X Y Z~~
After L? No

~~A B C D E F G H I J~~ K L ~~M N O P Q R S T U V W X Y Z~~
After K? Yes   The Letter is L

FIT 100 — What Are the Questions Doing?

❖ We use questions with the term "after" to keep our search algorithm simple

❖ Using "after" questions …. the probe should be:
  □ The middle item for odd-length ranges
  □ The last item in the first half for even-length ranges

❖ The questions stop when there is only one item left

    A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
    After M? No
    A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
    After G? Yes
    A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
    After J? Yes
    A B C D E F G H I J K L M N O P Q R S
    After L? No
    A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
    After K? Yes   The Letter is L

We always eliminate about ½ of the items. The best case scenario is ½ +1, the worst, ½ - 1

FIT 100 — Algorithm Analysis

❖ What are the inputs?
  □ The end points of an ordered sequence (lo, hi)
    ≡ In other words, both ends of the range to be searched
  □ Answers to a series of questions

❖ What are the outputs?
  □ A selected item

❖ How are the inputs transformed into outputs?
  □ The questions of the series are as follows:
    "Is the item you are looking for after item *x* ?"
    x is chosen to be the middle point in the current interval
    If the reply is yes, the new lo end of the interval is next after x
    If the reply is no, the new hi end of the interval is now x

❖ When does it end?
  □ When the interval contains only a single item

FIT 100 Example With Numbers

❖ Find a number in the range 1 to 20
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
After _ ?

FIT 100 Example With Numbers

❖ Find a number in the range 1 to 20
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
After 10 ? Yes
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
After _ ?

FIT 100 Example With Numbers

❖ Find a number in the range 1 to 20
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
After 10 ? Yes
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
After 15 ? No
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
After _ ?

FIT 100 Example With Numbers

❖ Find a number in the range 1 to 20
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
After 10 ? Yes
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
After 15 ? No
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
After 13 ? Yes
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
After _ ?

Example With Numbers

❖ Find a number in the range 1 to 20

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
After 10 ? Yes
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
After 15 ? No
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
After 13 ? Yes
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
After 14 ? Yes
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
The number is 15

Finding the Endpoints

❖ Find a number in the range of 1 to 20

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Probe = 10   Range Start = 1   Range End = 20   [1,20]

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Probe = 15   Range Start = 11   Range End = 20   [11,20]

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Probe = 13   Range Start = 11   Range End = 15   [11,15]

1 2 3 4 | One end of the range is changed after each answer
Probe = | The new end-point is either probe, or probe + 1 | 14,15]
        | Choice of probe or probe + 1 depends on a No or Yes answer

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Probe = 15   Range Start = 15   Range End = 15   [15,15]

Analysis of Probe

❖ The probe can be found by adding the endpoints of the range and dividing by 2

❖ Using the Integer division symbol (\ ) instead of the normal division symbol (/), will drop fractional digits and give the correct whole value

   probe value = (low endpoint + high endpoint) \ 2

❖ You now have enough information to compute the low end of any given range, the high end of any given range and the probe of any given range.

❖ How does this translate to the Day Finder project?

Month Extension Technique

❖ There is a slight hitch in our use of binary search with the Day Finder project. The complication is that our search for an exact day in a sign's range might span across 2 months

❖ How do we fix it? Logically extend the month:

Day Find

Enter Your Sign, Please

◯ Aries   ◯ Leo   ◯ Sagittarius
◯ Taurus   ◯ Virgo   ◯ Capricorn
◯ Gemini   ◯ Libra   ◯ Aquarius
◯ Cancer   ◯ Scorpio   ◯ Pisces

You were born between
August 23 And September 22
Y N Were you born after September 7

Days in August   31
Day in September +   22
Days in exAugust   53

August 53

The interval to be searched in 23 through 53

Visualize the Extended Month

❖ Think of the Zodiac sign as starting at its "start" day (23) and extending to the "last" day (22) + the number of days in the loMonth (31)

| ———— August ———— | ———— September ———— |

22 23 24 25 26 27 28 29 30 31 1 2 3 4 5 6 7 8 9 … 18 19 20 21 22

22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 … 49 50 51 52 53
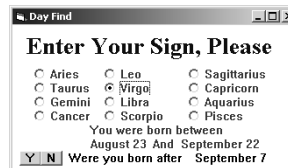
———————————— Virgo ————————————

❖ Any date that is less than or equal to the last day of the loMonth is in the loMonth

❖ Any date that is more than the last day of the loMonth is in the hiMonth. To get the correct day display, the number of days in the loMonth should be subtracted from the date

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Changing the Probe to a Date

❖ The initial interval for Virgo is: 23 to 53
❖ The probe, therefore, is: $(23 + 53) \setminus 2 = 38$
❖ What day is August 38th?
  ▫ The month is the next month, September
  ▫ The day is 38, from which we subtract the number of days in August, 38 –31 = 7

Day Find _ □ ×

Enter Your Sign, Please

○ Aries   ○ Leo   ○ Sagittarius
○ Taurus   ● Virgo   ○ Capricorn
○ Gemini   ○ Libra   ○ Aquarius
○ Cancer   ○ Scorpio ○ Pisces

You were born between
August 23 And September 22
Y N Were you born after September 7

Overall Flow

❖ What are the new variables needed?
  ▫ loEnd, hiEnd, midPt and numDays

❖ Where do the initial values come from?
  ▫ After the Zodiac Range is found, loEnd and hiEnd can be set
  ▫ Once you know the endpoints of the range, the probe (midPt) can be computed

❖ How are those values updated (loEnd, hiEnd, midPt)?
  ▫ In the Yes and No button click event handlers
  ▫ In the case of Yes being clicked, which end moves?
    ▪ loEnd moves past the midPt by one
  ▫ In the case of No, which end moves?
    ▪ hiEnd moves down to the midPt

❖ When do the questions end?
  ▫ When the end points are equal

Solution Structure

Declarations	--additional variable declarations
Private Sub optAri	
Private Sub optCan	*Inherit from Zodiac (part 1)*
Private Sub optCap	
...	
Private Sub cmdOK	--initialize, make first guess
Private Sub cmdYes	--revise interval, make guess
Private Sub cmdNo	-- revise interval, make guess