Binary Search -- A Fundamental Algorithm

FIT 100

Binary search is a clever, though common sense way to search an ordered set of items. Queries are made, called *probes*, asking whether the desired item is smaller or larger. If the probe is chosen in the middle of the sequence, 1/2 of the possibilities must be eliminated with any answer. Now the details...

FIT 100 Reminder … Algorithm vs Program

- The process just described on the title slide -- suitably embellished -- is the binary search *algorithm* … the idea given abstractly
- A *program* for binary search -- your goal -- will encode the algorithm for a specific situation, in a specific language, with specific assumptions

Today's Topics: Analyze the binary search algorithm
Review the Day Finder application
Reason through the logic of using binary search in the Day Finder context

FIT 100 An Example

- Use binary search to locate a letter in the alphabet

  A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

FIT 100 An Example

- Use binary search to locate a letter in the alphabet

  A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

  After M?

FIT 100 An Example

- Use binary search to locate a letter in the alphabet

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
After M? N
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

FIT 100 An Example

- Use binary search to locate a letter in the alphabet

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
After M? N
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
After G?

FIT 100 An Example

- Use binary search to locate a letter in the alphabet

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
After M? N
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
After G? Y
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
After J?

FIT 100 An Example

- Use binary search to locate a letter in the alphabet

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
After M? N
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
After G? Y
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
After J? Y
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
After L?

FIT 100 An Example

- Use binary search to locate a letter in the alphabet

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
After M? N
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
After G? Y
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
After J? Y
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
After L? N
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
After K?

FIT 100 An Example

- Use binary search to locate a letter in the alphabet

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
After M? N
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
After G? Y
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
After J? Y
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
After L? N
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
After K? Y
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
The letter is L

FIT 100 Analyzing Properties Of Probe

- Though "before" and "after" questions could be used, adopting one and staying with it simplifies the effort
- Using "after" questions … the probe should be
  - For odd-length ranges, the middle item
  - For even-length ranges, last item in first half
- Stop when there is only one item left

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
After M? N
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
After G? Y
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
After J? Y
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
After L? N
A B C D E F G H I J K L M N O
After K? Y
A B C D E F G H I J K L M N O
The letter is L

We always eliminate about 1/2 of the items, with best case 1/2+1 and worst cast 1/2-1

FIT 100 Algorithm Analysis

- Understanding the problem …
  - Inputs: The end points, (*lo*, *hi*), of an ordered sequence
    Answers to an series of questions
  - Outputs: A selected item
  - How the inputs are transformed to the outputs:

The questions of the series have the form

"Is the desired item after item x?"

so that the x^{th} item is chosen to be midway in the interval

If the reply is *yes*, the new interval (*next after x, hi*)

If the reply is *no*, the new interval is (*lo, x*)

The output is the item when the interval contains only a single item

FIT 100 Example -- End Points and Probe

- Find a number in the range 1 to 20

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
After __ ?

FIT 100 Example -- End Points and Probe

- Find a number in the range 1 to 20

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
After 10 Y
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
After __ ?

FIT 100 Example -- End Points and Probe

- Find a number in the range 1 to 20

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
After 10
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
After 15 N
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
After __ ?

FIT 100 Example -- End Points and Probe

- Find a number in the range 1 to 20

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
After 10
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
After 15
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
After 13 Y
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
After __ ?

Example -- End Points and Probe

- Find a number in the range 1 to 20

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
After 10
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
After 15
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
After 13
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
After 14 Y
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
The number is 15

Analysis of End Points

- Find a number in the range 1 to 20

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Probe = 10, Range Start = 1, Range End = 20 [1,20]
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Probe = 15, Range Start = 11, Range End = 20 [11,20]
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Probe = 13, Range Start = 11, Range End = 15 [11,15]
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Probe = 14, Range Start = 14, Range End = 15 [14,15]
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Range Start = 15, Range End = 15 [15,15]

One end of the range is changed after each answer
The new end-point is either the *probe* or *probe+1*
Choice of *probe* or *probe+1* depends on N or Y ans

Analysis of Probe

- Find a number in the range 1 to 20

The range is inclusive

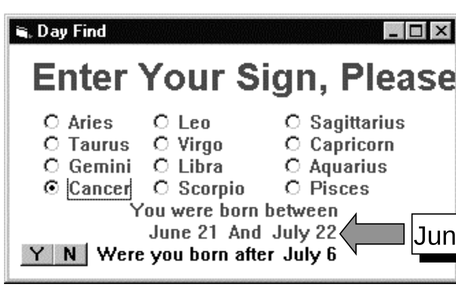
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Probe = 10, [1,20] (1+20)/2 = 10.5 (1+20)\2 = 10
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Probe = 15, [11,20] (11+20)/2 = 15.5 (11+20)\2 = 15
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Probe = 13, [11,15] (11+15)/2 = 13 (11+15)\2 = 13
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Probe = 14, [14,15] (14+15)/2 = 14.5 (14+15)\2 = 14
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
[15,15]

Probe is found by adding end points and dividing by 2

Integer divide (\) drops fractional digits, giving the right probe value

Month Extension Technique

- The “complicating” problem with searching for a birthday in a sign, is that the signs span parts of two months
- Not to worry … logically extend the starting month



June 52

Days in June	30
Day in July +	22
Day in exJune	52

The interval to be searched is 21 through 52

FIT 100 Visualize The Extended Month

- Think of the Zodiac sign as starting at its start day (21) and extending to the end day (22) + number of days in loMo (30)

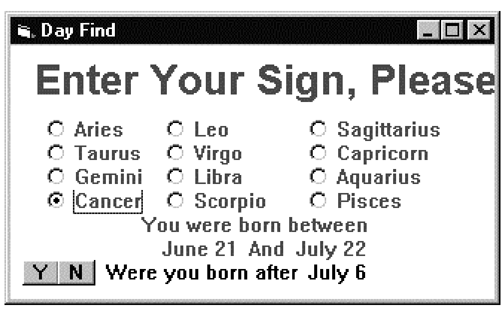
June										July						
21	22	23	24	25	26	27	28	29	30	1	2	3	4	5	...	21	22
21	22	23	24	25	26	27	28	29	30	31	32	33	34	34	...	51	52

Cancer

- Any date that is less than or equal to the last day of loMo is in the loMo
- Any date that is more than the last day of loMo is in hiMo, and is too large by the number of days in loMo

FIT 100 Transforming Probe To A Date

- The interval is: [21-52]
- The probe for the interval is: (21+52)\2 = 36
- What day is June 36?
  - The month is the next month, July
  - The day is 36 reduced by the number of days in June, 36-30 = 6



FIT 100 Overall Data Flow ...

- What are the new variables needed
  - loEnd, hiEnd, midPt and lastDay
- Where do the initial values come from?
  - After the Zodiac computation, loEnd and hiEnd can be set
  - Once interval set, probe (midPt) can be computed
- How are these values updated?
  - In the yes and no button event handlers
  - In the case of “yes,” which end moves?
    - loEnd moves up past the midPt
  - In the case of “no”, which end moves?
    - hiEnd moves down to the midPt
- When does the questioning terminate?
  - When the end points are equal

FIT 100 Structure Of Solution

Declarations	Inherit from Zodiac	-- *additional variable declarations*
Private Sub optAri		
Private Sub optTau		
Private Sub optGem		
Private Sub optCan		
Private Sub optLeo		
Private Sub optVir		
Private Sub optLib		
Private Sub optSco		
Private Sub optSag		
Private Sub optCap		
Private Sub optAqu		
Private Sub optPis		
Private Sub cmdOK		-- *initialize, make first guess*
Private Sub cmd Yes		-- *revise interval, make guess*
Private Sub cmd No		-- *revise interval, make guess*