Encryption

CSE 100

Keeping secrets in an electronic context is a challenge, but a new technology solves the problem using some very old mathematics

FIT 100 Keeping Secrets

- Some things should be kept private
  - Credit card numbers
  - Passwords
  - …
- There are cases where trust is needed in IT …

Secret Codes
and
Coin Flipping on the Phone

FIT 100

A Secret Code


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5815 Ann Arbor NE SEATTLE 98105 206 524-3371
John R Dr 8001 Sand Point Way NE SEATTLE 98115 206 523-8877
GATELY H A 3847 Woodlawn N SEATTLE 98103 206 634-2368
Joe & Kelley 15114 SE 224th KENT 98042 253 639-8073
Kimberlie 5815 Ann Arbor NE SEATTLE 98105 206 524-0179
Steve & Christina 31500 1st Ave S FEDWY 98003 253 946-4303
GATENS Clay M 206 352-1590
James 2008 SW 348th FEDWY 98023 253 838-3565
GATERS M 11300 3d NE SEATTLE 98125 206 363-1482
GATES A 721 17th SEATTLE 98122 206 323-3705
A & C Shoreline 206 542-4366
Abraham 820 NE 57th SEATTLE 98105 206 729-1580
Andrew R 14405 SE 15th BLVU 98007 425 957-7398
Barron 206 901-1947
Bertha 3014 NE 98th SEATTLE 98115 206 729-0714

524-0179 4
634-2368 2
634-2368 3
634-2368 6
352-1590 2

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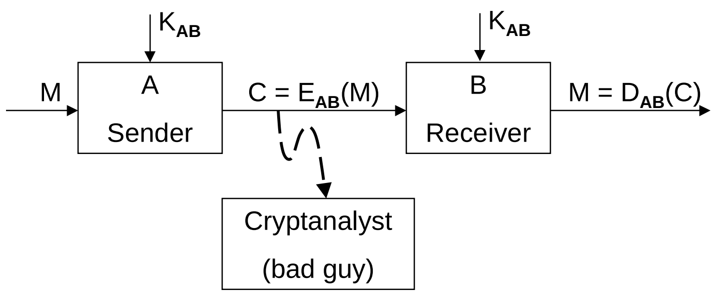
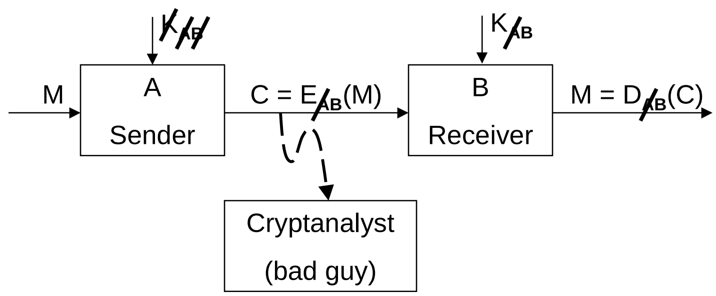
What Is a Cryptosystem?



M	C	K_{AB}
Message	Encryption	Key
Plaintext	Cyphertext	
Cleartext		

What Is a Public Key Cryptosystem?



M	C	K_B	E_B
Message	Encryption	Key	Public Key
Plaintext	Cyphertext	Private Key	
Cleartext			

The RSA Public Key Cryptosystem

- Invented by Rivest, Shamir, and Adleman in 1977.
- Has proven resilient to all cryptanalytic attacks since.

FIT 100 Receiver's Set-Up

- Choose 500-digit primes p and q (each 2 more than a multiple of 3).
  - $p = 5$, $q = 11$
- Let $n = pq$.
  - $n = 55$
- Let $s = (1/3)\,(2(p - 1)(q - 1) + 1)$.
  - $s = (1/3)\,(2 \cdot 4 \cdot 10 + 1) = 27$
- Publish n.
  - Keep p, q and s secret.

FIT 100 Encrypting a Message

- Break the message into chunks.
  - $\boxed{H}\boxed{I}\quad\boxed{C}\boxed{H}\boxed{R}\boxed{I}\boxed{S}$ …
- Translate each chunk into an integer M ($0 < M < n$).
  - $\boxed{8}\ \boxed{9}\quad\boxed{3}\ \boxed{8}\ \boxed{18}\ \boxed{9}\boxed{19}$ …
- Divide M^3 by n. $E(M)$ is the remainder.
  - $M = 8$, $n = 55$
  - $8^3 = 512 = 9 \times 55 + 17$
  - $E(8) = 17$

FIT 100 Decrypting A Cyphertext C

- Divide C^s by *n*. *D(C)* is the remainder.
  $C = 17$, $n = 55$, $s = 27$
  $17^{27} = 1{,}667{,}711{,}322{,}168{,}688{,}287{,}513{,}535{,}727{,}415{,}473$
  $= 30{,}322{,}024{,}039{,}430{,}696{,}136{,}609{,}740{,}498{,}463 \times 55 + 8$
  $D(17) = 8$
- Translate *D(C)* into letters.
  H

FIT 100 Why does it work?

Euler's Theorem (1736): Suppose

- p and q are distinct primes,
- $n = pq,$
- $0 \leq M < n,$ and
- $k > 0.$

If $M^{k(p-1)(q-1)+1}$ is divided by n, the remainder is M.

$$(M^3)^s = (M^3)^{(1/3)(2(p-1)(q-1)+1)}$$
$$= M^{2(p-1)(q-1)+1}$$

FIT 100 Leonhard Euler 1707-1783

FIT 100 Why Is It Secure?

- To find $M = D(C)$, you seem to need s.
- To find s, you seem to need p and q.
- All you have is $n = pq$.
- How hard is it to factor a 1000-digit number n?
  With the grade school method,
  doing 10,000,000 steps per second
  it would take … 10^{485} years.

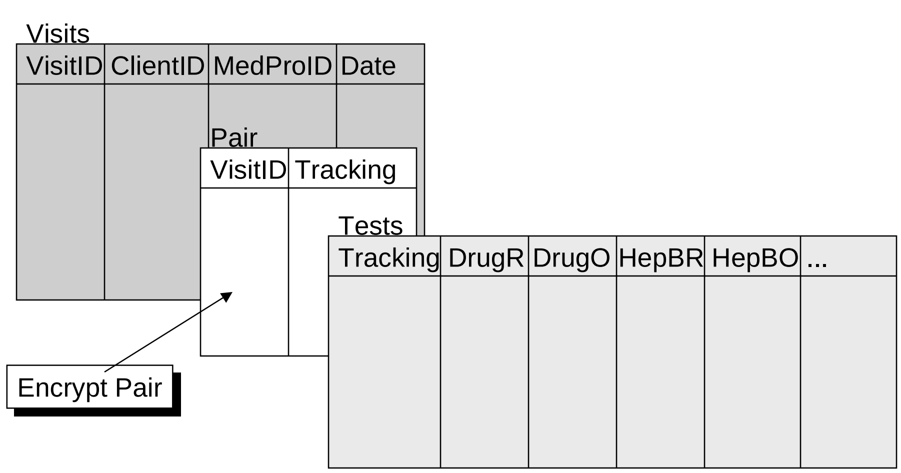
State of the Art in Factoring

- 1977: Inventors encrypt a challenge using “RSA129,” a 129-digit number *n = pq.*
- 1981: Pomerance invents a new factoring method.
- 1994: RSA129 factored over an 8 month period using 1000 computers on the Internet around the world.

- With this method, a 250-digit number would take 100,000,000 times as long.

Keeping Tracking Number Private

- Split Visits table into three parts



FIT 100 Flipping a Coin Over the Phone

A		B
Choose random *x*.		
$y = E_A(x)$		
	$\xrightarrow{y}$	
		Guess if *x* is even or odd.
	$\xleftarrow[\text{“odd”}]{\text{“even”}}$	
	$\xrightarrow{x}$	
		Check $y = E_A(x)$.

- B wins if the guess about *x* was right

FIT 100 Summary

- Encryption is a process of encoding information so it is not easily deciphered
- The strength of the encryption technique is based on the difficulty of a cryptanalyst to break the code
- A Public Key Encryption scheme like RSA allows sender-to-receiver secure transmission without sender and receiver first exchanging keys
- Encryption aids with other trust-related tasks