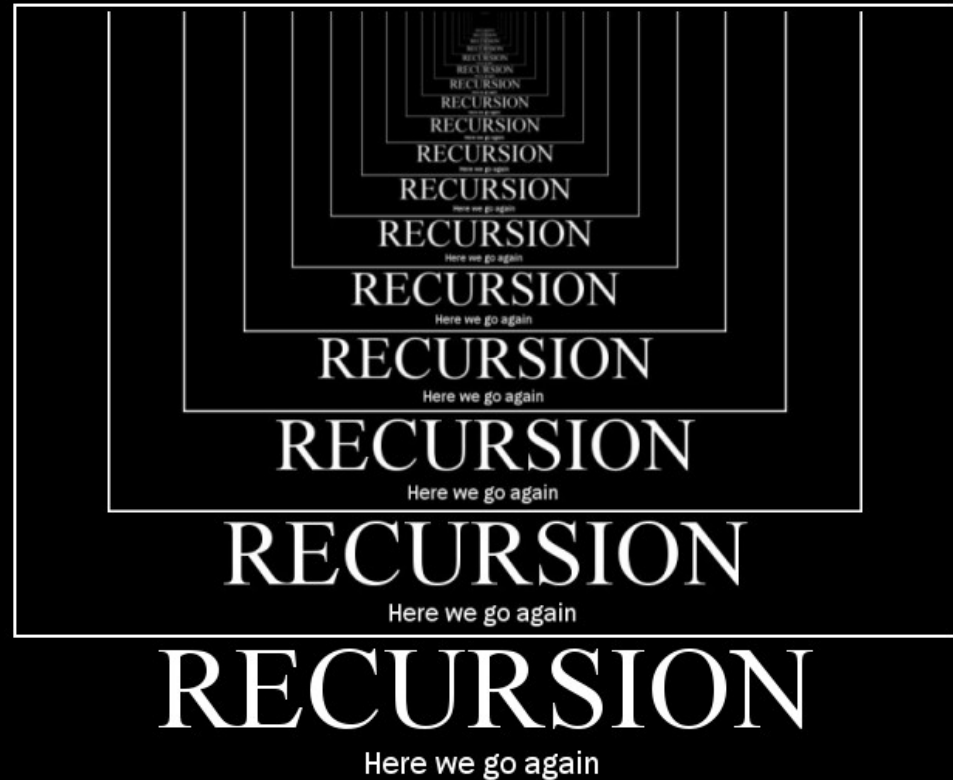


Use what you've got

Recursion

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University of Washington*



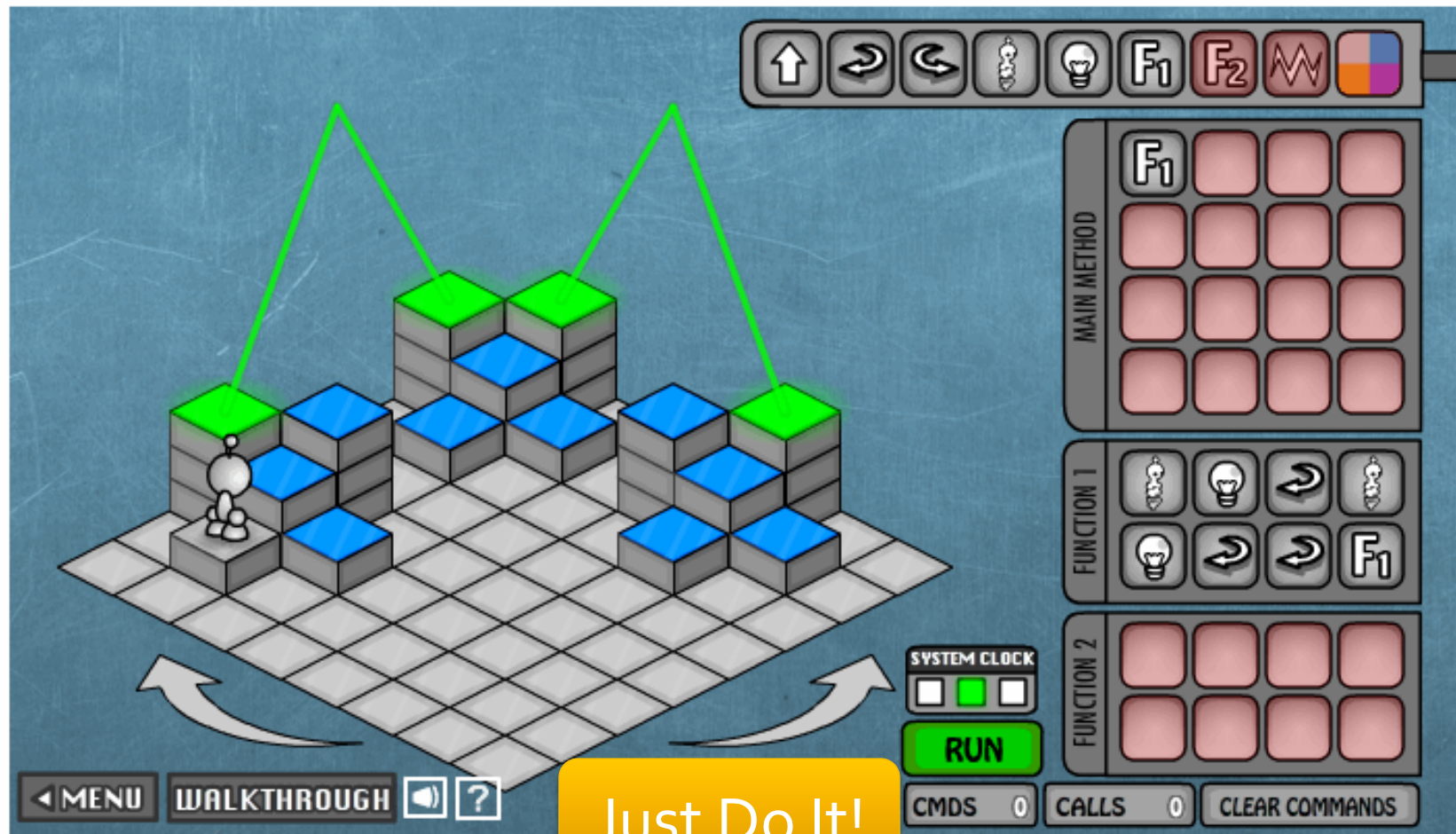
Recall Recursion In Lightbot 2.0

Recursion means to call a function in its own definition

The screenshot shows the Lightbot 2.0 interface. On the left, a 3D maze is displayed with a small robot on a grey block. The maze has several blue and green blocks. A yellow arrow points from the text 'Recursion means to call a function in its own definition' to the 'F1' icon in the 'FUNCTION 1' slot of the function editor. The function editor has three slots: 'MAIN METHOD', 'FUNCTION 1', and 'FUNCTION 2'. The 'MAIN METHOD' slot contains an 'F1' icon. The 'FUNCTION 1' slot contains an upward arrow, a lightbulb, and an 'F1' icon. The 'FUNCTION 2' slot is empty. A 'SYSTEM CLOCK' indicator shows a green light, and a 'RUN' button is at the bottom.

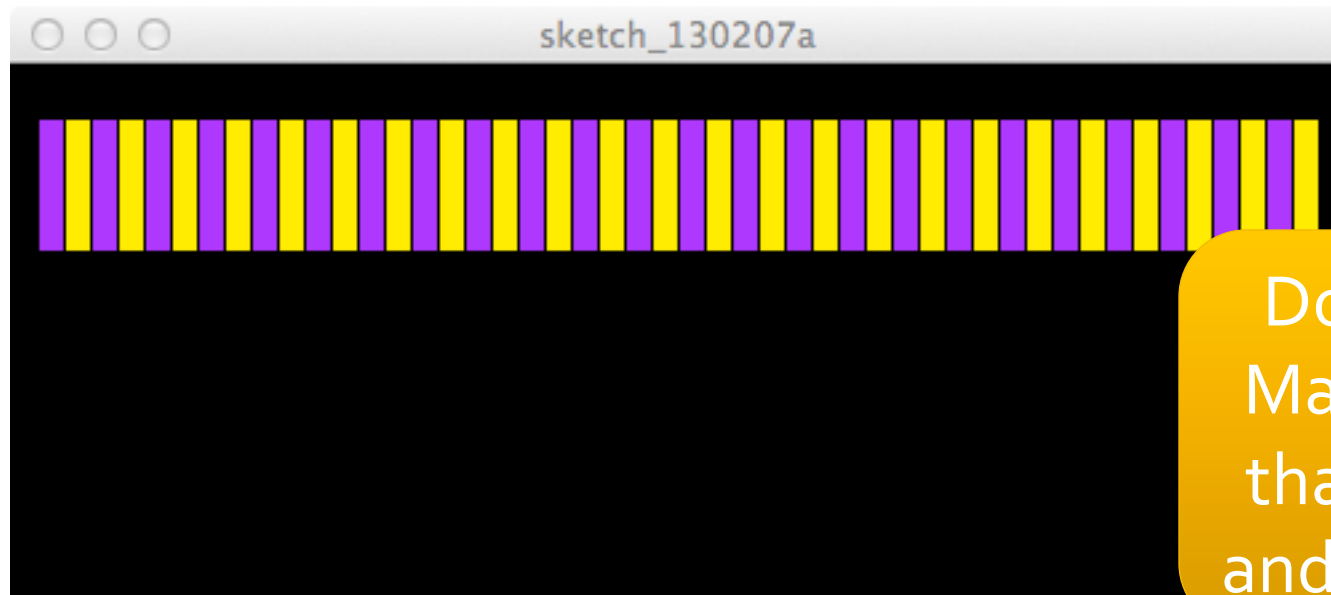
Recursion

- If the “concept applies,” use it



Often, Recursive Is Easiest Solution

- Specification: Draw alternating purple & gold bars across the top, leaving 10 px at each end
 - How large is canvas?
 - How many bars are needed?
 - What color to start/end with?



Don't Know?
Make solution
that is flexible
and adjust later

Recursion: Draw A Pair, Ask "More?"

```
void setup( ) {  
  size(500,500);  
  background(0);  
}
```

```
void draw( ) {  
  husky(10, 100);  
}
```



```
void husky(int xpos, int ypos) {  
  fill(157,0,255);  
  rect(xpos, ypos, 10,50);  
  fill(255,235,0);  
  rect(xpos+10, ypos, 10, 50);  
  if (xpos+20 <= width-30) {  
    husky(xpos+20, ypos);  
  }  
}
```

Draw A Purple Bar

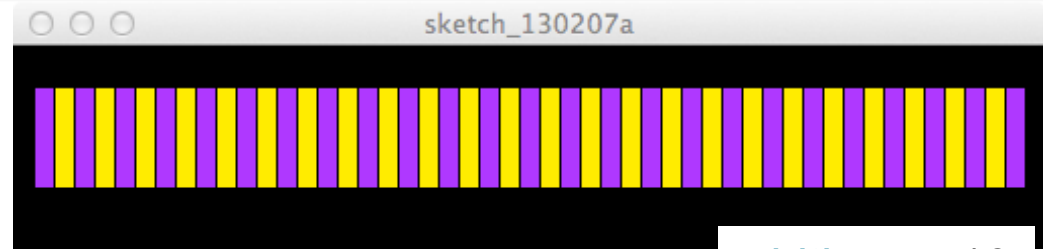
Draw A Gold Bar

Is There Space For One
More Pair Before End?

Yup, Do It Again

Another: Draw 1 Bar, Flip Colors

```
void draw( ) {  
  husky(10, 20, true);  
}
```



```
void husky(int xpos, int ypos, boolean pORg) {  
  if (pORg) {  
    fill(157,0,255); // Pick Purple  
  } else {  
    fill(255,235,0); // Pick Gold  
  }  
  rect(xpos, ypos, 10,50); // Draw bar  
  if (xpos+10 <= width-20) { // Keep Going?  
    husky(xpos+10, ypos, !pORg); // Yes  
  }  
}
```

One More Example: Factorial

- Math people say $n! = n * (n-1) * (n-2) * \dots * 2 * 1$
- CS people say $n! = \text{if } n == 1, \text{ then } 1, \text{ else } n * (n-1)!$

```
int fact( int n) {  
    int soFar = 1;  
    for(int i = 2; i <= n; i = i +1) {  
        soFar= soFar* i;  
    }  
}
```

```
int fact( int n ) {  
    if (n <= 1) {  
        return 1;  
    } else {  
        return n*fact(n-1);  
    }  
}
```

Most Recursions Have 2 Cases

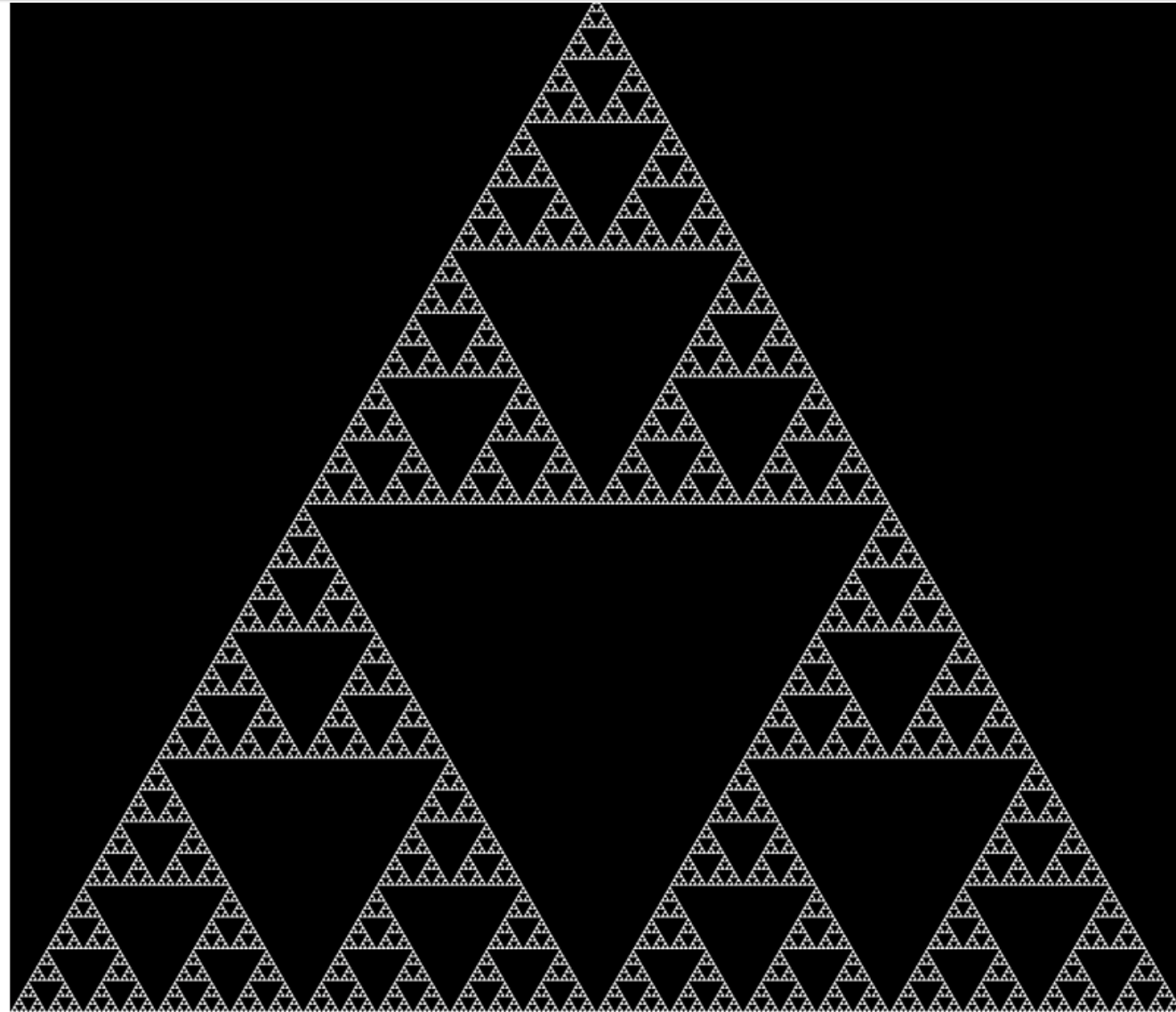
- Generally, in recursion, the program will show two cases: Base and Recursive ... you need both

```
int fact( int n ) {  
    if (n <= 1) {  
        return 1;  
    } else {  
        return n*fact(n-1);  
    }  
}
```

Basis Case – It Stops
The Recursion

Recursive Case – It Keeps
The Recursion Going

Check it Out: Sierpinski Triangle



Sierpinski Triangle

- What is it?
- Abstracting, we have
 - “A Sierpinski Triangle is an equilateral triangle”
 - “A Sierpinski Triangle can also be three copies of a Sierpinski Triangle, touching at their corners”



Sierpinski Triangle

- What is it?
- Abstracting, we have
 - “A Sierpinski Triangle is an equilateral triangle”
 - “A Sierpinski Triangle can also be three copies of a Sierpinski Triangle, touching at their corners”



- What's the base case? What's the recursive case?

Sierpinski Triangle Code (tiny)

```
1 // Sierpinski.pde by Martin Prout
2 float T_HEIGHT = sqrt(3)/2;
3 float TOP_Y = 1/sqrt(3);
4 float BOT_Y = sqrt(3)/6;
5 float triangleSize = 800;
6
7 void setup(){
8   size(int(triangleSize),int(T_HEIGHT*triangleSize));
9   smooth();
10  fill(255);
11  background(0);
12  noStroke();
13  drawSierpinski(width/2, height * (TOP_Y/T_HEIGHT), triangleSize);
14 }
15
16 void drawSierpinski(float cx, float cy, float sz){
17   if (sz < 5){ // Limit no of recursions on size
18     drawTriangle(cx, cy, sz); // Only draw terminals
19     noLoop();
20   }
21   else{
22     float cx0 = cx;
23     float cy0 = cy - BOT_Y * sz;
24     float cx1 = cx - sz/4;
25     float cy1 = cy + (BOT_Y/2) * sz;
26     float cx2 = cx + sz/4;
27     float cy2 = cy + (BOT_Y/2) * sz;
28     drawSierpinski(cx0, cy0, sz/2);
29     drawSierpinski(cx1, cy1, sz/2);
30     drawSierpinski(cx2, cy2, sz/2);
31   }
32 }
33
34 void drawTriangle(float cx, float cy, float sz){
35   float cx0 = cx;
36   float cy0 = cy - TOP_Y * sz;
37   float cx1 = cx - sz/2;
38   float cy1 = cy + BOT_Y * sz;
39   float cx2 = cx + sz/2;
40   float cy2 = cy + BOT_Y * sz;
41   triangle(cx0, cy0, cx1, cy1, cx2, cy2);
42 }
```

Sierpinski Triangle Code

```
1 // Sierpinski.pde by Martin Prout
2 float T_HEIGHT = sqrt(3)/2;

7 void setup(){
8   size(int(triangleSize),int(T_HEIGHT*triangleSize));
9   smooth();
10  fill(255);
11  background(0);
12  noStroke();
13  drawSierpinski(width/2, height * (TOP_Y/T_HEIGHT), triangleSize);
14 }

15
16 void drawSierpinski(float cx, float cy, float sz){
17   if (sz < 5){ // Limit no of recursions on size
18     drawTriangle(cx, cy, sz); // Only draw terminals
19     noLoop();
20   }
21   else{
22     float cx0 = cx;
23     float cy0 = cy - BOT_Y * sz;
24     float cx1 = cx - sz/4;
25     float cy1 = cy + (BOT_Y/2) * sz;
26     float cx2 = cx + sz/4;
27     float cy2 = cy + (BOT_Y/2) * sz;
28     drawSierpinski(cx0, cy0, sz/2);
29     drawSierpinski(cx1, cy1, sz/2);
30     drawSierpinski(cx2, cy2, sz/2);
31   }
32 }
33
```

```
41   triangle(cx0, cy0, cx1, cy1, cx2, cy2);
42 }
```

Recursion Often Uses Less Thinking

- Often we can solve a problem “top down”
- Fibonacci numbers –

1, 1, 2, 3, 5, 8, 13, 21, 34, ...

i^{th} item is $i-1^{\text{st}} + i-2^{\text{nd}}$ except the first two, both 1

- Translate definition directly:

$$fib(n) = \begin{cases} 1 & \text{if } n < 2 \\ fib(n-1) + fib(n-2) & \text{otherwise} \end{cases}$$

- It works like all functions work

The Fib Code In Processing ...

- If anyone actually cared about Fibonacci numbers, they could be computed ...

```
int fib(int n) {  
    if (n < 2 ) {  
        return 1;  
    }else{  
        return fib(n-1) + fib(n-2);  
    }  
}
```

Leave The Thinking To The Agent ...

$$fib(n) = \begin{cases} 1 & \text{if } n < 2 \\ fib(n-1) + fib(n-2) & \text{otherwise} \end{cases}$$

$$fib(4) = fib(3) + fib(2)$$

- $fib(3) = fib(2) + fib(1)$
 - $fib(2) = fib(1) + fib(0) = 1 + 1 = 2$
 $= 2 + 1 = 3$

$$\text{So, } fib(4) = 3 + fib(2)$$

- $fib(2) = fib(1) + fib(0) = 1 + 1 = 2$

$$\text{So, } fib(4) = 3 + 2 = 5$$

Programmers don't need to worry about the details if the definition is right and the termination is right; the computer does the rest

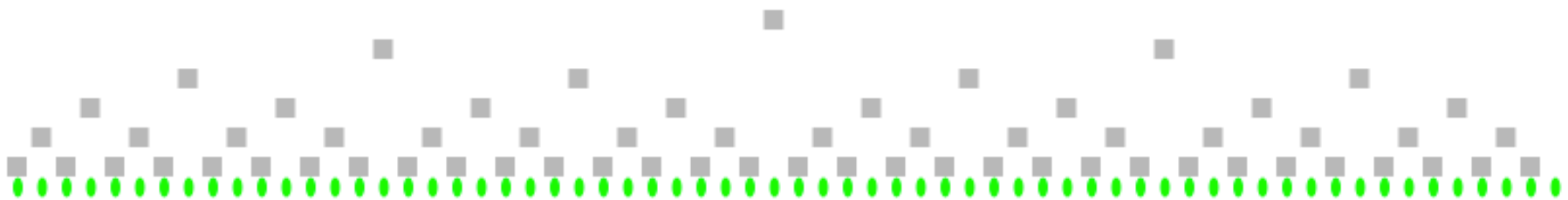
See The Progression of Calls ...



```
    box(6);  
}  
void box(int level) {  
    if (level > 0) {  
        level = level - 1;  
        box(level);  
        fill(170);  
        rect(xdist, 200-level*30, 20, 20);  
        xdist = xdist + 25;  
        box(level);  
    }  
}
```

The boxes are drawn in order, left to right

See The Progression of Calls ...

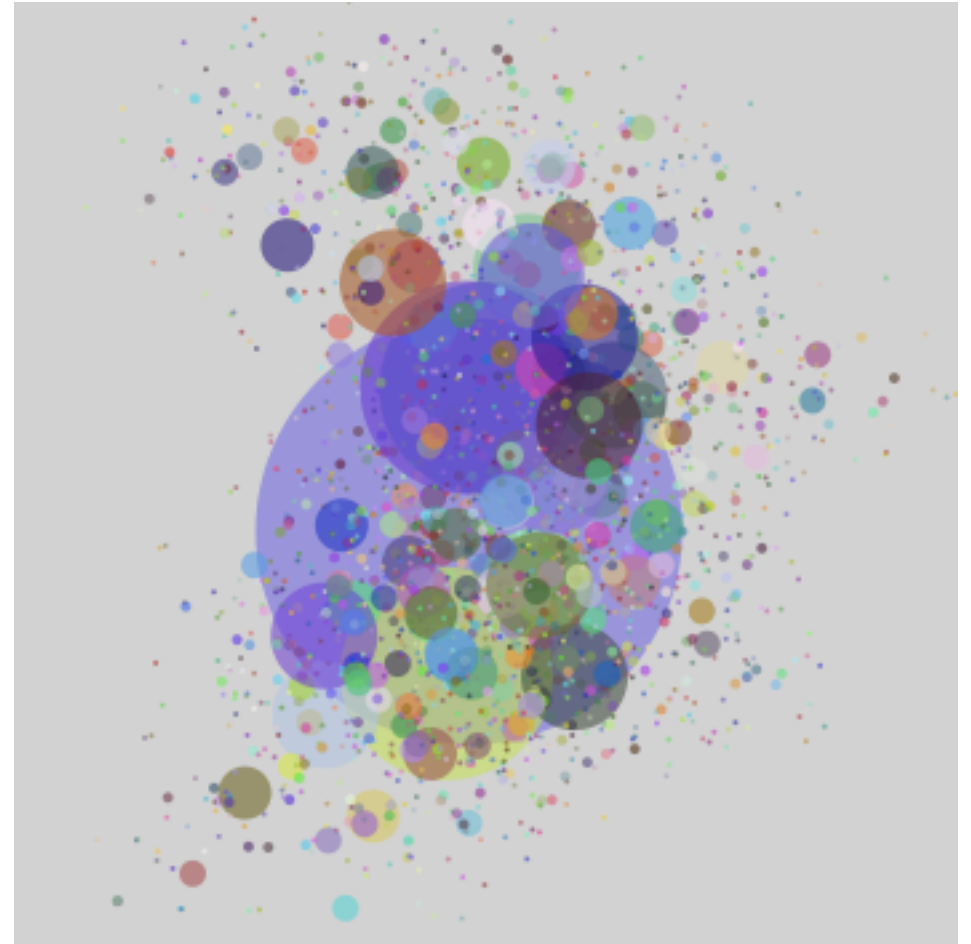


```
void box(int level) {  
    if (level > 0) {  
        level = level - 1;  
        box(level);  
        fill(170);  
        rect(xdist, 200-level*30, 20, 20);  
        xdist = xdist + 25;  
        box(level);  
    } else {  
        fill(0,255,0);  
        ellipse(xdist+10, 230, 10, 20);  
    }  
}
```

Each level 0 call
draws a leaf

Wrap Up

- Recursion often simplifies programming
- It's only a big deal to CS people, and for them only because it is so "elegant" (?)
- See Processing Ref for this cute program



All Circles From 1 Call Are 1 Color

