Recursion II

CSE 120 Spring 2017

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Administrivia

- Assignments:
  - Mid-Quarter Survey due tonight (5/3)
  - Recursive Tree due Thursday (5/4)
  - Color Checker due Saturday (5/6)
  - Living Computers Museum Report (5/14)

- Guest lecture on Friday: Proofs and Computation
  - Reading Check (5/4): mathematics

- Midterm re-grade requests due tonight (5/3)
  - Adjusted scores will be uploaded to Canvas after regrade requests are handled

Recursion Review

- A *recursive* function calls itself to solve its problem

- Base Case:
  - What happens for special/simple inputs
  - Need base case(s) to prevent infinite recursion

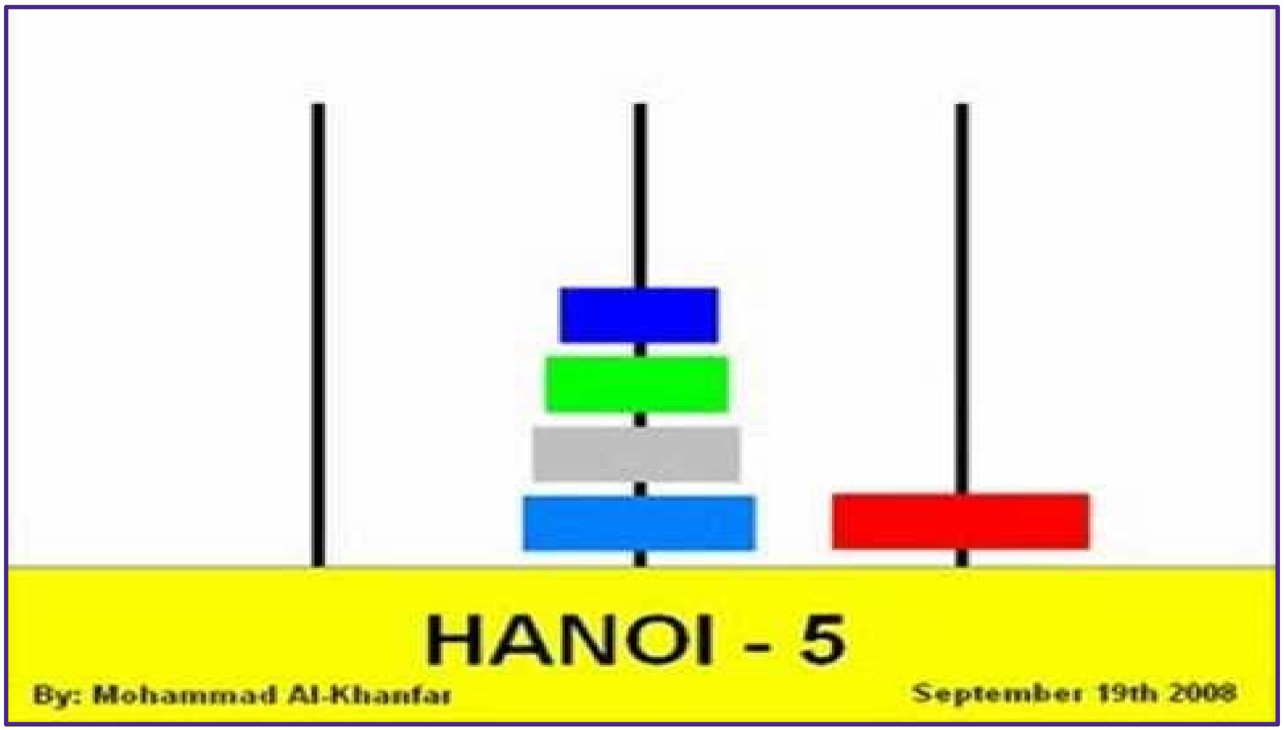
- Recursive Case:
  - Function calls itself one or more times on “smaller” problems
    - How to make the problem smaller varies ← this is the tricky part!

Outline

- **Example: Tower of Hanoi**
- Variable Scope Revisited
- Example: Fibonacci
- Example: Snowflake Fractal

Tower of Hanoi

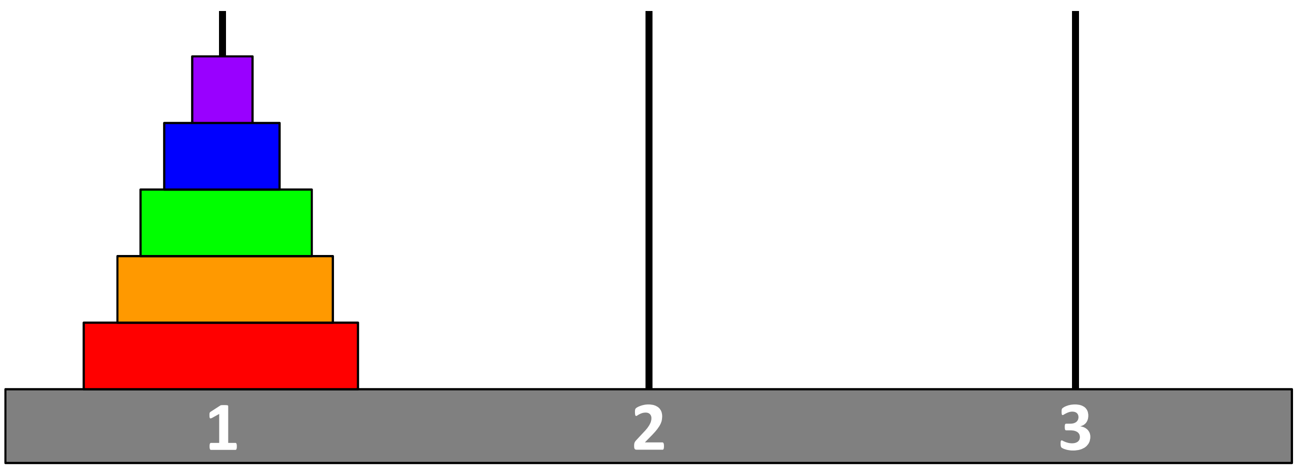
- Mathematical puzzle/game
  - Goal is to move entire stack from one peg to any other peg
- Rules:
  - There are only 3 available pegs
  - Can only move one disk at a time
  - A disk cannot sit on top of a smaller one



Solving the Tower of Hanoi

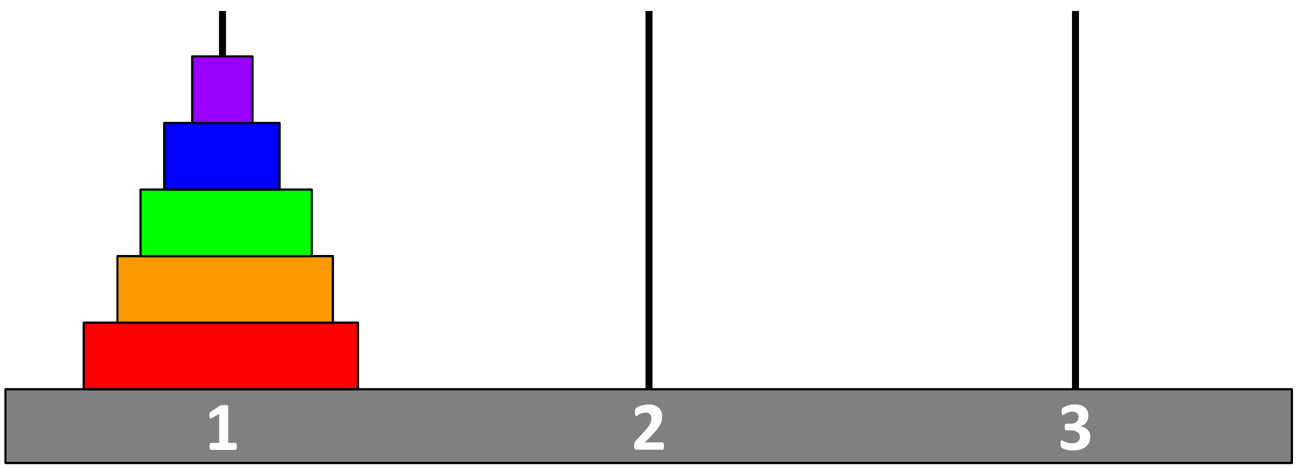
- The animation was probably daunting, but the recursive solution is surprisingly clean
  - Can still be mind-blowingly confusing to understand
  - For illustrative purposes – you're not responsible for knowing this

- Goal: Move the tower of height 5 from peg 1 to peg 3
  - Let's assume our solution looks something of the form:
    `moveTower(int height, int startPeg, int endPeg)`



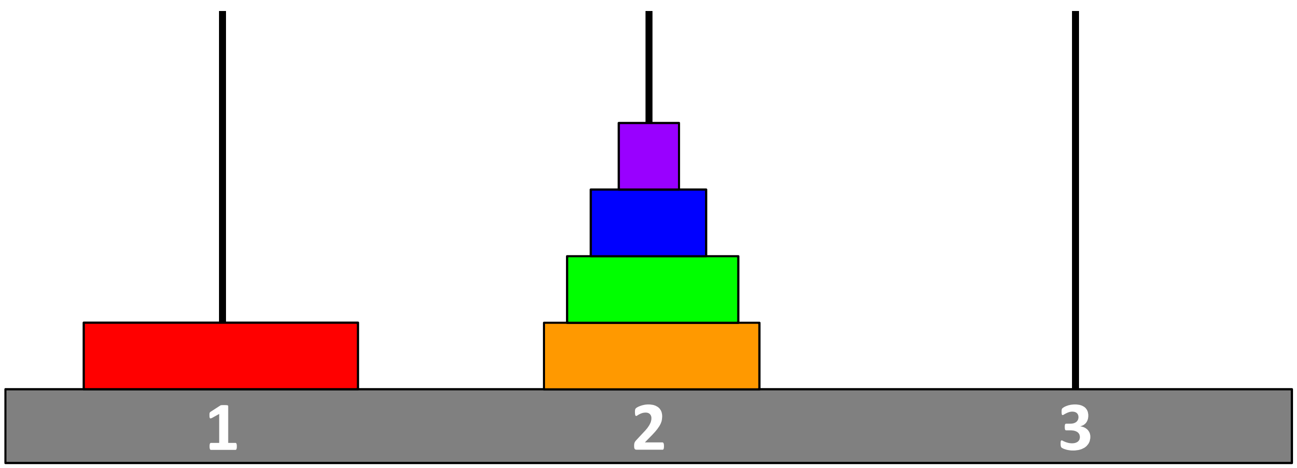
Solving the Tower of Hanoi

- To reconstruct the tower on peg 3, we first need to get the largest disk (red) onto peg 3
  - Can't do this while the other disks are on top
  - Solution: First move the 4 disks on top to peg 2



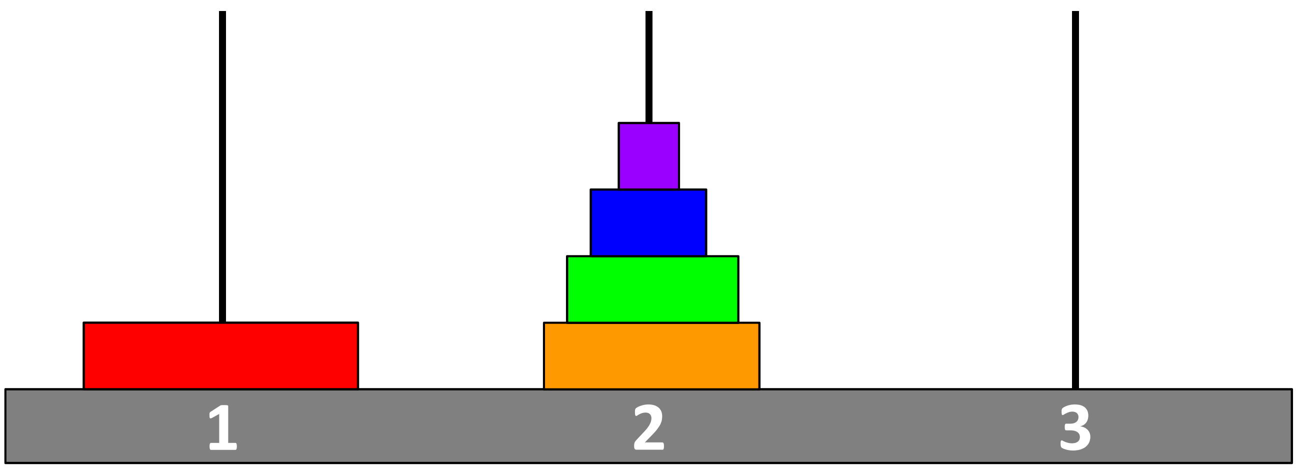
Solving the Tower of Hanoi

- To reconstruct the tower on peg 3, we first need to get the largest disk (red) onto peg 3
  - Can’t do this while the other disks are on top
  - Solution: First move the 4 disks on top to peg 2
    - `moveTower(4,peg1,peg2);` ← just assume it works!



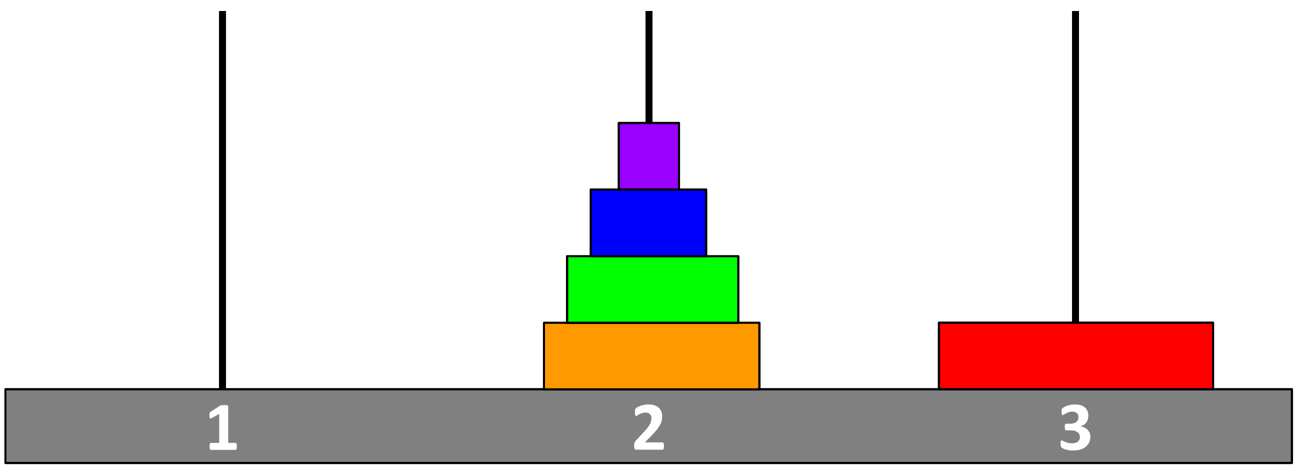
Solving the Tower of Hanoi

- Now we can move the red disk to peg 3



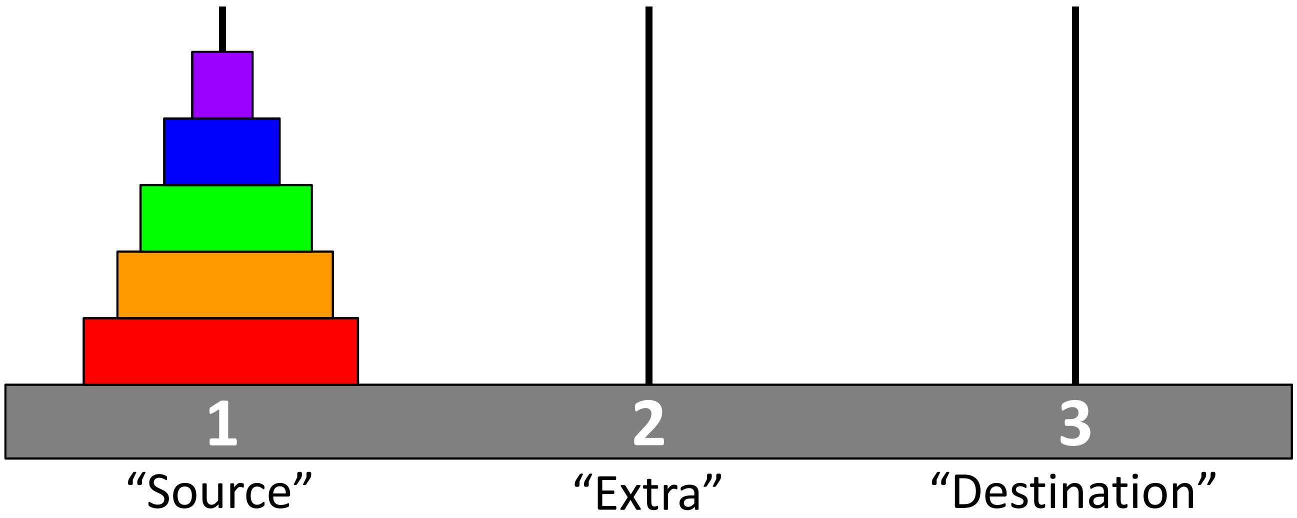
Solving the Tower of Hanoi

- Now we can move the red disk to peg 3
  - `moveTower(1,peg1,peg3);`
- Next Goal: Move the tower of height 4 from peg 2 to peg 3
  - Solution: First move the 3 disks on top to peg 1, then move the orange disk to peg 3



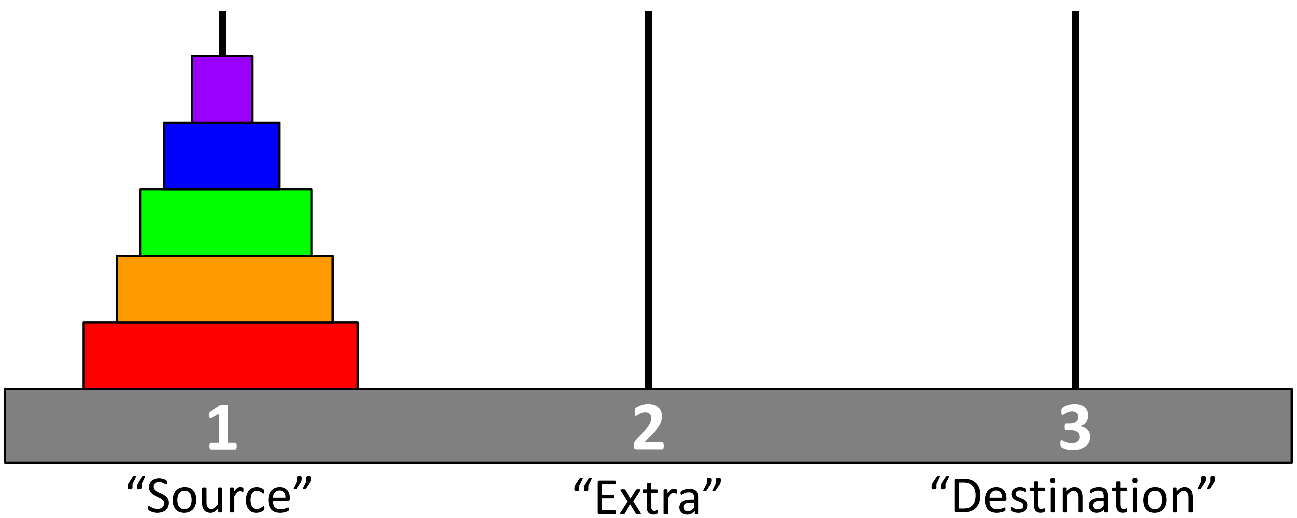
Solving the Tower of Hanoi

- Generalized recursive solution to move tower of height H from *source* peg to *destination* peg:
  - Move tower of height H-1 from *source* peg to *extra* peg
  - Move the remaining bottom disk from *source* peg to *destination* peg
  - Move tower of height H-1 from *extra* peg to *destination* peg



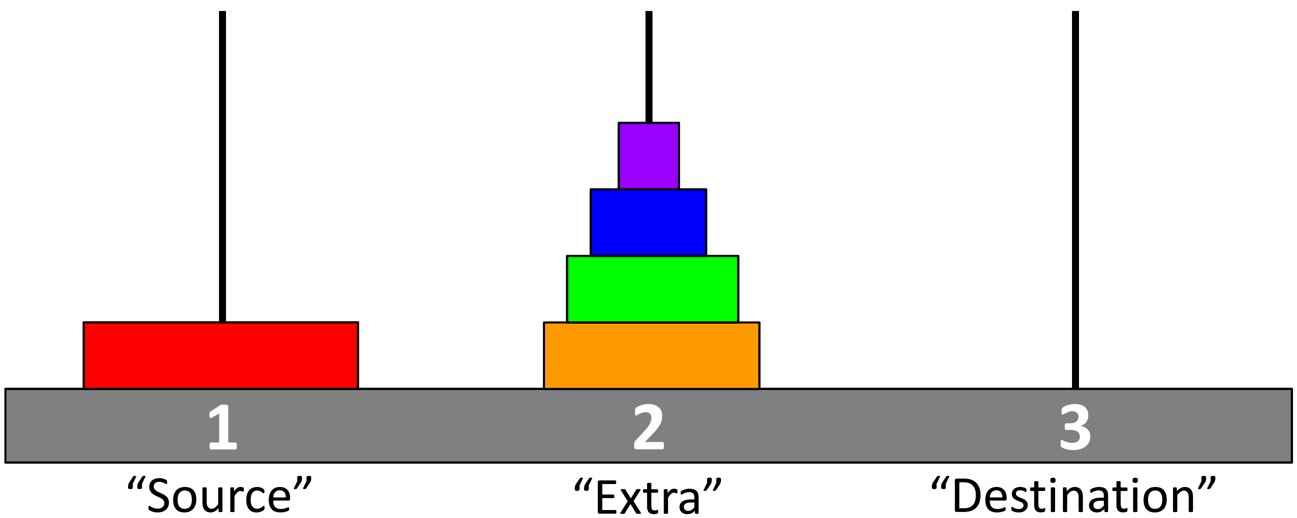
Solving the Tower of Hanoi

- Generalized recursive solution to move tower of height H from *source* peg to *destination* peg:
  - `moveTower(H-1,peg1,peg2);`
  - `moveTower(1,peg1,peg3);`
  - `moveTower(H-1,peg2,peg3);`



Solving the Tower of Hanoi

- What's the base case?
  - Don't recurse (but still move disk) when $\mathrm{H} == 1$



Outline

- Example: Tower of Hanoi
- **Variable Scope Revisited**
- Example: Fibonacci
- Example: Snowflake Fractal

Variable Scope Revisited

- Internal variables (*i.e.* parameters) only exist within the function they are declared
  - The variables "cease to exist" when the function returns
- Each individual call of a recursive function contains a *separate* set of parameters, even though they have the same variable names
  - Parameters have initial values set by the passed arguments

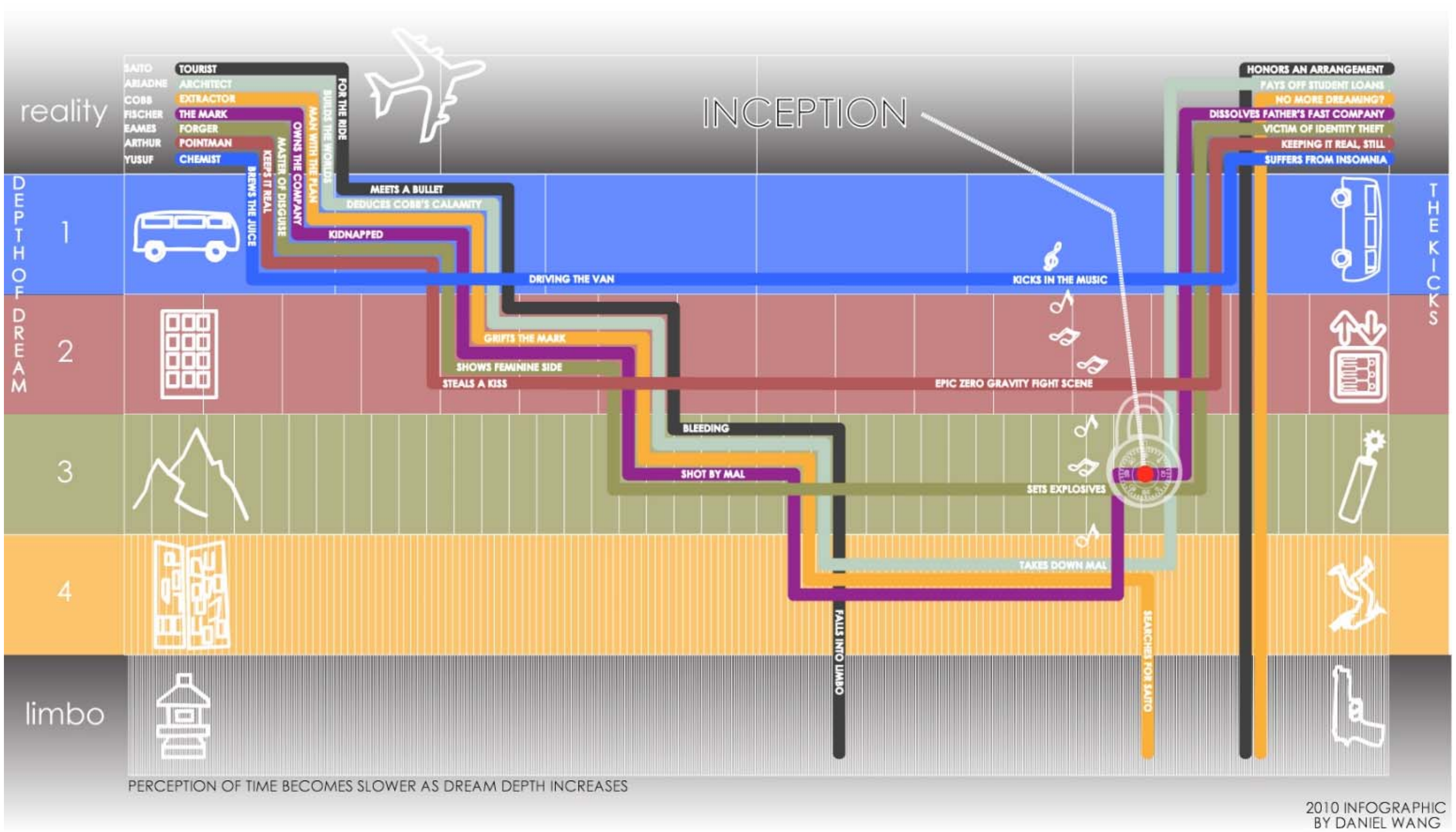
Variable Scope Revisited

- Local variables take precedent over variables of the same name
  - Detail Removal: internal variable names are independent of external variable names, even if the same names are used

- We can think of every function call as creating a new function *environment*, which later disappears once the function returns
  - Global variables exist outside of these environments and are accessible to all of them

'Inception' Analogy (2010 film)

- Each dream is a function call, each "kick" is a function return
  - *e.g.* the 'reality' function calls the 'Robert Fischer dream' function
  - Characters are the parameters – they may have the same names, but are different (clothes?) in every layer



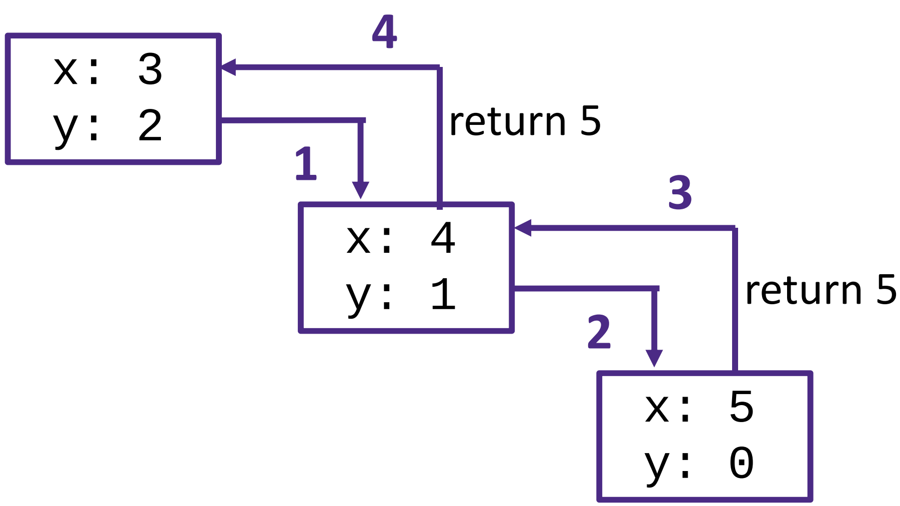
http://images.fastcompany.com/upload/INCEPTION%20infographic%20v3.5.2_dwang.jpg

Add Example

- Recursive `add()`:

```
int add(int x, int y) {
  if(y==0) {
    return x;
  } else {
    return add(x+1,y-1);
  }
}
```

- Environment diagram if we call `add(3,2)`:



Peer Instruction Question

- In the shown code, what will be printed after `"3: "`?
  - Vote at http://PollEv.com/justinh

A. 0
B. 1
C. 4
D. 5

```
int x = 0;

void setup() {
  x = 1;
}
void draw() {
  println("1: " + x);
  foo(4);
  println("3: " + x);
  noLoop();
}
void foo(int x) {
  x = x + 1;
  println("2: " + x);
}
```

Outline

- Example: Tower of Hanoi
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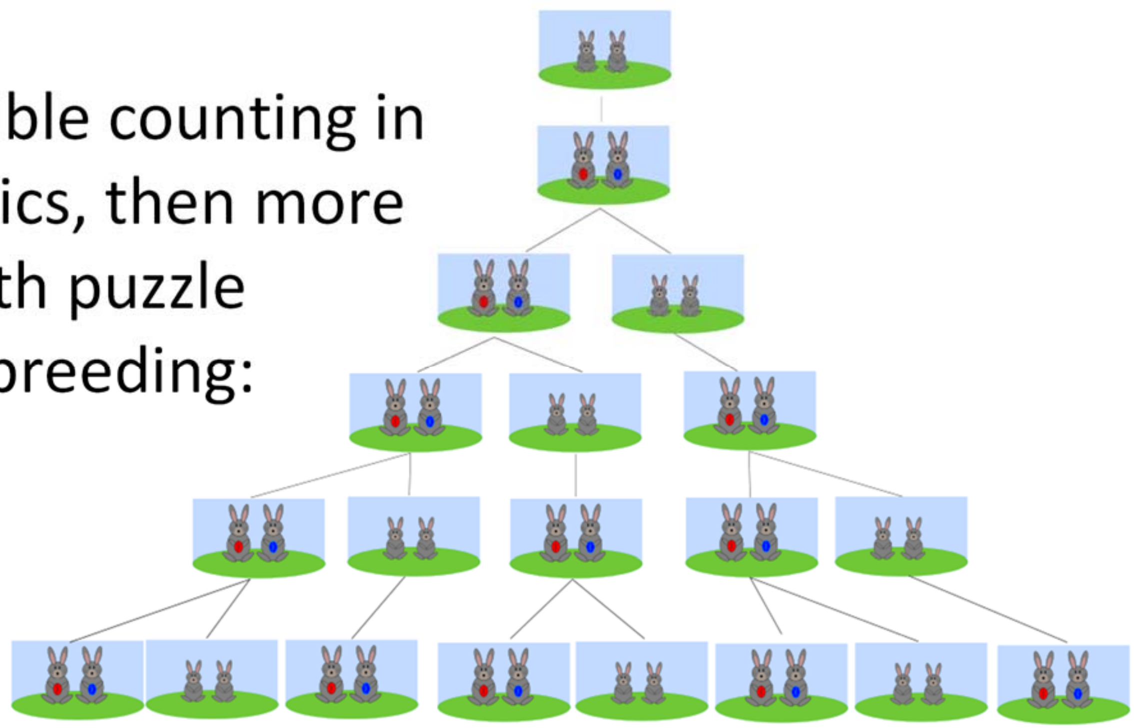
- **Example: Fibonacci**
- Example: Snowflake Fractal

Fibonacci

- The Fibonacci Sequence is as follows:
  - 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, …
    - The first two numbers are 0 and 1
    - All following numbers are the sum of the previous two numbers
  - https://en.wikipedia.org/wiki/Fibonacci_number
  - Appeared as syllable counting in Indian mathematics, then more famously in a math puzzle regarding rabbit breeding:

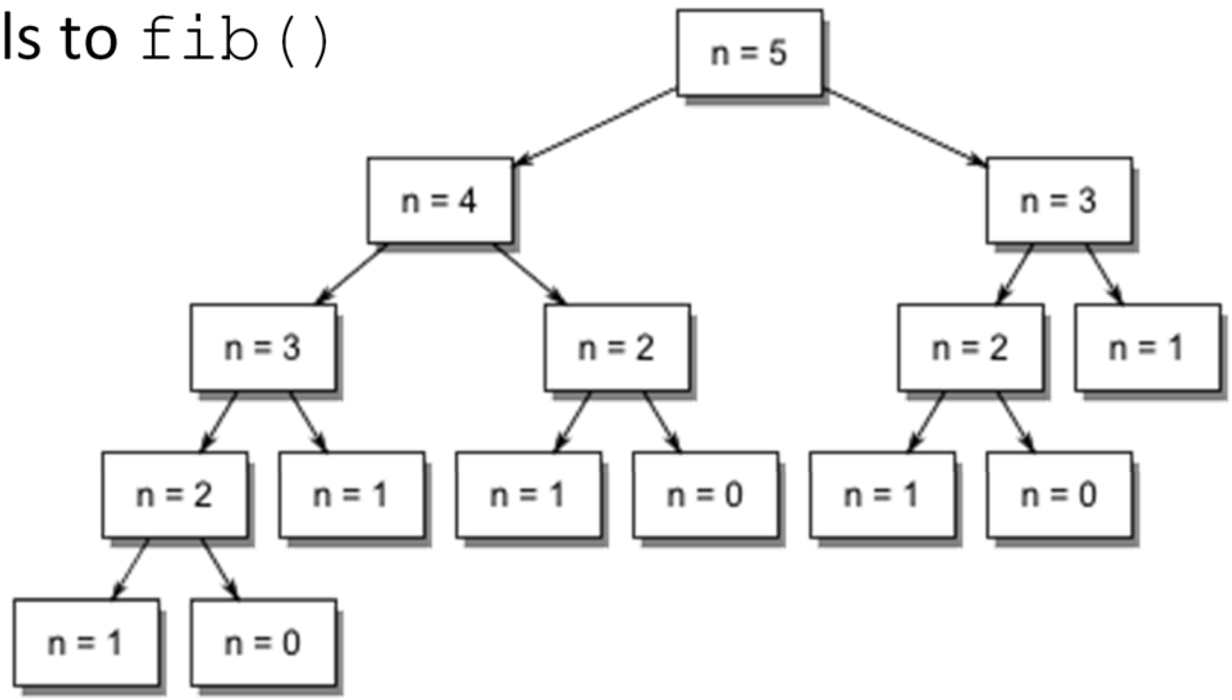


Fibonacci

- The Fibonacci Sequence is as follows:
  - 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, …
    - `fib(0) = 0`, `fib(1) = 1`
    - Otherwise, `fib(n) = fib(n-1) + fib(n-2)`

Fibonacci Call Structure

- Call structure of `add()` looked like a call list
  - It contained one recursive call: `add(x+1,y-1)`
- Fibonacci makes how many recursive calls?
  - `fib()` looks like a call *tree* – each recursive case makes two calls to `fib()`



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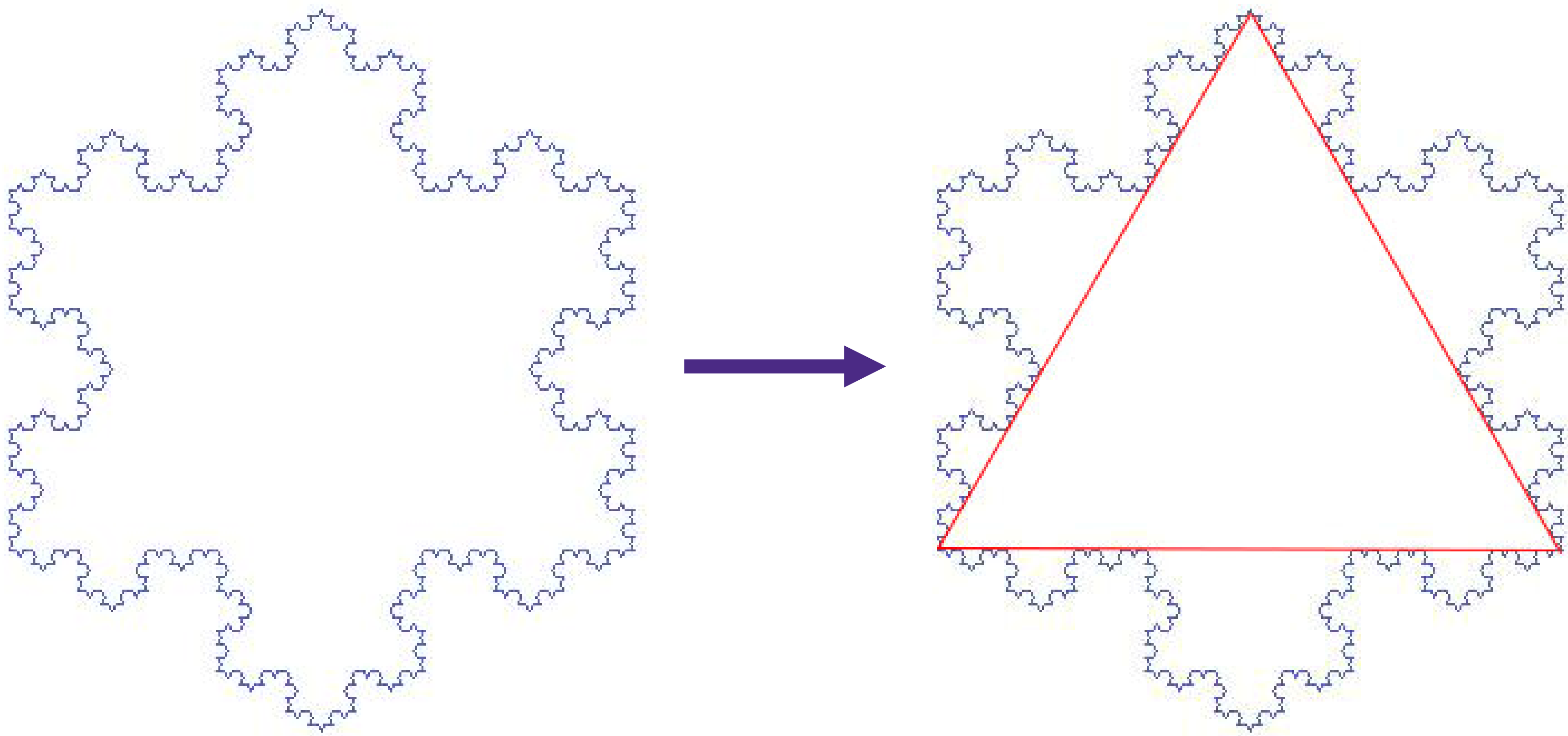
- **Example: Snowflake Fractal**

The following exercise is from the Beauty and Joy of Computing (BJC) curriculum:
http://bjc.berkeley.edu/bjc-r/cur/programming/recur/fractals/snowflake.html

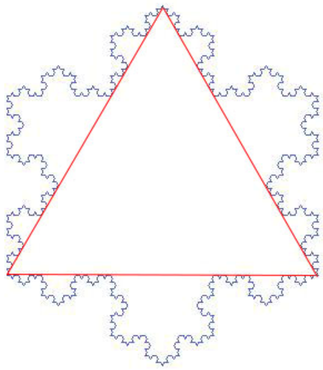
Koch Snowflake

- A mathematical curve that is one of the earliest fractal curves to have been described
  - https://en.wikipedia.org/wiki/Koch_snowflake
  - 3 arranged copies of the same *fractal*



Code: Triangle

- Copies of fractal arranged in a triangle:

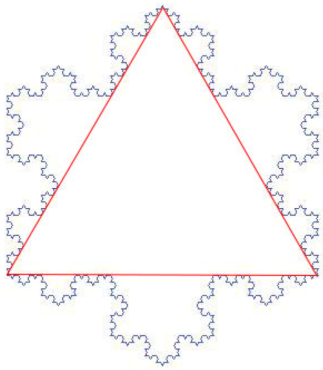


```
void draw() {




    noLoop();
}
```

Code: Triangle



- Copies of fractal arranged in a triangle:

```
void draw() {
  translate(250,100);  // start at top point
  rotate(radians(60));
  for(int i=0; i<3; i=i+1) {
    line(0,0,len,0);    // replace with fractal
    translate(len,0);
    rotate(radians(120));
  }
  noLoop();
}
```

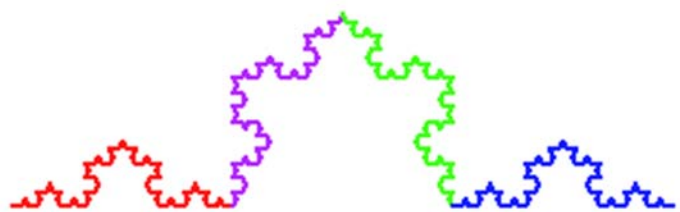
Drawing the Fractal

- Break each segment into 4 segments of equal length
  - First call:

  - Second call:

  - Third call:

  - Fourth call:



Code: Fractal

- First call:

```
void snowflake_fractal(float len) {
  line(0,0,len/3,0);
  translate(len/3,0);
  rotate(radians(-60));
  line(0,0,len/3,0);
  translate(len/3,0);
  rotate(radians(120));
  line(0,0,len/3,0);
  translate(len/3,0);
  rotate(radians(-60));
  line(0,0,len/3,0);
  translate(len/3,0);
}
```

Code: Make It Recursive

- Recursive case
  - Instead of drawing a line, draw the fractal!
    - Each smaller segment is 1/3 the length of the larger segment
    - Replace `line()` and `translate()` command pairs with calls to `snowflake_fractal()`
- Base case
  - Introduce `level` variable
    - Arbitrarily tells us how deep to recurse
  - When `level==0`, just draw line instead of fractal

The Result

- Can draw snowflake fractal of arbitrary depth!