Reference information about many standard Java classes appears at the end of the test. You might want to tear off those pages to make them easier to refer to while solving the programming problems.

Question 1. (6 points) (a) Prove that $3n^2 + 25n + 17$ is $O(n^2)$.

To prove this we need to find some constant c such that $3n^2 + 25n + 17 \leq cn^2$ for all sufficiently large values of n. If we pick c to be 4, for example, that will be sufficient.

(b) Prove that $3n^2 + 25n + 17$ is $O(n^3)$.

The proof is almost the same as in part (a): we need to find some constant c such that $3n^2 + 25n + 17 \leq cn^3$ for all sufficiently large values of n. If we pick c to be any positive number, that will be sufficient.

(c) Which of the two proofs in parts (a) and (b) of this question provides the most useful information and why?

The proof from part (a) that $3n^2 + 25n + 17$ is $O(n^2)$ is more useful since it provides a tighter (smaller) bound on the size.

Question 2. (2 points) When defining a new class in Java, the standard advice is that if we override the inherited `equals(Object)` method we should also override the inherited `hashCode()` method. What relationship should be true between the methods `equals` and `hashCode`?

The basic rule is if `o1.equals(o2)` then `o1.hashCode()` should equal `o2.hashCode()`.

Question 3. (4 points) A good implementation of a hash table (`HashMap` or `HashSet`, for example) provides *O*(1) (constant time) insert and contains operations. Give *two* ***distinct*** reasons why these operations might be significantly slower, possibly proportional to the number of items in the hash table.

(i) If the hash function does not generate widely distributed hash codes and instead assigns the same hash codes to many of the objects, then there will be too many collisions, resulting in many objects being assigned to the same slot (bucket) in the table.

(ii) If the number of slots (buckets) in the table is small compared to the number of objects stored in the table, then operations will take time proportional to the number of items in the table and will not be constant time.

Question 4. (4 points) Suppose we have two packages `Cowboy` and `Graphics`, both of which contain a class named `Draw`.

(a) (2 points) Will the following code compile? Why or why not?

```
import Cowboy.*;
import Graphics.*;

public class Test {
   Draw d;

   /** Constructor */
   public Test() {
      d = new Draw();
   }
}
```

The code will not compile because the reference to class `Draw` is ambiguous.

(b) (2 points) If the code in part (a) won’t compile, how could we fix it so it will? [If the code in part (a) does compile, leave this part of the question blank for 2 free points!]

Use the full name of the class in the declaration and `new` operation, either `Cowboy.Draw` or `Graphics.Draw`.

Question 5. (2 points) One of your colleagues is implementing a simple list class, which contains the following method specification.

```
/** Return the object at the given position
 * @param pos position of the desired object in the list
 * @return the selected object
 * @throws IndexOutOfBoundsException if pos is invalid
 */
public Object get(int pos)throws IndexOutOfBoundsException { … }
```

Is the “`throws IndexOutOfBoundsException`” clause in the method heading required, or can it be omitted? Why?

It can be omitted. `IndexOutOfBoundsException` is an unchecked exception, so it doesn’t need to be declared in the method heading.

The next few questions involve single-linked lists of integers. The nodes in the linked lists are instances of class `Link`, defined as it was in lecture and in section.

```
public class Link {      // one node in a linked list
   public int item;      // data associated with this link
   public Link next;     // next node in the list; null if none

   /** Construct a new node referring to the given object */
   public Link(int item, Link next) { … }
}
```

The state of a `SimpleLinkedList` is represented by these instance variables:

```
Link first;      // first node in the list; null if empty
Link last;       // last node in the list, null if empty
```

Question 6. (5 points) You've been hired to work on Java 6 (or 1.6, or whatever marketing will eventually call it) and have been asked to write a method to sum up the values in a list, but for whatever reason your boss says that you can't use a loop. However, it's fine if you use recursion.

To compute the sum of the list we call the following method:

```
/** return the sum of the ints in the list */
public int sum() {
   return sumFrom(first);
}
```

Complete the definition of the following method so it returns the sum of the list starting at the given node. For full credit, you **may not** use iteration – use recursion instead.

```
/** return the sum of the ints in the list starting at node p */
public int sumFrom(Link p) {

   if (p == null) {

      return 0;

   } else {

      return p.item + sumFrom(p.next);

   }

}
```

Question 7 (9 points) Our simple array-based list implementation included an iterator class that provided the standard `hasNext()` and `next()` operations. For this question, complete the following methods to provide an iterator for the `SimpleLinkedList` class implemented as described on the previous page. You should assume that this iterator class definition is *nested inside* the `SimpleLinkedList` class definition, so it has direct access to any instance variables and methods in that class that it needs.

```
/** Iterator class for a SimpleLinkedList */
public class SimpleLinkedListIterator {

   // declare any instance variables you need here

   private Link nextItem;      // next item to be returned by
                               // next() or null if no more items
                               // in the iteration

   /** Construct a new SimpleLinkedListIterator */
   public SimpleLinkedListIterator() {

      nextItem = first;

   }

   /** Return true if there are more items in this iteration */
   public boolean hasNext() {

      return nextItem != null;

   }

   /** Return the next item in this iteration.
    * @throws NoSuchElementException if no more elements */
   public int next() {

      if (nextItem == null) {

         throw new NoSuchElementException();

      }

      int result = nextItem.item;

      nextItem = nextItem.next;

      return result;

   }
}
```

Question 8 (8 points) Another possible use of linked data structures is to implement stacks (the same behavior as an array-based implementation, but with a different underlying data structure). For reference, the definition of a link is repeated here.

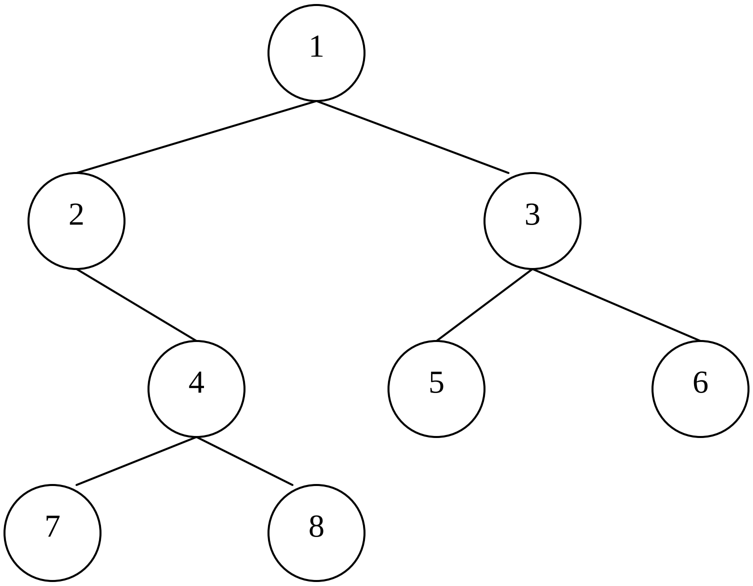
```
public class Link {         // one node in a linked list
   public int item;         // data associated with this link
   public Link next;        // next node in the list; null if none

   /** Construct a new node referring to the given object */
   public Link(Object item, Link next) { … }
}
```

Complete the definitions of the constructor and methods `push` and `pop` below to implement a stack. The constructor and one instance variable are provided for you. You can add additional instance variables and modify the constructor if you need to.

```
public class SimpleStack {
   // instance variables
   private Link top;           // top of stack or null if the
                               //   stack is empty


   /** Construct a new empty stack */
   public SimpleStack() {

      top = null;

   }

   /** Push item onto the top of the stack */
   public void push(int item) {

      top = new Link(item, top);

   }

   /** Return the top item on the stack and delete it.
    * @throws NoSuchElementException if the stack is empty. */
   public int pop() throws NoSuchElementException {

      if (top == null) {
         throw new NoSuchElementException();
      }
      int result = top.item;
      top = top.next;
      return result;

   }
}
```

Question 9 (8 points) The tree traversals that we have looked at are known as *depth-first* traversals. The essential idea is that we completely visit a subtree of a node, including all the nodes in the subtree, before we visit the other subtrees of that node.

Another possible strategy is a *breadth-first* traversal. In this strategy, we first visit the root, then we visit all of the root's immediate children, then all the children at the next level of the tree, and so forth. For example, the nodes in the following tree are numbered in the order they would be reached in a breadth-first traversal.



Assume that we have a binary tree containing String data values whose nodes are represented as follows.

```
public class BTNode {       // one node in a binary tree
   public String value;    // value stored in this node
   public BTNode left;     // left subtree or null if empty
   public BTNode right;    // right subtree or null if empty
}
```

Complete the definition of method `btraversal` on the next page so it does a breadth-first traversal of the tree with the given root and prints out all the strings contained in the tree in breadth-first order. If you need additional helper methods, feel free to declare them. You can declare any instance variables that you need outside the `btraversal` method.

Hint: One useful strategy for doing this is to keep a queue of nodes that you have visited, but whose children have not been visited. Initially put the root in this queue, then visit the nodes in the queue one by one. Each time you visit a node, print out its contents and add its children to the end of the queue.

Second hint: Recursion may or may not be your friend. Take a couple of minutes to think about your solution strategy before you start coding.

Question 9 (cont).

```
// Declare any instance variables you need here




/** Perform a breadth-first traversal of tree t and
 *  print all of the strings in the nodes in the order
 *  they are reached */
public void btraversal(BTNode t) {

   LinkedList toVisit = new LinkedList();  // nodes to visit
   if (t == null) {
      return;            // nothing to do
   }
   // t not null – add it to the list to visit
   toVisit.add(t);
   // visit nodes on the list while there are any
   while (toVisit.size()) > 0) {
      BTNode p = (BTNode)toVisit.removeFirst();
      System.out.println(p.value);
      if (p.left != null) {
         toVisit.add(p.left);
      }
      if (p.right != null) {
         toVisit.add(p.right);
      }
   }
}
```

Question 10. (10 points) This question involves processing a text stream to accumulate some statistics. The input data consists of one or more lines that each contain a last name, a first name, and the letter M or F to indicate the person's sex. For example:

```
Smith  Sue   F
Jones  Ralph   M
Moose Bullwinkle  M
Bird  Tweety  F
```

Complete the method `percentFemale`, below, so it returns the percentage of the number of lines in the stream where the sex is `F`. You can assume that the sex is always capitalized so you only need to check for `M` or `F`. You can also assume that there are no extra leading or trailing blanks or other characters at the beginning or end of the input lines, and you can assume that there is at least one line in the input stream.

Hint: The last couple of pages of the exam contain some summary information about Java stream and string classes that you might find useful.

```
/** Return the percentage of lines in the input stream where
 *  the third entry on the line is the string “F”.
 * @param in the input stream to read from */
public double percentFemale(BufferedReader in) {

   int nLines = 0;       // # of lines in the stream

   int nF = 0;           // # of lines ending in ‘F’

   try {

      String line = in.readLine();

      while (line != null) {

         nLines++;

         if (line.substring(line.length()-1).equals(“F”)) {

            nF++;

         }

         line = in.readLine();

      }

   } catch (IOException e) {}

   return (double)nF/nLines;

}
```

There is still a potential bug in this code: if an exception is thrown when reading the first line, then the division at the end would divide 0/0. This and similar issues were ignored during grading, since the problem was worded to imply that it would be possible to read at least one line from the stream.

Question 11. (8 points) We looked at several implementations of collections – arrays, linked lists, trees, binary search trees, and hash tables. Different operations on these data structures had different expected and worst-case times. Complete the tables below with the expected and worst case times (using O()-notation) for the different operations on the given data structures. You should assume that suitable instance variables are used to make operations reasonably fast instead of having to, for example always traverse the complete data structure to count the number of items in it.

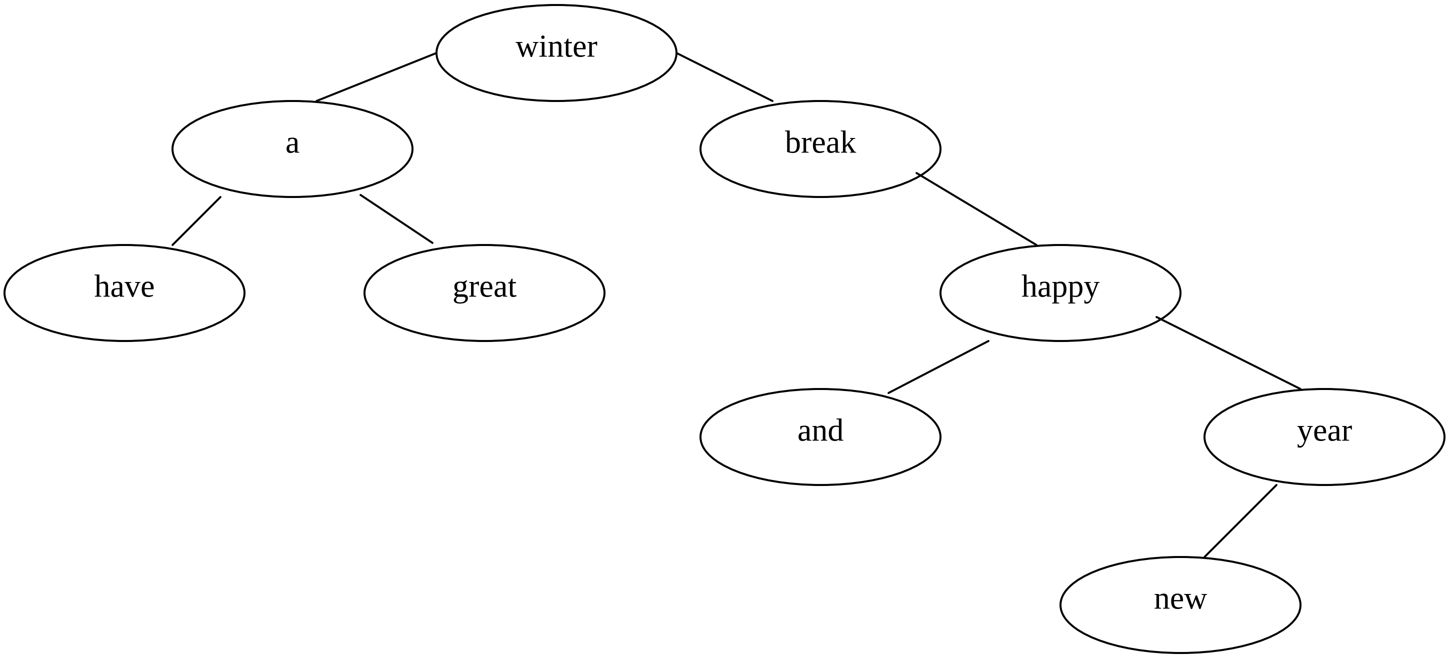
(a) **Expected times**

Operation	Sorted array-based list	Unsorted Linked list	Binary search tree	Hash set
Add item to the collection	**O(n)**	**O(1)**	**O(log n)**	**O(1)**
Search for item	**O(log n)**	**O(n)**	**O(log n)**	**O(1)**
Return size of the collection	**O(1)**	**O(1)**	**O(1)**	**O(1)**

(b) **Worst-case times**

Operation	Sorted array-based list	Unsorted Linked list	Binary search tree	Hash set
Add item to the collection	**O(n)**	**O(1)**	**O(n)**	**O(n)**
Search for item	**O(log n)**	**O(n)**	**O(n)**	**O(n)**
Return size of the collection	**O(1)**	**O(1)**	**O(1)**	**O(1)**

Question 12. (2 points) Write down the words in the following tree in the order they are reached in an ***inorder*** traversal.



have a great winter break and happy new year (!)