

---

# CSE 143 Java

## Trees

12/5/2004

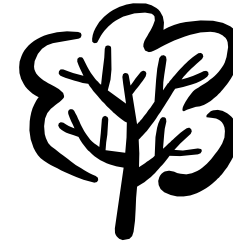
(c) 2001-4, University of Washington

20-1

---

## Overview

- Topics
  - Trees: Definitions and terminology
  - Binary trees
  - Tree traversals



12/5/2004

(c) 2001-4, University of Washington

20-2

---

## Trees

- Most of the structures we've looked at so far are linear
  - Arrays
  - Linked lists
- There are many examples of structures that are not linear, e.g. hierarchical structures
  - Organization charts
  - Book contents (chapters, sections, paragraphs)
  - Class inheritance diagrams
- *Trees* can be used to represent hierarchical structures

12/5/2004

(c) 2001-4, University of Washington

20-3

---

## Looking Ahead To An Old Goal

- Finding algorithms and data structures for fast searching
  - A key goal
  - Sorted arrays are faster than unsorted arrays, for searching
    - Can use binary search algorithm
    - Not so easy to keep the array in order
  - **LinkedLists were faster than arrays (or ArrayLists), for insertion and removal operations**
    - The extra flexibility of the "next" pointers avoided the cost of sliding
    - But... LinkedLists are hard to search, even if sorted
- Is there an analogue of LinkedLists for sorted collections??
- The answer will be...Yes: a particular type of *tree*!

12/5/2004

(c) 2001-4, University of Washington

20-4

## Tree Definitions

- A *tree* is a collection of *nodes* connected by *edges*
- A *node* contains
  - Data (e.g. an Object)
  - References (edges) to two or more *subtrees* or *children*
- Trees are hierarchical
  - A node is said to be the *parent* of its *children* (subtrees)
  - There is a single unique *root* node that has no parent
  - Nodes with no children are called *leaf nodes*
  - A tree with no nodes is said to be *empty*

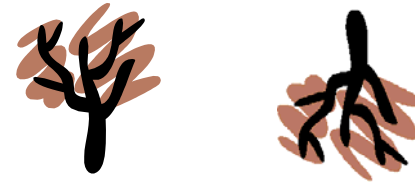
12/5/2004

(c) 2001-4, University of Washington

20-5

## Drawing Trees

- For whatever reason, computer sciences trees are normally drawn upside down: root at the top

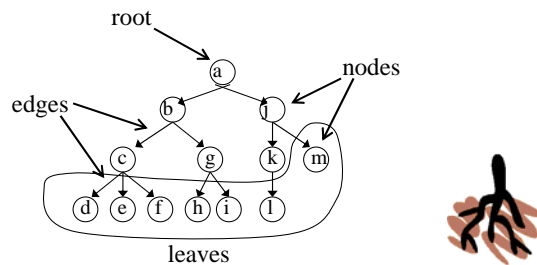


12/5/2004

(c) 2001-4, University of Washington

20-6

## Tree Terminology



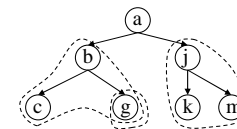
12/5/2004

(c) 2001-4, University of Washington

20-7

## Subtrees

- A *subtree* in a tree is any node in the tree together with all of its descendants (its children, and their children, recursively)



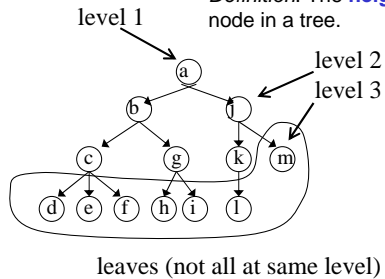
12/5/2004

(c) 2001-4, University of Washington

20-8

## Level and Height

*Definition:* The root has **level 1**  
Children have level 1 greater than their parent  
*Definition:* The **height** is the highest level of any node in a tree.



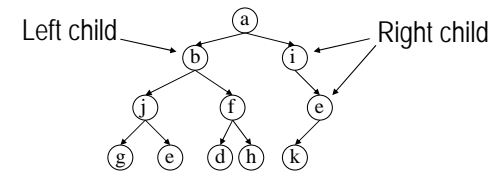
12/5/2004

(c) 2001-4, University of Washington

20-9

## Binary Trees

- A **binary tree** is a tree each of whose nodes has no more than two children
  - The two children are called the **left child** and **right child**
  - The subtrees belonging to those children are called the **left subtree** and the **right subtree**



12/5/2004

(c) 2001-4, University of Washington

20-10

## Binary Tree Nodes

- A node for a binary tree holds the item and references to its subtrees

```
class BTreeNode {
    public Object item;           // data item in this node
    public BTreeNode left;       // left subtree, or null if none
    public BTreeNode right;      // right subtree, or null if none
    public BTreeNode(Object item, BTreeNode left, BTreeNode right) { ... }
}
```

- This class can be public (if other code needs access), or it can be a private nested class inside the class that uses it

12/5/2004

(c) 2001-4, University of Washington

20-11

## Binary Tree Implementation

- The whole tree can be represented just by a pointer to the root node, or null if the tree is empty

```
public class BinTree {
    private BTreeNode root;           // root of tree, or null if empty
    public BinTree() { root = null; }
    ...
}
```

12/5/2004

(c) 2001-4, University of Washington

20-12

## Tree Algorithms

- The definition of a tree is naturally recursive:
  - A tree is either null (empty),  
or data + left (sub-)tree + right (sub-)tree
  - Base case(s)?
  - Recursive case(s)?
- Given a recursively defined data structure, recursion is often a very natural technique for algorithms on that data structure
  - Don't fight it!

12/5/2004

(c) 2001-4, University of Washington

20-13

## A Typical Tree Algorithm: size()

```
public class BinTree {
    ...
    /** Return the number of items in this tree */
    public int size() {
        return subtreeSize(root);
    }
    // Return the number of nodes in the (sub-)tree with root n
    private int subtreeSize(BTNode n) {
        if (n == null) {
            return _____;
        } else {
            return _____;
        }
    }
}
```

12/5/2004

(c) 2001-4, University of Washington

20-14

## Tree Traversal

- Functions like subtreeSize systematically “visit” each node in a tree
  - This is called a *traversal*
  - We also used this word in connection with lists
- Traversal is a common pattern in many algorithms
  - The processing done during the “visit” varies with the algorithm
- What order should nodes be visited in?
  - Many are possible
  - Three have been singled out as particularly useful for binary trees: *preorder*, *postorder*, and *inorder*

12/5/2004

(c) 2001-4, University of Washington

20-15

## Traversals

- **Preorder** traversal:
    - “Visit” the (current) node first  
i.e., do what ever processing is to be done
    - Then, (recursively) do preorder traversal on its children, left to right
  - **Postorder** traversal:
    - First, (recursively) do postorder traversals of children, left to right
    - Visit the node itself last
  - **Inorder** traversal:
    - (Recursively) do inorder traversal of left child
    - Then visit the (current) node
    - Then (recursively) do inorder traversal of right child
- Footnote: pre- and postorder make sense for all trees; inorder only for binary trees

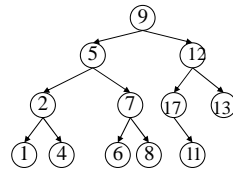
12/5/2004

(c) 2001-4, University of Washington

20-16

## Example of Tree Traversal

In what order are the nodes visited, if we start the process at the root?



Preorder:

Inorder:

Postorder:

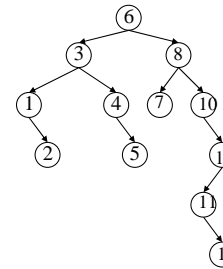
12/5/2004

(c) 2001-4, University of Washington

20-17

## More Practice

What about this tree?



Preorder:

Inorder:

Postorder:

12/5/2004

(c) 2001-4, University of Washington

20-18

## New Algorithm: *contains*

- Return whether or not a value is an item in the tree

```

public class BinTree {
    ...
    /** Return whether item is in tree */
    public boolean contains(Object item) {
        return subtreeContains(root, item);
    }
    // Return whether item is in (sub-)tree with root r
    private boolean subtreeContains(BTNode r, Object item) {
        if (r == null) {
            return _____;
        } else if (r.item.equals(item)) {
            return _____;
        } else {
            return _____;
        }
    }
}
    
```

12/5/2004

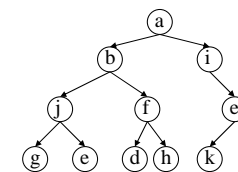
(c) 2001-4, University of Washington

20-19

## Test

contains(d)

contains(c)



12/5/2004

(c) 2001-4, University of Washington

20-20

## Cost of *contains*

---

- Work done at each node:
- Number of nodes visited:
- Total cost:
- Can we do better?