CSE 143

Searching and Recursion

Reading: Ch. 14 & Secs. 19.1-19.2

Overview

- Topics
  - Sequential and binary search
  - Recursion

Problem: A Word Dictionary

- Suppose we want to maintain a list of words
  - "aardvark"
  - "apple"
  - "tomato"
  - "orange"
  - "banana"
  - etc.
- Use the same basic representation as in SimpleArrayList

```
String[ ] words;    // the list of words is stored in words[0..size-1]
int size;           // number of words currently in the list
```

- We would like to be able to determine efficiently if a particular word is in the list

Sequential (Linear) Search

- If we don't know anything about the order of the words in the list, we basically have to use a *linear search* to look for a word

```
// return location of word in words, or –1 if found
int find(String word) {
    int k = 0;
    while (k < size && !word.equals(words[k]) {
       k++
    }
    if (k < size) { return k; } else { return –1; }     // lousy indenting to fit on slide
}                                                       // don't do this at home
```

- Search time for list of size n:
  - Can we do better?

Can we do better?

- Yes if the list is in alphabetical order

```
0 aardvark      // instance variable of the Ordered List class
1 apple         String[ ] words;   // list is stored in words[0..size-1]
2 banana                           //   and words are in ascending
3 cherry        int size;          //   order
4 kumquat
5 orange
6 pear
7 rutabaga
```


Binary Search

- Key idea: to search a section of the array,
  - Examine middle element
  - Search either left or right half depending on whether desired word precedes or follows middle word alphabetically
- A *precondition* for binary search is that the list is sorted
  - The algorithm is not guaranteed (or required) to give the correct answer if the precondition is violated

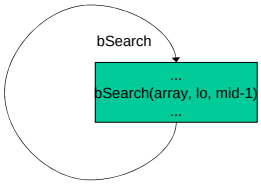
Binary Search Sketch (not quite legal Java)

```
/** Return the location of word in words[lo..hi], or -1 if not found */
int bSearch(String word, int lo, int hi) {
    if (lo > hi) { return -1; }                    // empty interval
    int mid = (lo + hi) / 2;
    if (word "==" words[mid]) { return mid; }      // found it!
    else if (word "<=" words[mid]) {
       // look for word in the left half
       return _____________________________;
    } else {  // word ">=" words[mid]
       // look for word in the right half
       return _____________________________;
    }
```

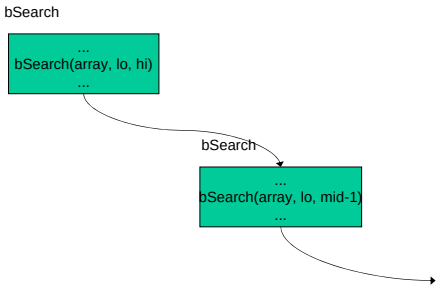

Recursion

- A method (function) that calls itself is *recursive*
- Nothing really new here
- Method call review:
  - Evaluate argument expressions
  - Allocate space for parameters and local variables of function being called
  - Initialize parameters with argument values
  - Then execute the function body
- What if the function being called is the same one that is doing the calling?
  - Answer: no difference at all!

Wrong Way to Think About It



Right Way to Think About It



Recursive Definitions

- We see these all the time in mathematics
- Simple example: factorial function

$$n! = \begin{cases} 1, & \text{if } n \le 1 \\ n \times (n-1)! & \text{otherwise} \end{cases}$$

Trace – Recursion as Substitution

- Execution of: result = fact(4);

Recursive Implementation in Java

- We can use the definition directly to create a java method to compute factorial

```
/** Return n! */
int fact(int n) {
    if (n <= 1) {
        return _____ ;
    } else {
        return _____ ;
```


Trace – Recursion as Method Calls

- Execution of: result = fact(4);

Recursive Cases, Base Cases, and Termination

- A recursive definition needs to have two parts
  - One or more *base cases* that are not recursive
    if (n <= 1) { return 1; }
  - One or more *recursive cases* that handle a "smaller" instance of the problem
    else { return n * fact(n-1); }
- The recursive cases must "make progress" towards a base case
  - If not, or if no base case(s) – infinite recursion

Back to Binary Search – Real Java This Time

```
/** Return word loc. in the list or –1 if not found */
int find(String word) { return bSearch(0, size-1); }
// Return location of word in words[lo..hi] or –1 if
// not found
int bSearch(String word, int lo, int hi) {
    // return –1 if interval lo..hi is empty
    if (lo > hi)  { return  –1; }
    // search words[lo..hi]
    int mid = (lo + hi) / 2;
    int comp = word.compareTo(words[mid]);
    if (comp == 0)  { return mid; }
    else if (comp < 0)  {
        return bSearch(word, lo, mid-1) ;
    } else /* comp > 0 */ {
        return bSearch(word, mid+1, hi) ;
    }
}
```

- Which are the
  - Base case(s)?
  - Recursive case(s)?
- How do the recursive case(s) make progress towards the base case(s)?

Trace

- Trace execution of find("orange")

0 aardvark
1 apple
2 banana
3 cherry
4 kumquat
5 orange
6 pear
7 rutabaga

Trace

- Trace execution of find("kiwi")

0 aardvark
1 apple
2 banana
3 cherry
4 kumquat
5 orange
6 pear
7 rutabaga

Analysis of Binary Search

- Time (number of steps) per each recursive call:
- Number of recursive calls:
- Total time:

How Many Calls Needed for a List of Size n?

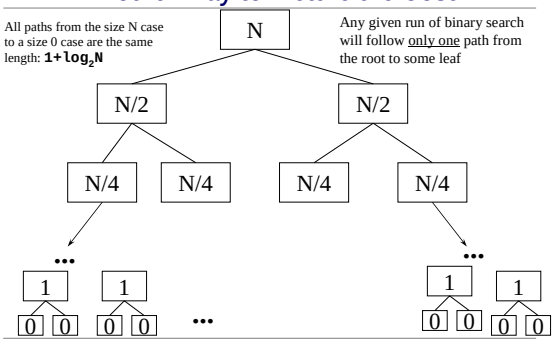
of recursive calls needed (*t*) List size (*n*)

Graph: Linear vs Binary Search

Another Way to Picture the Cost

All paths from the size N case to a size 0 case are the same length: $1+\log_2 N$

Any given run of binary search will follow only one path from the root to some leaf



Linear Search vs. Binary Search

- What is incremental cost if size of list is doubled?
  - Linear search:
  - Binary search:
- Why is Binary search faster?
  - The data structure is the same
  - The precondition on the data structure is different: stronger
  - Recursion itself is *not* an explanation

    One could code linear search using recursion, or binary search with a loop

Recursion vs. Iteration

- Recursion can completely replace iteration
- Some rewriting of the algorithm is necessary
  - usually minor
- Some languages have recursion only
- Recursion is often more elegant but has some extra overhead (often not a major issue, but can be)
- Recursion is a natural for certain algorithms and data structures
  - Useful in "divide and conquer" situations

- Iteration can completely replace recursion
- Some rewriting of the algorithm is necessary
  - often major
- A few (mostly older languages) have iteration only
- Iteration is not always elegant but is usually efficient
- Iteration is natural for linear (non-branching) algorithms and data structures

Recursion Summary

- Recursive definition: a definition that is (partially) given in terms of itself
- Recursive method (function): a method that is (partially) implemented by calling itself
- Need base case(s) and recursive case(s)
  - Recursive cases must make progress towards reaching a base case – must solve "smaller" subproblems
- Often a very elegant way to formulate a problem
  - Let the method call mechanism handle the bookkeeping behind the scenes for you
- A powerful technique – add it to your toolbag