CSE 143

Sorting

Reading: Sec. 19.3

Sorting

- Binary search is a huge speedup over sequential search
  - But requires the list be sorted
- Slight Problem: How do we get a sorted list?
  - Maintain the list in sorted order as each word is added
  - Sort the entire list when needed
- Many, many algorithms for sorting have been invented and analyzed
  - Let's invent some!

Insert for a Sorted List

- One possibility: ensure the list is always sorted as it is created
- Exercise: Assume that words[0..size-1] is sorted. Place new word in correct location so modified list remains sorted
  - Assume that there is spare capacity for the new word
- Before coding:
  - Draw pictures of an example situation, before and after
  - Write down the postconditions for the operation

```
// given existing list words[0..size-1], insert word in correct place and increase size
void insertWord(String word) {



    size++;
}
```


Picture

- Draw your picture here

Insertion Sort

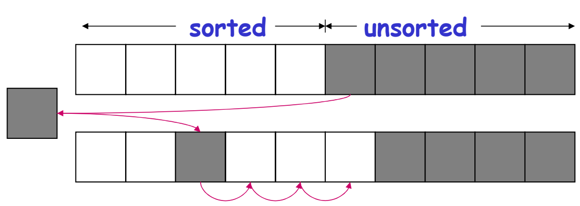
- Once we have insertWord working...
- We can sort a list in place by repeating the insertion operation

```
void insertionSort( ) {
    int finalSize = size;
    size = 1;
    for (int k = 1; k < finalSize; k++) {
        insertWord(words[k]);
    }
}
```

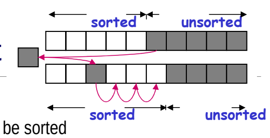

Insertion Sort As A Card Game Operation

- A bit like sorting a hand full of cards dealt one by one:
  - Pick up 1st card – it's sorted, the hand is sorted
  - Pick up 2nd card; *insert* it after or before 1st – both sorted
  - Pick up 3rd card; *insert* it after, between, or before 1st two
  - ...
- Each time:
  - Determine where new card goes
  - Make room for the newly inserted card and place it there

Insertion Sort As Invariant Progression



Insertion Sort




```
// instance variable
int[ ] list;            // list[0..size-1] is the list to be sorted
int size;
// Sort list[0..size-1]
public void  sort {
    for (int j=1 ; j < size; j++) {
        // pre: 1 <= j && j < size && list[0 ... j-1] is in sorted order
        int temp = list[ j ];
        for (int i = j -1 ; i >= 0 && list[i] > temp ; i-- ) {
            list[i+1] = list[i] ;
        }
        list[i+1] = temp ;
        // post: 1 <= j && j < size && list[0 ... j] in sorted order
    }
}
```

Insertion Sort Trace

- Initial array contents
  - 0 pear
  - 1 orange
  - 2 apple
  - 3 rutabaga
  - 4 aardvark
  - 5 cherry
  - 6 banana
  - 7 kumquat

Insertion Sort Performance

- Cost of each insertWord operation:
- Number of times insertWord is executed:
- Total cost:
- Can we do better?

Analysis

- Why was binary search so much more effective than sequential search?
  - Answer: binary search divided the search space in half each time; sequential search only reduced the search space by 1 item per iteration
- Why is insertion sort O(n^2)?
  - Each insert operation only gets 1 more item in place at cost O(n)
  - O(n) insert operations
- Can we do something similar for sorting?

Where are we on the chart?

N	$\log_2 N$	5N	N $\log_2 N$	N^2	2^N
8	3	40	24	64	256
16	4	80	64	256	65536
32	5	160	160	1024	$\sim 10^9$
64	6	320	384	4096	$\sim 10^{19}$
128	7	640	896	16384	$\sim 10^{38}$
256	8	1280	2048	65536	$\sim 10^{76}$
10000	13	50000	10^5	10^8	$\sim 10^{3010}$

Divide and Conquer Sorting

- Idea: something similar to binary search
  1. divide the sorting problem into two subproblems;
  2. recursively sort each subproblem;
  3. combine results
- Want division and combination at the end to be fast
- Want to be able to sort two halves independently
- This algorithm strategy is called *divide and conquer*

Quicksort

- Invented by C. A. R. Hoare (1962)
- Idea
  - Pick an element of the list: the *pivot*
  - Place all elements of the list smaller than the pivot in the half of the list to its left; place larger elements to the right
  - Recursively sort each of the halves
- Before looking at any code, see if you can draw pictures based just on the first two steps of the description

Code for QuickSort

```
// Sort words[0..size-1]
void quickSort( ) {
    qsort(0, size-1);
}

// Sort words[lo..hi]
void qsort(int lo, int hi) {
    // quit if empty partition
    if (lo > hi) { return; }
    int pivotLocation = partition(lo, hi);      // partition array and return pivot loc
    qsort(lo, pivotLocation-1);
    qsort(pivotLocation+1, hi);
}
```


Recursion Analysis

- Base case? Yes.

```
// quit if empty partition
if (lo > hi) { return; }
```

- Recursive cases? Yes

```
qsort(lo, pivotLocation-1);
qsort(pivotLocation+1, hi);
```

  - Each recursive cases work on a smaller subproblem, so algorithm will terminate

A Small Matter of Programming

- *Partition* algorithm
  - Pick pivot
  - Rearrange array so all smaller element are to the left, all larger to the right, with pivot in the middle
- *Partition* is not recursive
- Fact of life: *partition* can be a bit tricky to get right
  - Pictures and invariants are your friends here
- How do we pick the pivot?
  - For now, keep it simple – use the first item in the interval
  - Better strategies exist

Partition design

- We need to partition words[lo..hi]
- Pick words[lo] as the pivot
- Picture:

A `Partition` Implementation

- Use first element of array section as the pivot
- Invariant:

	lo		L	R	hi
words	x	<=x	unprocessed	>x	

pivot

Partition Algorithm: PseudoCode

```
// Partition words[lo..hi]; return location of pivot in range lo..hi
int partition(int lo, int hi) {

}
```

Partition Test

- Check: partition(0,7)
  - 0 orange
  - 1 pear
  - 2 apple
  - 3 rutabaga
  - 4 aardvark
  - 5 cherry
  - 6 banana
  - 7 kumquat

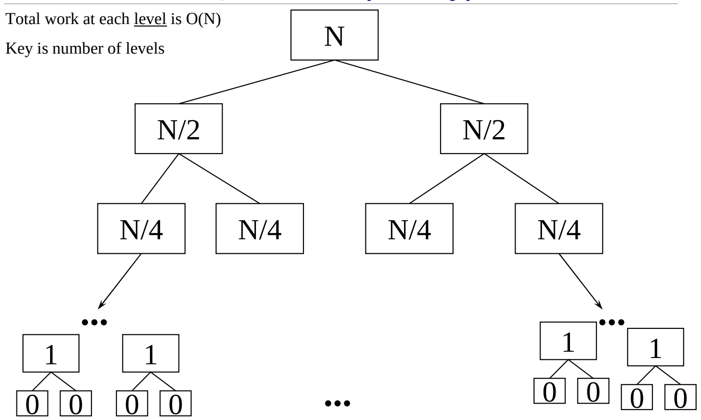
Complexity of QuickSort

- Each call to Quicksort (ignoring recursive calls):
  - Each call of partition( ) is O(n) where n is size of the *part* of array being sorted
    - Note: This n is smaller than the N of the original problem
  - Some O(1) work
  - Total = O(n) (n is the size of array part being sorted)
- Including recursive calls:
  - Two recursive calls at each level of recursion, each partitions “half” the array at a cost of O(n/2)
  - How many levels of recursion?

QuickSort (Ideally)

Total work at each level is O(N)

Key is number of levels

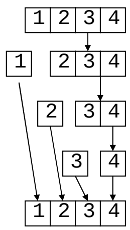


QuickSort Performance (Ideal Case)

- Each partition divides the list parts in half
  - Sublist sizes on recursive calls: n, n/2, n/4, n/8....
  - Total depth of recursion: ________________
  - Total work at each level: O(n)
  - Total cost of quicksort: ______________ !
- For a list of 10,000 items
  - Insertion sort: O(n^2): 100,000,000
  - Quicksort: O(n log n): 10,000 $\log_2$ 10,000 = 132,877

Worst Case for QuickSort

- If we're very unlucky, each pass through partition removes only a *single* element.



- In this case, we have N levels of recursion rather than $\log_2 N$. What's the total complexity?

QuickSort Performance (Worst Case)

- Each partition manages to pick the largest or smallest item in the list as a pivot
  - Sublist sizes on recursive calls:
  - Total depth of recursion: _______________
  - Total work at each level: O(n)
  - Total cost of quicksort: ______________ !

Worst Case vs Average Case

- QuickSort has been shown to work well in the average case (mathematically speaking)
- In practice, Quicksort works well, provided the pivot is picked with some care
- Some strategies for choosing the pivot:
  - Compare a small number of list items (3-5) and pick the *median* for the pivot

    (Typically check the first, middle, last, and a couple of items in between – works well even if the original array is almost sorted)
  - Pick a pivot element *randomly* (!) in the range lo..hi

QuickSort as an Instance of Divide and Conquer

Generic Divide and Conquer	QuickSort
1. Divide	Pick an element of the list: the *pivot* Place all elements of the list smaller than the pivot in the half of the list to its left; place larger elements to the right
2. Solve subproblems separately (and recursively)	Recursively sort each of the halves
3. Combine subsolutions to get overall solution	Surprise! Nothing to do

Another Divide-and-Conquer Sort: Mergesort

- 1. Split array in half
  - just take the first half and the second half of the array, *without* rearranging
- 2. Sort the halves separately
- 3. Combining the sorted halves ("merge")
  - repeatedly pick the least element from each array
  - compare, and put the smaller in the resulting array
  - example: if the two arrays are

    1 12 15 20

    5 6 13 21 30

    The "merged" array is

    1 5 6 12 13 15 20 21 30
  - note: we will need a second array to hold the result

3/3/2005 (c) 2001-5, University of Washington 23-29

Quicksort vs MergeSort

- Mergesort always has subproblems of size n/2
  - Which means guaranteed O(n log n)
- But mergesort requires an extra array for the result
  - No problem if you're sorting disk or tape files
  - Can be a problem if you're trying to sort large lists in main memory
- In practice, quicksort is the most commonly used general-purpose sort
  - Pretty easy to pick pivots well, so expected time is O(n log n)
  - Doesn't require extra space for a copy of the data

3/3/2005 (c) 2001-5, University of Washington 23-30

Summary

- Divide and Conquer
  - Algorithm design strategy that exploits recursion
  - Divide original problem into subproblems
  - Solve each subproblem recursively
  - Can sometimes yield dramatic performance improvements
- Sorting
  - Quicksort, mergesort: classic divide and conquer algorithms

3/3/2005 (c) 2001-5, University of Washington 23-31