CSE 143

Sorting

Reading: Sec. 19.3

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Sorting

- Binary search is a huge speedup over sequential search
  - But requires the list be sorted
- Slight Problem: How do we get a sorted list?
  - Maintain the list in sorted order as each word is added
  - Sort the entire list when needed
- Many, many algorithms for sorting have been invented and analyzed
  - Let's invent some!

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Insert for a Sorted List

- One possibility: ensure the list is always sorted as it is created
- Exercise: Assume that words[0..size-1] is sorted. Place new word in correct location so modified list remains sorted
  - Assume that there is spare capacity for the new word
- Before coding:
  - Draw pictures of an example situation, before and after
  - Write down the postconditions for the operation

```
// given existing list words[0..size-1], insert word in correct place and increase size
void insertWord(String word) {



    size++;
}
```

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Picture

- Draw your picture here

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Insertion Sort

- Once we have insertWord working...
- We can sort a list in place by repeating the insertion operation

```
void insertionSort( ) {
    int finalSize = size;
    size = 1;
    for (int k = 1; k < finalSize; k++) {
        insertWord(words[k]);
    }
}
```

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Insertion Sort As A Card Game Operation

- A bit like sorting a hand full of cards dealt one by one:
  - Pick up 1st card – it's sorted, the hand is sorted
  - Pick up 2nd card; *insert* it after or before 1st – both sorted
  - Pick up 3rd card; *insert* it after, between, or before 1st two
  - ...
- Each time:
  - Determine where new card goes
  - Make room for the newly inserted card and place it there

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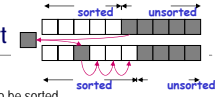
Insertion Sort As Invariant Progression
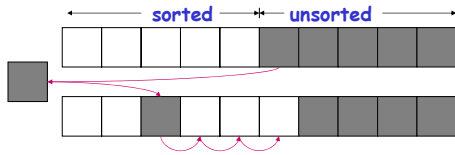
Insertion Sort


```
// instance variable
int[ ] list;                    // list[0..size-1] is the list to be sorted
int size;
// Sort list[0..size-1]
public void  sort {
    for (int j=1 ; j < size; j++) {
        // pre: 1 <= j && j < size && list[0 ... j-1] is in sorted order
        int temp = list[ j ];
        for (int i = j -1 ; i >= 0 && list[i] > temp ; i-- ) {
            list[i+1] = list[i] ;
        }
        list[i+1] = temp ;
        // post: 1 <= j && j < size && list[0 ... j] in sorted order
    }
}
```

Insertion Sort Trace

- Initial array contents
    - 0 pear
    - 1 orange
    - 2 apple
    - 3 rutabaga
    - 4 aardvark
    - 5 cherry
    - 6 banana
    - 7 kumquat

Insertion Sort Performance

- Cost of each insertWord operation:

- Number of times insertWord is executed:

- Total cost:

- Can we do better?

Analysis

- Why was binary search so much more effective than sequential search?
    - Answer: binary search divided the search space in half each time; sequential search only reduced the search space by 1 item per iteration
- Why is insertion sort $O(n^2)$?
    - Each insert operation only gets 1 more item in place at cost $O(n)$
    - $O(n)$ insert operations
- Can we do something similar for sorting?

Where are we on the chart?

N	$\log_2 N$	5N	$N \log_2 N$	N^2	2^N
8	3	40	24	64	256
16	4	80	64	256	65536
32	5	160	160	1024	$\sim 10^9$
64	6	320	384	4096	$\sim 10^{19}$
128	7	640	896	16384	$\sim 10^{38}$
256	8	1280	2048	65536	$\sim 10^{76}$
10000	13	50000	10^5	10^8	$\sim 10^{3010}$

Divide and Conquer Sorting

- Idea: something similar to binary search
  1. divide the sorting problem into two subproblems;
  2. recursively sort each subproblem;
  3. combine results
- Want division and combination at the end to be fast
- Want to be able to sort two halves independently
- This algorithm strategy is called *divide and conquer*

Quicksort

- Invented by C. A. R. Hoare (1962)
- Idea
  - Pick an element of the list: the *pivot*
  - Place all elements of the list smaller than the pivot in the half of the list to its left; place larger elements to the right
  - Recursively sort each of the halves
- Before looking at any code, see if you can draw pictures based just on the first two steps of the description

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Code for QuickSort

```
// Sort words[0..size-1]
void quickSort( ) {
    qsort(0, size-1);
}

// Sort words[lo..hi]
void qsort(int lo, int hi) {
    // quit if empty partition
    if (lo > hi) { return; }
    int pivotLocation = partition(lo, hi);    // partition array and return pivot loc
    qsort(lo, pivotLocation-1);
    qsort(pivotLocation+1, hi);
}
```

Recursion Analysis

- Base case? Yes.
  ```
  // quit if empty partition
  if (lo > hi) { return; }
  ```
- Recursive cases? Yes
  ```
  qsort(lo, pivotLocation-1);
  qsort(pivotLocation+1, hi);
  ```
  - Each recursive cases work on a smaller subproblem, so algorithm will terminate

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A Small Matter of Programming

- *Partition* algorithm
  - Pick pivot
  - Rearrange array so all smaller element are to the left, all larger to the right, with pivot in the middle
- *Partition* is not recursive
- Fact of life: *partition* can be a bit tricky to get right
  - Pictures and invariants are your friends here
- How do we pick the pivot?
  - For now, keep it simple – use the first item in the interval
  - Better strategies exist

Partition design

- We need to partition words[lo..hi]
- Pick words[lo] as the pivot
- Picture:

A `Partition` Implementation

- Use first element of array section as the pivot
- Invariant:

```
         lo            L           R        hi
words  x     <=x     unprocessed       >x
        ↑
      pivot
```
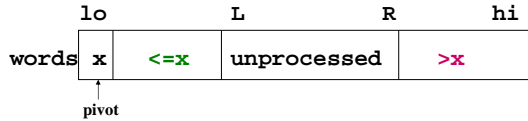
Partition Algorithm: PseudoCode

```
// Partition words[lo..hi]; return location of pivot in range lo..hi
int partition(int lo, int hi) {



}
```

Partition Test

- Check: partition(0,7)

```
0  orange
1  pear
2  apple
3  rutabaga
4  aardvark
5  cherry
6  banana
7  kumquat
```

Complexity of QuickSort

- Each call to Quicksort (ignoring recursive calls):
  - Each call of partition( ) is O(n) where n is size of the *part* of array being sorted
    - Note: This n is smaller than the N of the original problem
  - Some O(1) work
  - Total = O(n) (n is the size of array part being sorted)
- Including recursive calls:
  - Two recursive calls at each level of recursion, each partitions "half" the array at a cost of O(n/2)
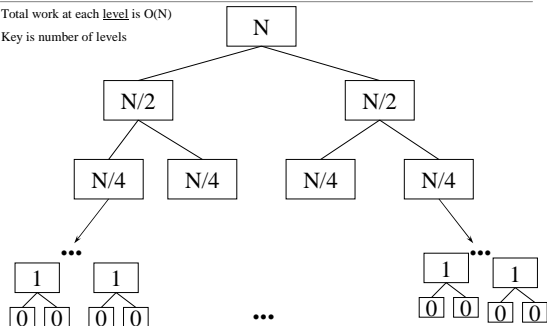 - How many levels of recursion?

QuickSort (Ideally)

Total work at each <u>level</u> is O(N)

Key is number of levels

QuickSort Performance (Ideal Case)

- Each partition divides the list parts in half
  - Sublist sizes on recursive calls: n, n/2, n/4, n/8....
  - Total depth of recursion: _____
  - Total work at each level: O(n)
  - Total cost of quicksort: _____ !

- For a list of 10,000 items
  - Insertion sort: $O(n^2)$: 100,000,000
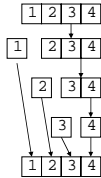 - Quicksort: O(n log n): $10,000 \log_2 10,000 = 132,877$

Worst Case for QuickSort

- If we're very unlucky, each pass through partition removes only a *single* element.

| 1 | 2 | 3 | 4 |

| 1 | | 2 | 3 | 4 |

| 2 | 3 | 4 |

| 3 | 4 |

| 1 | 2 | 3 | 4 |

- In this case, we have N levels of recursion rather than $\log_2 N$. What's the total complexity?

QuickSort Performance (Worst Case)

- Each partition manages to pick the largest or smallest item in the list as a pivot
  - Sublist sizes on recursive calls:
  - Total depth of recursion: _____
  - Total work at each level: O(n)
  - Total cost of quicksort: _____ !

Worst Case vs Average Case

- QuickSort has been shown to work well in the average case (mathematically speaking)
- In practice, Quicksort works well, provided the pivot is picked with some care
- Some strategies for choosing the pivot:
  - Compare a small number of list items (3-5) and pick the *median* for the pivot
    (Typically check the first, middle, last, and a couple of items in between – works well even if the original array is almost sorted)
  - Pick a pivot element *randomly* (!) in the range lo..hi

QuickSort as an Instance of Divide and Conquer

Generic Divide and Conquer	QuickSort
1. Divide	Pick an element of the list: the *pivot* Place all elements of the list smaller than the pivot in the half of the list to its left; place larger elements to the right
2. Solve subproblems separately (and recursively)	Recursively sort each of the halves
3. Combine subsolutions to get overall solution	Surprise! Nothing to do

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Another Divide-and-Conquer Sort: Mergesort

- 1. Split array in half
  - just take the first half and the second half of the array, *without* rearranging
- 2. Sort the halves separately
- 3. Combining the sorted halves ("merge")
  - repeatedly pick the least element from each array
  - compare, and put the smaller in the resulting array
  - example: if the two arrays are

    1   12   15   20
    5   6   13   21   30
    The "merged" array is
    1 5 6 12 13 15 20 21 30
  - note: we will need a second array to hold the result

Quicksort vs MergeSort

- Mergesort always has subproblems of size n/2
  - Which means guaranteed O(n log n)
- But mergesort requires an extra array for the result
  - No problem if you're sorting disk or tape files
  - Can be a problem if you're trying to sort large lists in main memory
- In practice, quicksort is the most commonly used general-purpose sort
  - Pretty easy to pick pivots well, so expected time is O(n log n)
  - Doesn't require extra space for a copy of the data

Summary

- Divide and Conquer
  - Algorithm design strategy that exploits recursion
  - Divide original problem into subproblems
  - Solve each subproblem recursively
  - Can sometimes yield dramatic performance improvements
- Sorting
  - Quicksort, mergesort: classic divide and conquer algorithms