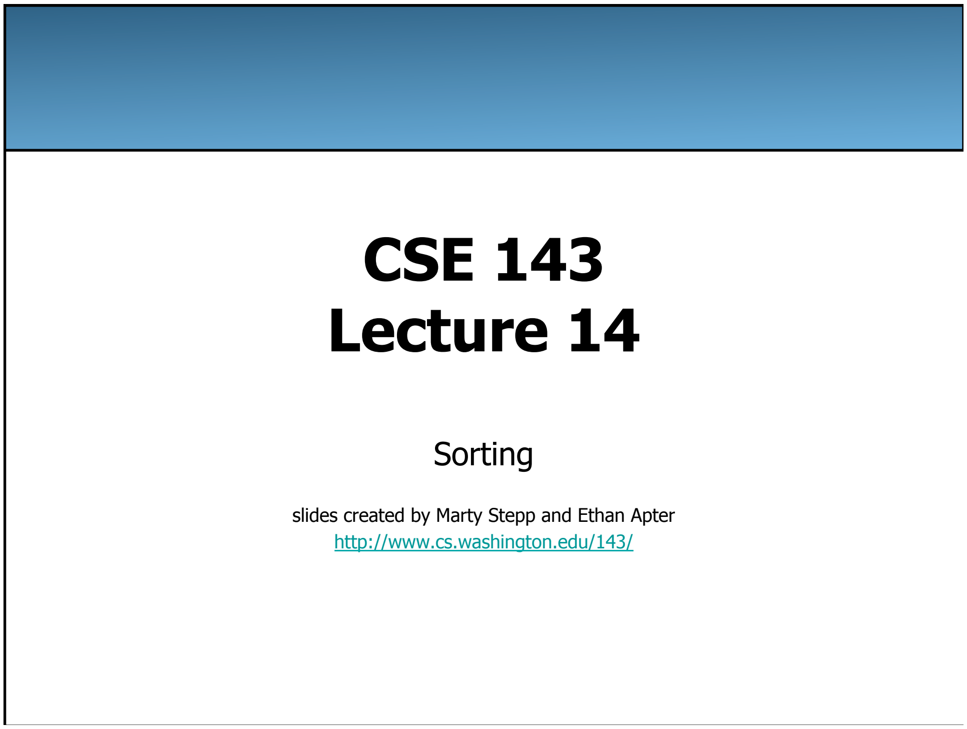
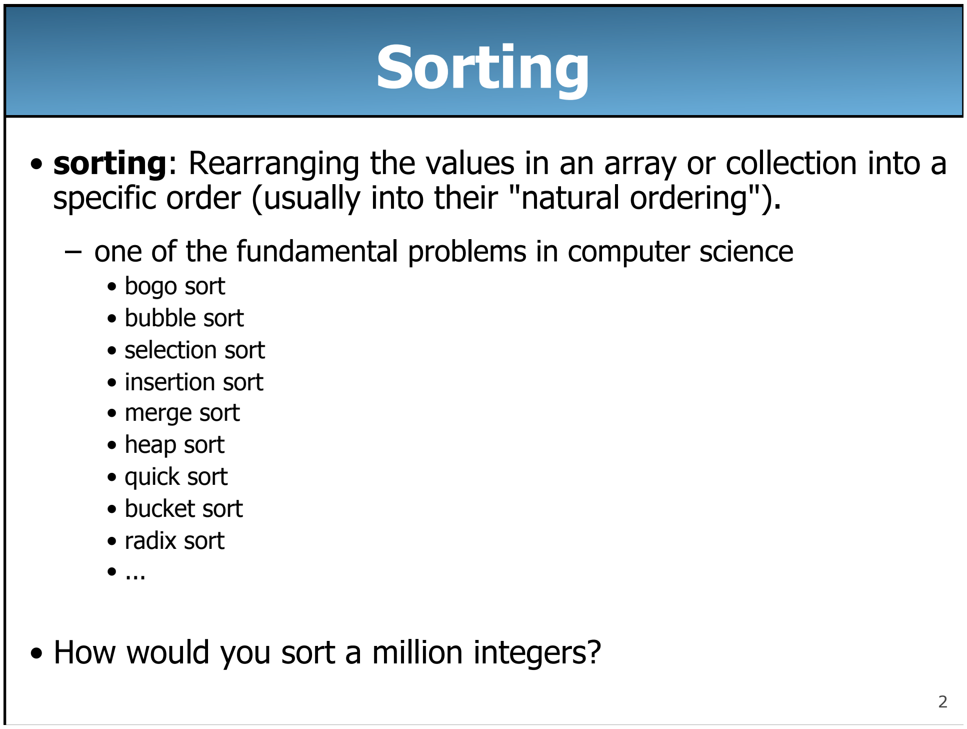


CSE 143
Lecture 14

Sorting

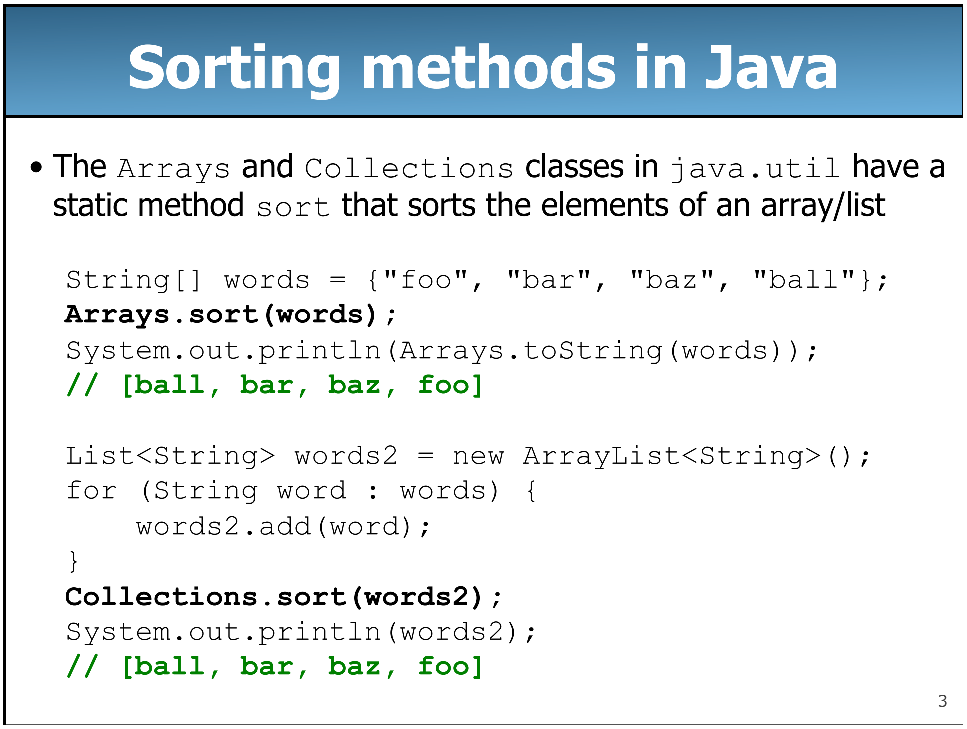
slides created by Marty Stepp and Ethan Apter
http://www.cs.washington.edu/143/



Sorting

- **sorting**: Rearranging the values in an array or collection into a specific order (usually into their "natural ordering").
  - one of the fundamental problems in computer science
    - bogo sort
    - bubble sort
    - selection sort
    - insertion sort
    - merge sort
    - heap sort
    - quick sort
    - bucket sort
    - radix sort
    - ...

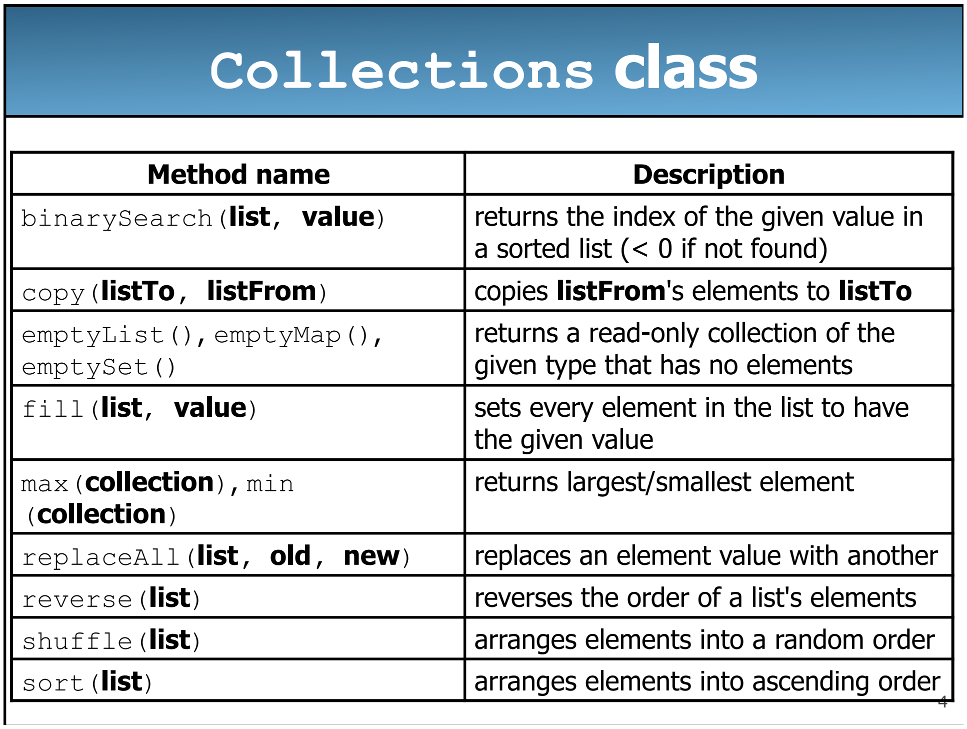
- How would you sort a million integers?



Sorting methods in Java

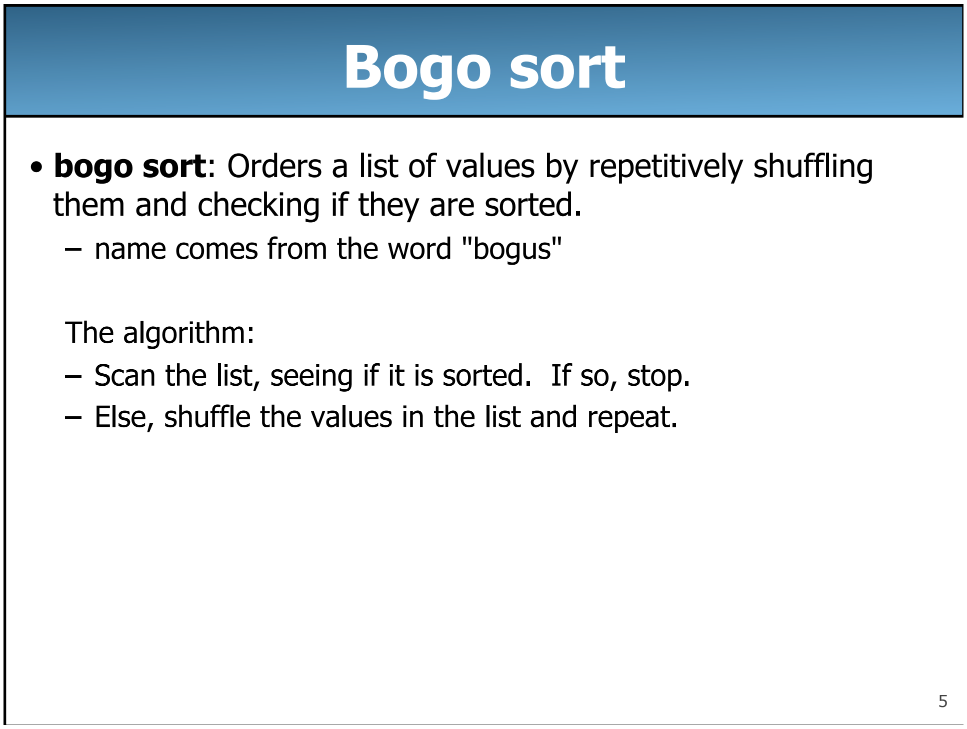
- The `Arrays` and `Collections` classes in `java.util` have a static method `sort` that sorts the elements of an array/list

```
String[] words = {"foo", "bar", "baz", "ball"};
Arrays.sort(words);
System.out.println(Arrays.toString(words));
// [ball, bar, baz, foo]

List<String> words2 = new ArrayList<String>();
for (String word : words) {
    words2.add(word);
}
Collections.sort(words2);
System.out.println(words2);
// [ball, bar, baz, foo]
```



`Collections` class

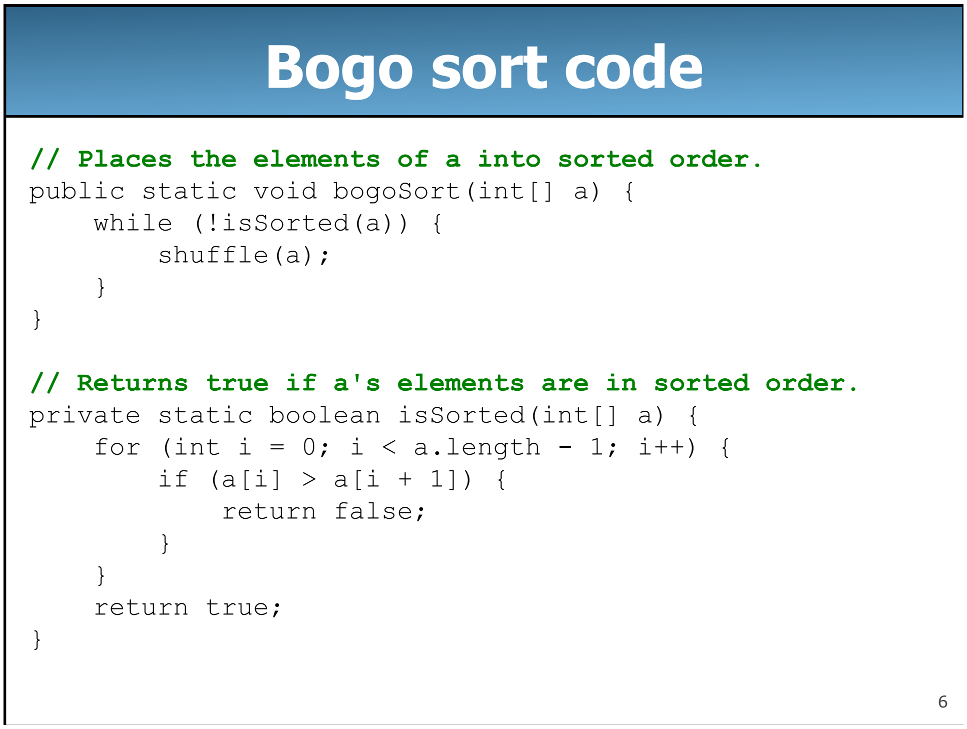
Method name	Description
`binarySearch(`**list**`, `**value**`)`	returns the index of the given value in a sorted list (< 0 if not found)
`copy(`**listTo**`, `**listFrom**`)`	copies **listFrom**'s elements to **listTo**
`emptyList(), emptyMap(), emptySet()`	returns a read-only collection of the given type that has no elements
`fill(`**list**`, `**value**`)`	sets every element in the list to have the given value
`max(`**collection**`), min(`**collection**`)`	returns largest/smallest element
`replaceAll(`**list**`, `**old**`, `**new**`)`	replaces an element value with another
`reverse(`**list**`)`	reverses the order of a list's elements
`shuffle(`**list**`)`	arranges elements into a random order
`sort(`**list**`)`	arranges elements into ascending order



Bogo sort

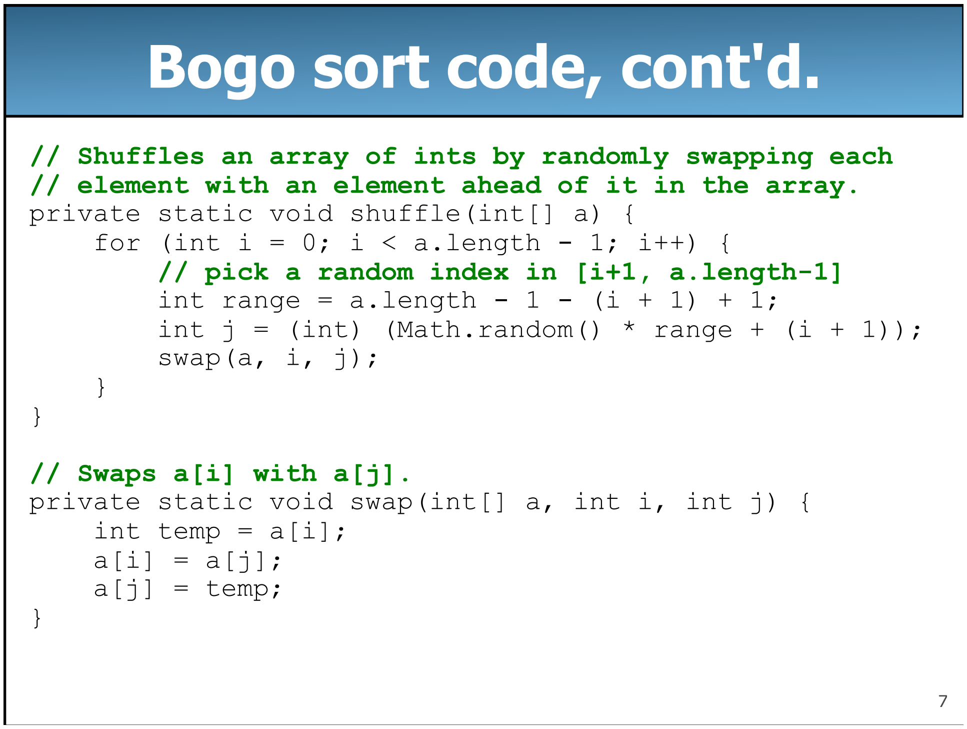
- **bogo sort**: Orders a list of values by repetitively shuffling them and checking if they are sorted.
  - name comes from the word "bogus"

  The algorithm:
  - Scan the list, seeing if it is sorted. If so, stop.
  - Else, shuffle the values in the list and repeat.



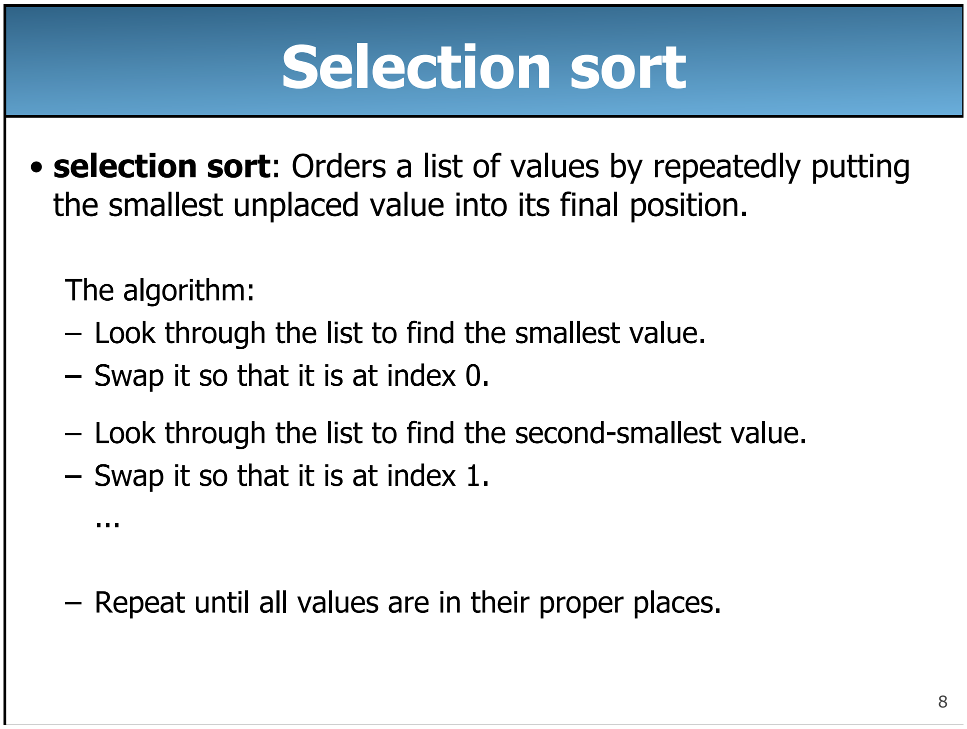
Bogo sort code

```
// Places the elements of a into sorted order.
public static void bogoSort(int[] a) {
    while (!isSorted(a)) {
        shuffle(a);
    }
}

// Returns true if a's elements are in sorted order.
private static boolean isSorted(int[] a) {
    for (int i = 0; i < a.length - 1; i++) {
        if (a[i] > a[i + 1]) {
            return false;
        }
    }
    return true;
}
```



Bogo sort code, cont'd.

```
// Shuffles an array of ints by randomly swapping each
// element with an element ahead of it in the array.
private static void shuffle(int[] a) {
    for (int i = 0; i < a.length - 1; i++) {
        // pick a random index in [i+1, a.length-1]
        int range = a.length - 1 - (i + 1) + 1;
        int j = (int) (Math.random() * range + (i + 1));
        swap(a, i, j);
    }
}

// Swaps a[i] with a[j].
private static void swap(int[] a, int i, int j) {
    int temp = a[i];
    a[i] = a[j];
    a[j] = temp;
}
```



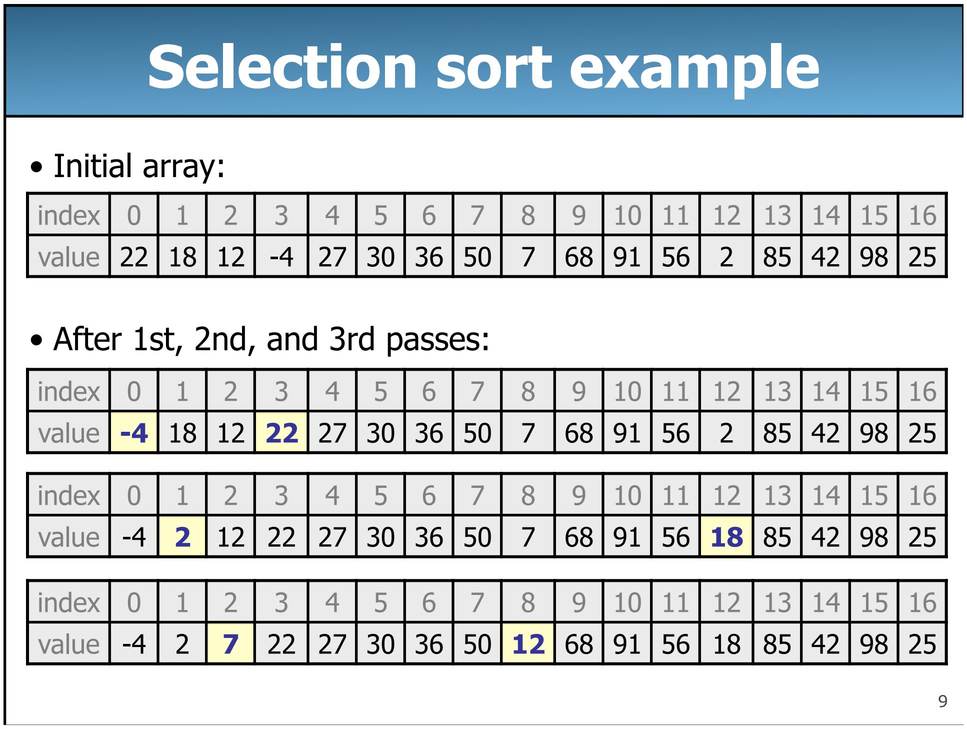
Selection sort

- **selection sort**: Orders a list of values by repeatedly putting the smallest unplaced value into its final position.

  The algorithm:
  - Look through the list to find the smallest value.
  - Swap it so that it is at index 0.
  - Look through the list to find the second-smallest value.
  - Swap it so that it is at index 1.

    ...

  - Repeat until all values are in their proper places.



Selection sort example

- Initial array:

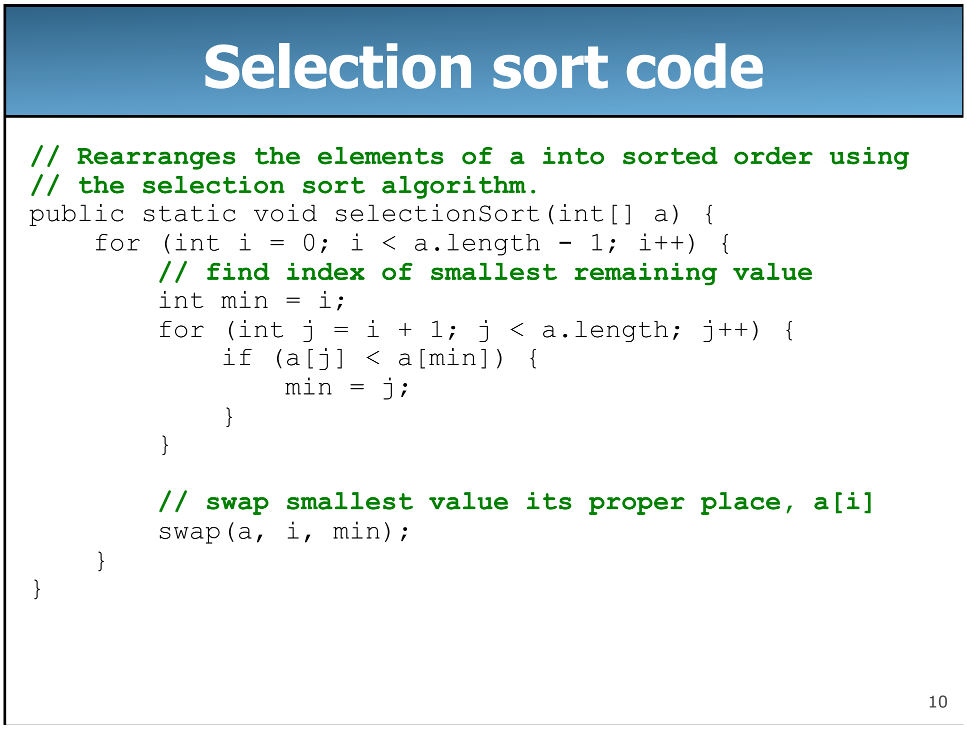
index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	22	18	12	-4	27	30	36	50	7	68	91	56	2	85	42	98	25

- After 1st, 2nd, and 3rd passes:

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	**-4**	18	12	**22**	27	30	36	50	7	68	91	56	2	85	42	98	25

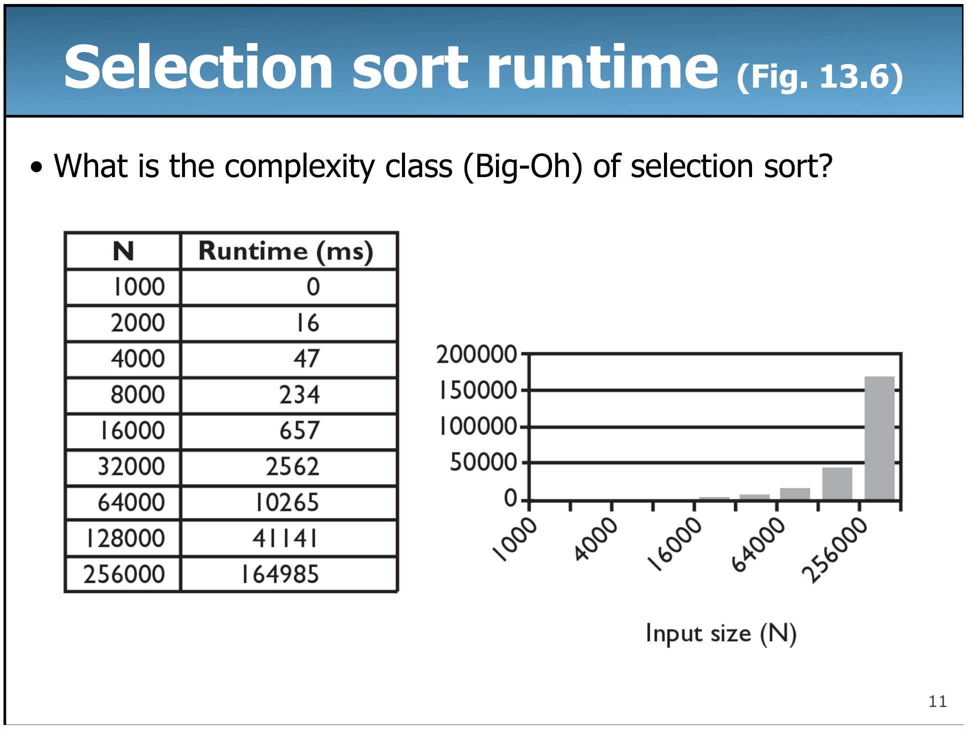
index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	**2**	12	22	27	30	36	50	7	68	91	56	**18**	85	42	98	25

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	2	**7**	22	27	30	36	50	**12**	68	91	56	18	85	42	98	25



Selection sort code

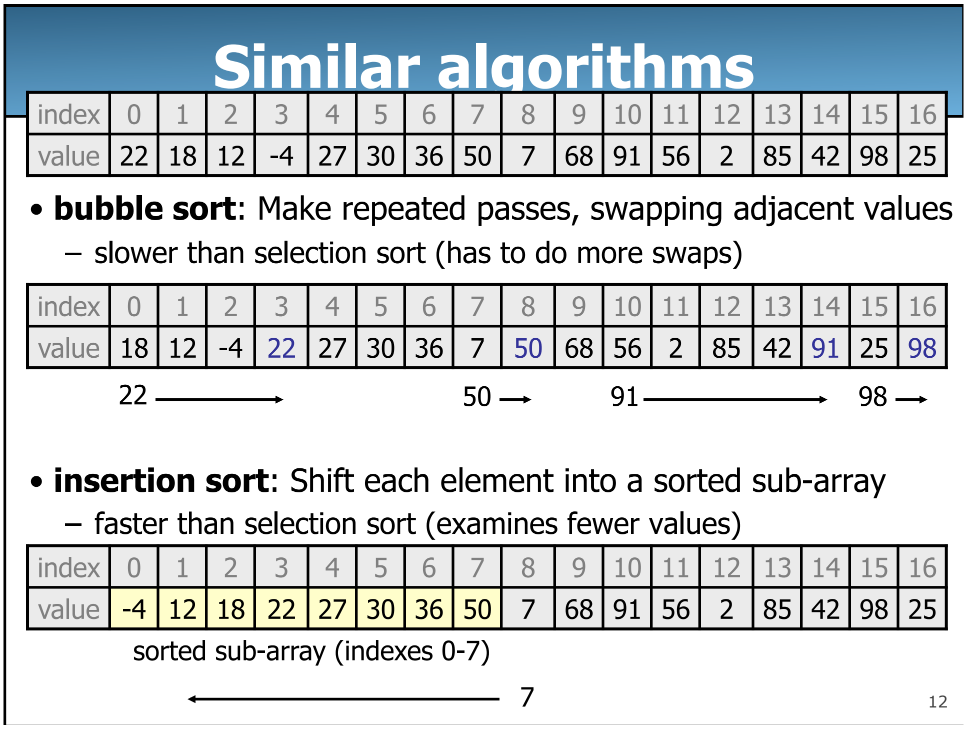
```
// Rearranges the elements of a into sorted order using
// the selection sort algorithm.
public static void selectionSort(int[] a) {
    for (int i = 0; i < a.length - 1; i++) {
        // find index of smallest remaining value
        int min = i;
        for (int j = i + 1; j < a.length; j++) {
            if (a[j] < a[min]) {
                min = j;
            }
        }

        // swap smallest value its proper place, a[i]
        swap(a, i, min);
    }
}
```



Selection sort runtime (Fig. 13.6)

- What is the complexity class (Big-Oh) of selection sort?

N	Runtime (ms)
1000	0
2000	16
4000	47
8000	234
16000	657
32000	2562
64000	10265
128000	41141
256000	164985



Similar algorithms

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	22	18	12	-4	27	30	36	50	7	68	91	56	2	85	42	98	25

- **bubble sort**: Make repeated passes, swapping adjacent values
  - slower than selection sort (has to do more swaps)

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	18	12	-4	22	27	30	36	7	50	68	56	2	85	42	91	25	98

22 → 50 → 91 → 98 →

- **insertion sort**: Shift each element into a sorted sub-array
  - faster than selection sort (examines fewer values)

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	12	18	22	27	30	36	50	7	68	91	56	2	85	42	98	25

sorted sub-array (indexes 0-7)

← 7

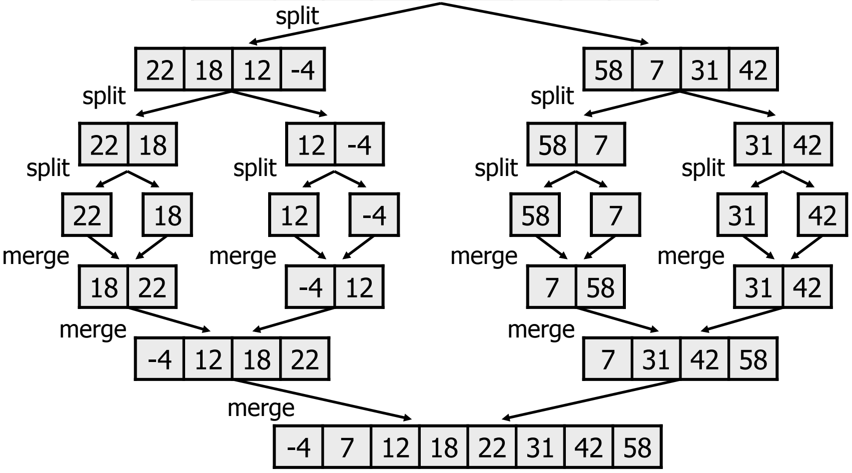
Merge sort

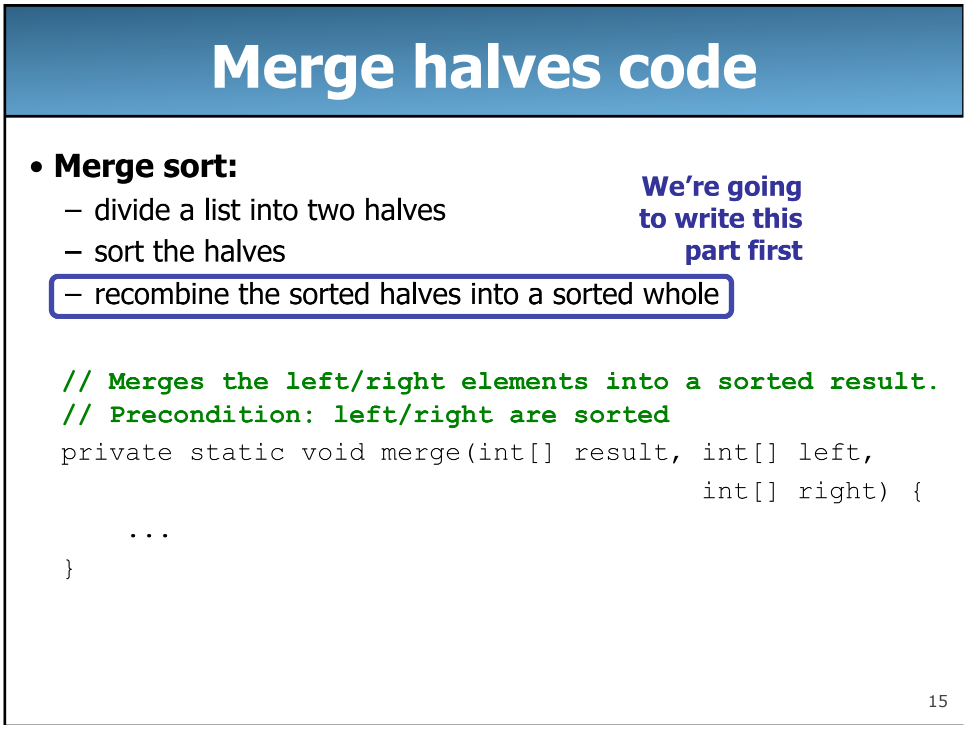
- **merge sort**: Repeatedly divides the data in half, sorts each half, and combines the sorted halves into a sorted whole.

  The algorithm:
  - Divide the list into two roughly equal halves.
  - Sort the left half.
  - Sort the right half.
  - Merge the two sorted halves into one sorted list.

Merge sort example

index	0	1	2	3	4	5	6	7
value	22	18	12	-4	58	7	31	42



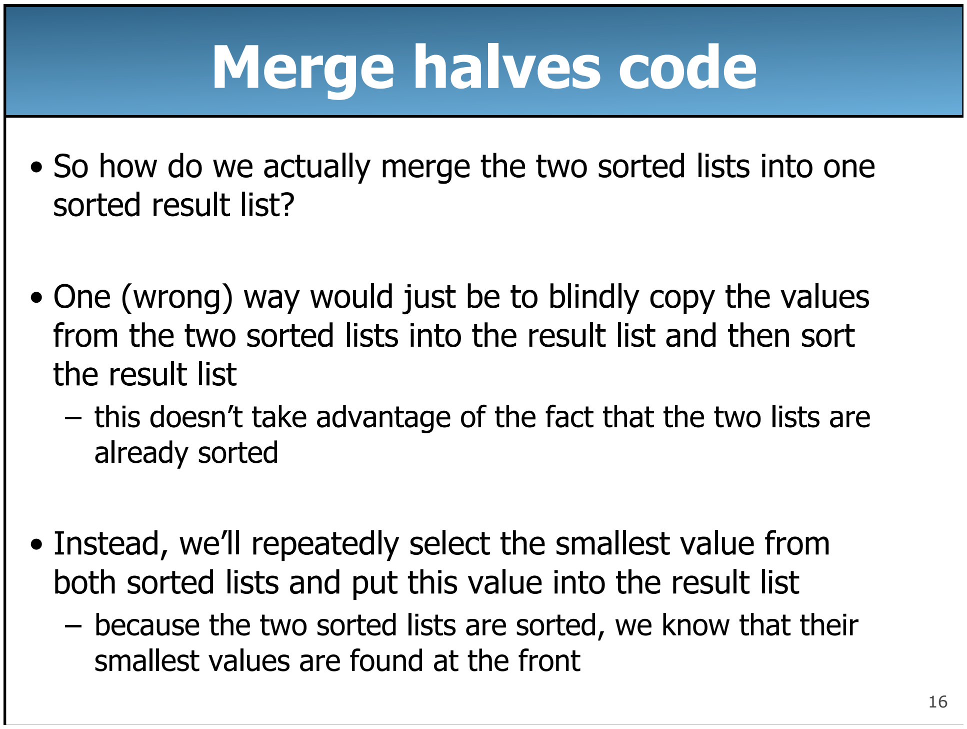


Merge halves code

- **Merge sort:**
  - divide a list into two halves
  - sort the halves
  - recombine the sorted halves into a sorted whole

We're going to write this part first

```
// Merges the left/right elements into a sorted result.
// Precondition: left/right are sorted
private static void merge(int[] result, int[] left,
                                        int[] right) {
    ...
}
```

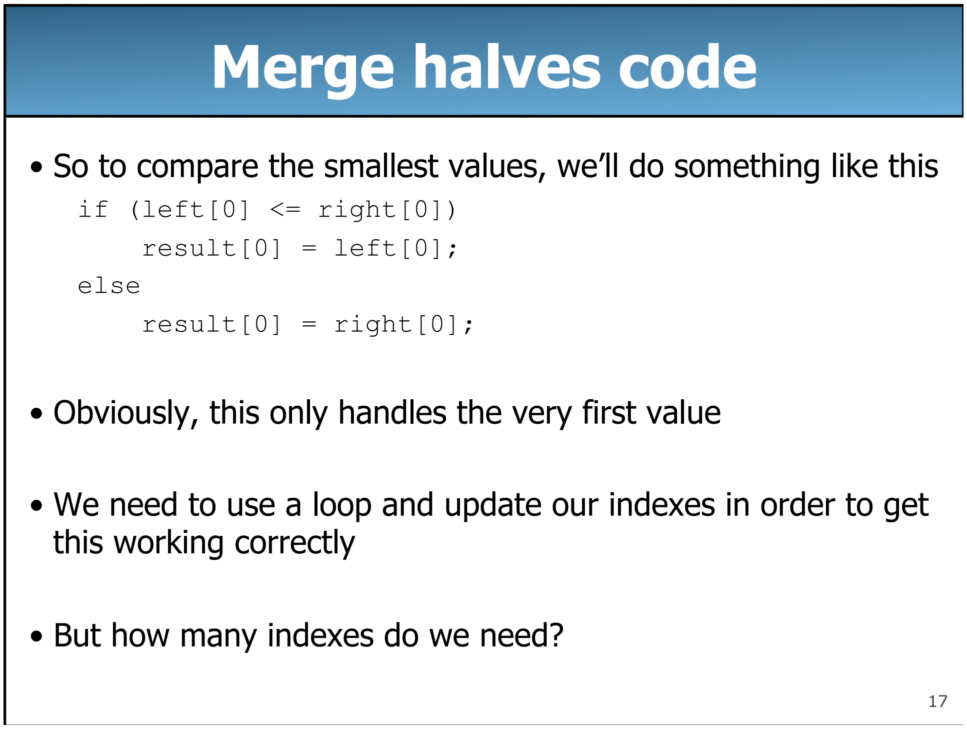


Merge halves code

- So how do we actually merge the two sorted lists into one sorted result list?

- One (wrong) way would just be to blindly copy the values from the two sorted lists into the result list and then sort the result list
  - this doesn't take advantage of the fact that the two lists are already sorted

- Instead, we'll repeatedly select the smallest value from both sorted lists and put this value into the result list
  - because the two sorted lists are sorted, we know that their smallest values are found at the front

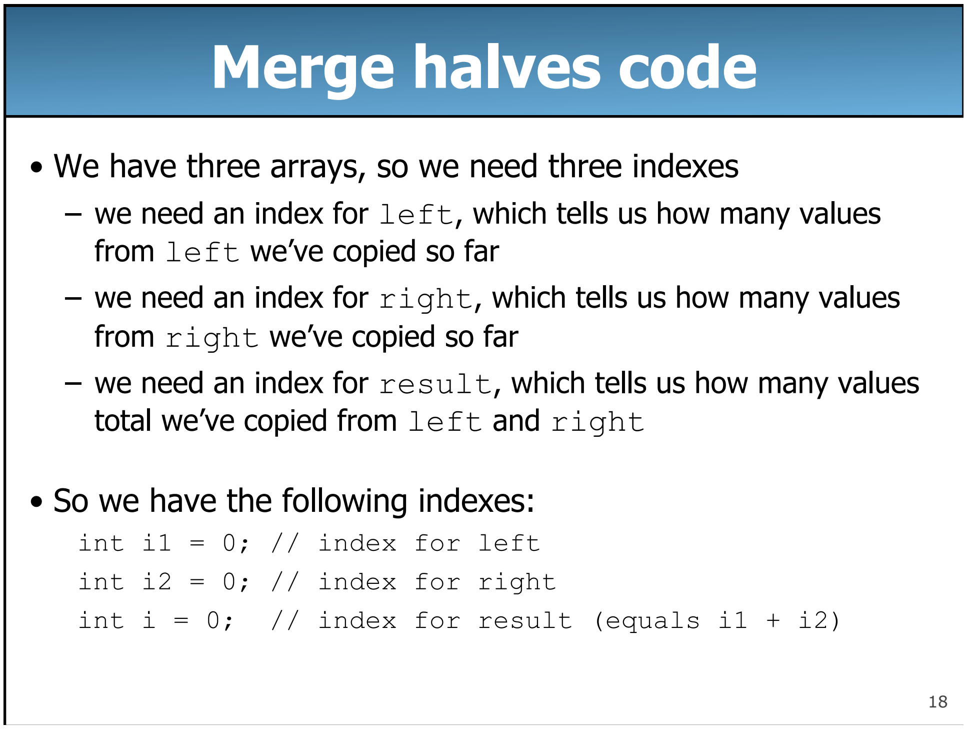


Merge halves code

- So to compare the smallest values, we'll do something like this

```
if (left[0] <= right[0])
    result[0] = left[0];
else
    result[0] = right[0];
```

- Obviously, this only handles the very first value
- We need to use a loop and update our indexes in order to get this working correctly
- But how many indexes do we need?

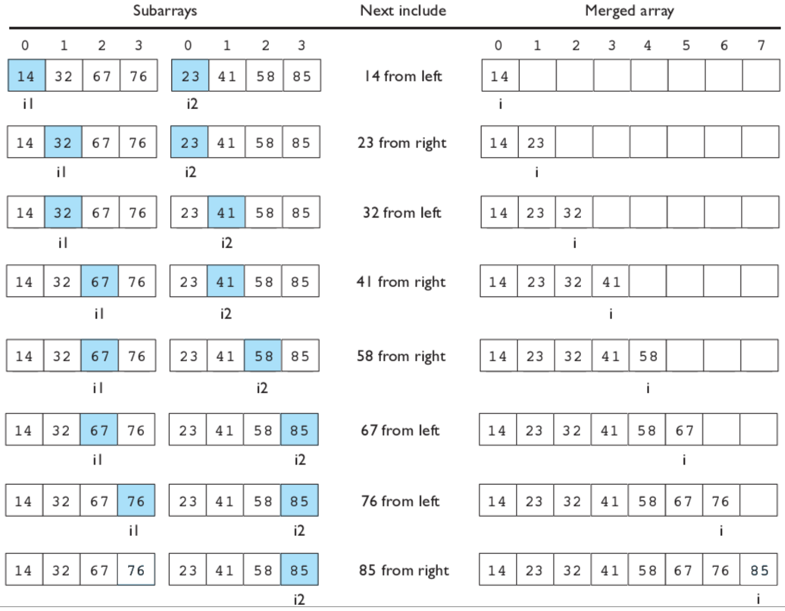


Merge halves code

- We have three arrays, so we need three indexes
  - we need an index for `left`, which tells us how many values from `left` we've copied so far
  - we need an index for `right`, which tells us how many values from `right` we've copied so far
  - we need an index for `result`, which tells us how many values total we've copied from `left` and `right`
- So we have the following indexes:

```
int i1 = 0; // index for left
int i2 = 0; // index for right
int i = 0;  // index for result (equals i1 + i2)
```

Merging sorted halves



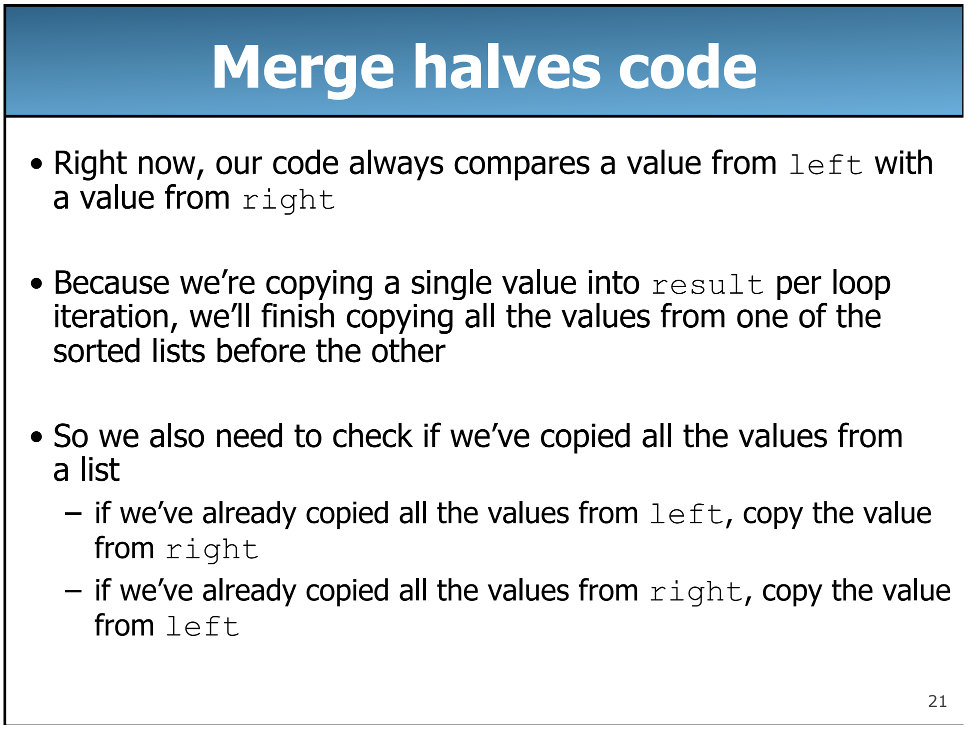
Subarrays								Next include	Merged array							
0	1	2	3	0	1	2	3		0	1	2	3	4	5	6	7
14	32	67	76	23	41	58	85	14 from left	14							
i1				i2					i							
14	32	67	76	23	41	58	85	23 from right	14	23						
	i1			i2						i						
14	32	67	76	23	41	58	85	32 from left	14	23	32					
	i1				i2						i					
14	32	67	76	23	41	58	85	41 from right	14	23	32	41				
		i1			i2							i				
14	32	67	76	23	41	58	85	58 from right	14	23	32	41	58			
		i1				i2							i			
14	32	67	76	23	41	58	85	67 from left	14	23	32	41	58	67		
		i1					i2							i		
14	32	67	76	23	41	58	85	76 from left	14	23	32	41	58	67	76	
			i1				i2								i	
14	32	67	76	23	41	58	85	85 from right	14	23	32	41	58	67	76	85
							i2									i

Merge halves code

- So now we'll update our previous code to use a loop and our indexes

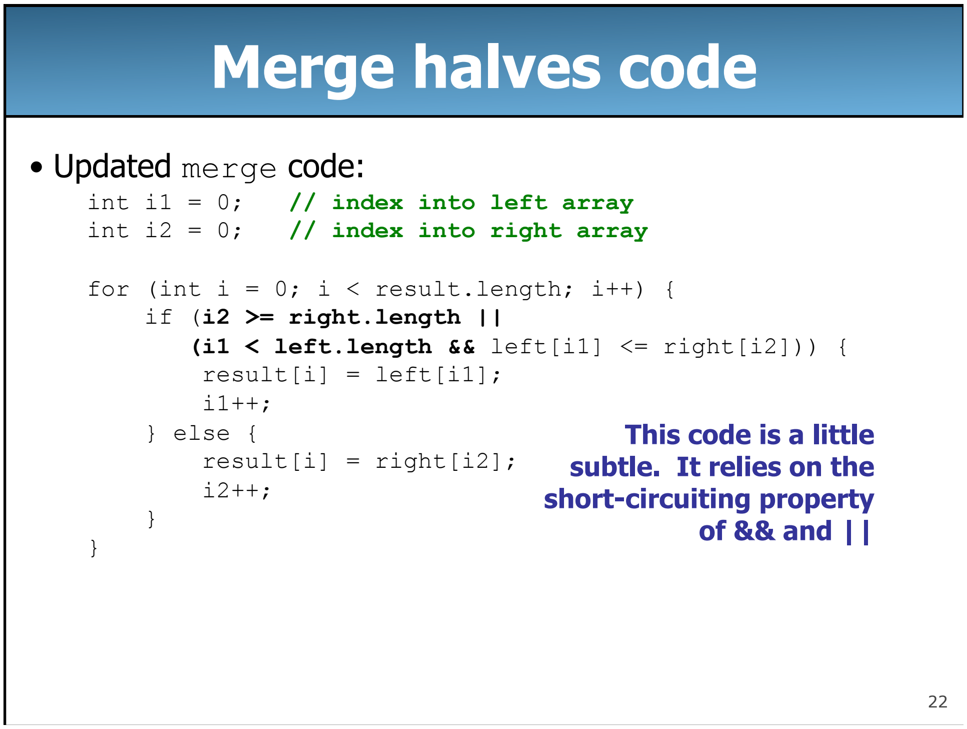
```
int i1 = 0;
int i2 = 0;
for (int i = 0; i < result.length; i++) {
    if (left[i1] <= right[i2]) {
        result[i] = left[i1];
        i1++;
    } else {
        result[i] = right[i2];
        i2++;
    }
}
```

But this doesn't quite work!



Merge halves code

- Right now, our code always compares a value from `left` with a value from `right`
- Because we're copying a single value into `result` per loop iteration, we'll finish copying all the values from one of the sorted lists before the other
- So we also need to check if we've copied all the values from a list
  - if we've already copied all the values from `left`, copy the value from `right`
  - if we've already copied all the values from `right`, copy the value from `left`



Merge halves code

- Updated `merge` code:

```
int i1 = 0;    // index into left array
int i2 = 0;    // index into right array

for (int i = 0; i < result.length; i++) {
    if (i2 >= right.length ||
       (i1 < left.length && left[i1] <= right[i2])) {
        result[i] = left[i1];
        i1++;
    } else {
        result[i] = right[i2];
        i2++;
    }
}
```

This code is a little subtle. It relies on the short-circuiting property of && and ||

Merge halves code

```
// Merges the left/right elements into a sorted result.
// Precondition: left/right are sorted
private static void merge(int[] result, int[] left,
                                        int[] right) {
    int i1 = 0;   // index into left array
    int i2 = 0;   // index into right array

    for (int i = 0; i < result.length; i++) {
        if (i2 >= right.length ||
           (i1 < left.length && left[i1] <= right[i2])) {
            result[i] = left[i1];    // take from left
            i1++;
        } else {
            result[i] = right[i2];   // take from right
            i2++;
        }
    }
}
```

`merge`'s Preconditions

- Our **`merge`** method has some preconditions
  - Both **`left`** and **`right`** must already be sorted
  - **`result`**'s length must equal **`left`**'s length plus **`right`**'s length
- What if the client violates the preconditions?
  - This is a private method—only we can use the method! We can ensure that the method is only called with valid input.
  - We're just going to settle for only commenting these preconditions
- Updated comment:

```
// pre:  left and right are sorted
//       result.length == left.length + right.length
// post: copies the values from left and right
//       into result so that the values in result
//       are in sorted order
```

Merge Sort

- Recall that merge sort consists of the following steps:
  - divide a list into two halves
  - sort the halves
  - recombine the sorted halves into a sorted whole

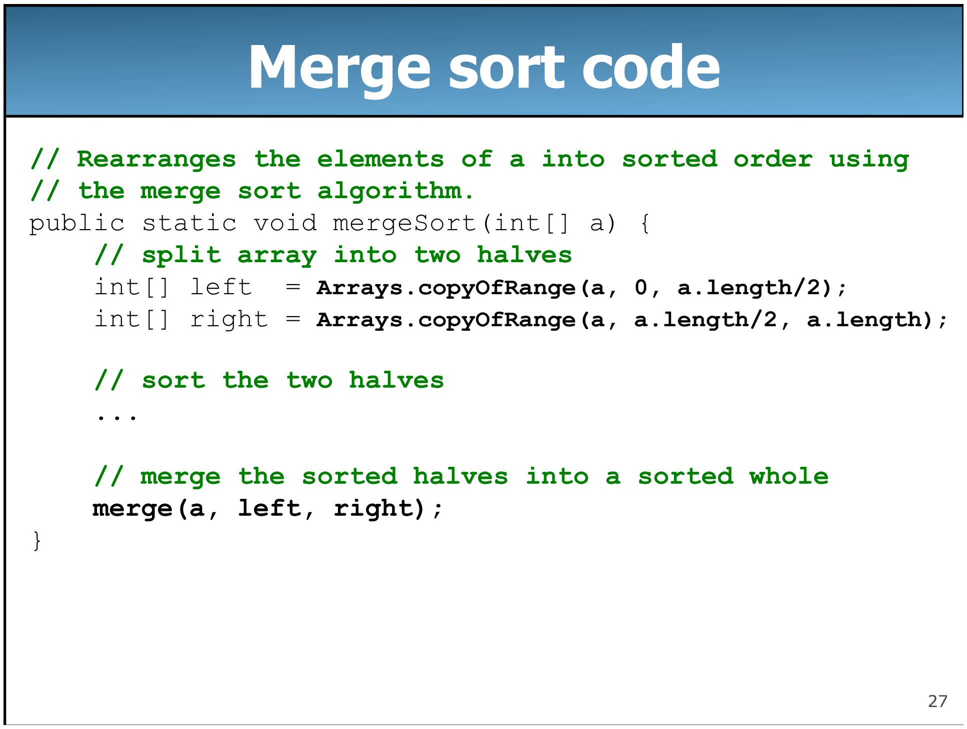
- Let's define our public sort method:

```
// post: sorts the given array into non-decreasing order
public static void mergeSort(int[] list) {
    ...
}
```

Dividing the List in Half

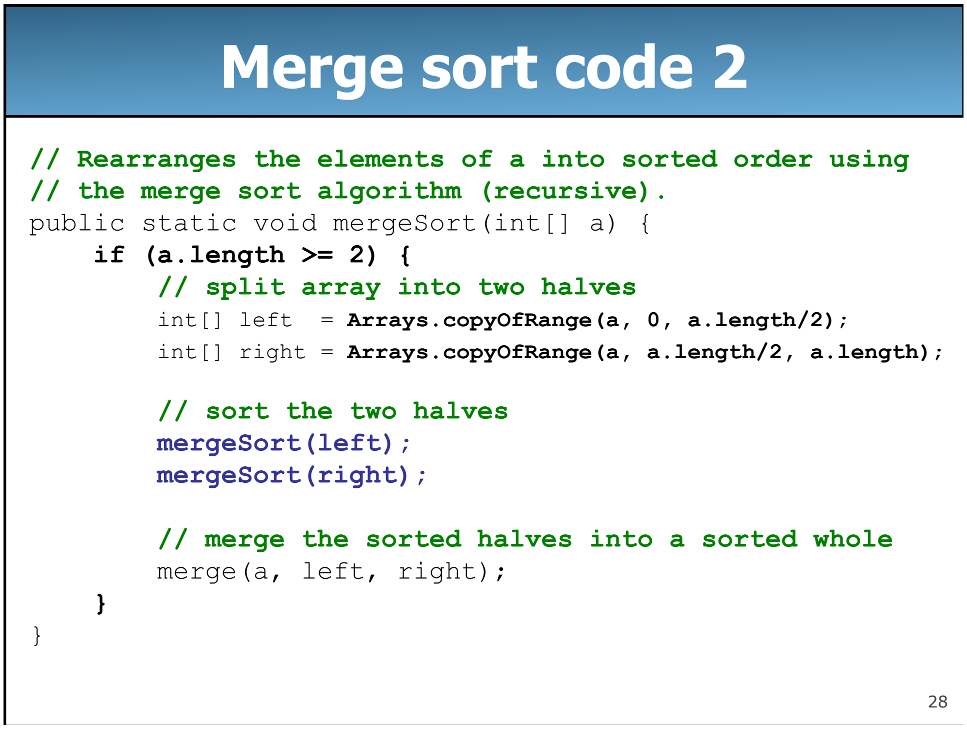
- The `Arrays` class has a useful method called `copyOfRange`:

```
// split array into two halves
int[] left  = Arrays.copyOfRange(a, 0, a.length/2);
int[] right = Arrays.copyOfRange(a, a.length/2, a.length);
```



Merge sort code

```
// Rearranges the elements of a into sorted order using
// the merge sort algorithm.
public static void mergeSort(int[] a) {
    // split array into two halves
    int[] left  = Arrays.copyOfRange(a, 0, a.length/2);
    int[] right = Arrays.copyOfRange(a, a.length/2, a.length);

    // sort the two halves
    ...

    // merge the sorted halves into a sorted whole
    merge(a, left, right);
}
```



Merge sort code 2

```
// Rearranges the elements of a into sorted order using
// the merge sort algorithm (recursive).
public static void mergeSort(int[] a) {
    if (a.length >= 2) {
        // split array into two halves
        int[] left  = Arrays.copyOfRange(a, 0, a.length/2);
        int[] right = Arrays.copyOfRange(a, a.length/2, a.length);

        // sort the two halves
        mergeSort(left);
        mergeSort(right);

        // merge the sorted halves into a sorted whole
        merge(a, left, right);
    }
}
```

Complexity of Merge Sort

- To determine the time complexity, let’s break our merge sort into pieces and analyze the pieces

- Remember, merge sort consists of:
  - divide a list into two halves
  - sort the halves
  - recombine the sorted halves into a sorted whole

- Dividing the list and recombining the lists are pretty easy to analyze
  - both have O(n) time complexity

- But what about sorting the halves?

Complexity of Merge Sort

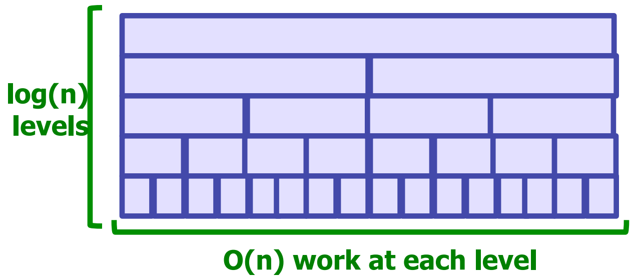
- We can think of merge sort as occurring in levels
  - at the first level, we want to sort the whole list
  - at the second level, we want to sort the two half lists
  - at the third level, we want to sort the four quarter lists
  - ...

- We know there’s O(n) work at each level from dividing/ recombining the lists

- But how many levels are there?
  - if we can figure this out, our time complexity is just O(n * num_levels)

Complexity of Merge Sort

- Because we divide the array in half each time, there are log(n) levels



log(n) levels

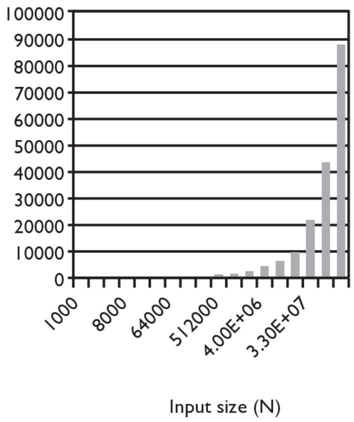
O(n) work at each level

- So merge sort is an O(n log(n)) algorithm
  - this is a big improvement over the O(n^2) sorting algorithms

Merge sort runtime

- What is the complexity class (Big-Oh) of merge sort?

N	Runtime (ms)
1000	0
2000	0
4000	0
8000	0
16000	0
32000	15
64000	16
128000	47
256000	125
512000	250
1e6	532
2e6	1078
4e6	2265
8e6	4781
1.6e7	9828
3.3e7	20422
6.5e7	42406
1.3e8	88344


4.00E+06
3.30E+07
Input size (N)