CSE 143
Lecture 20

Binary Search Trees

read 17.3

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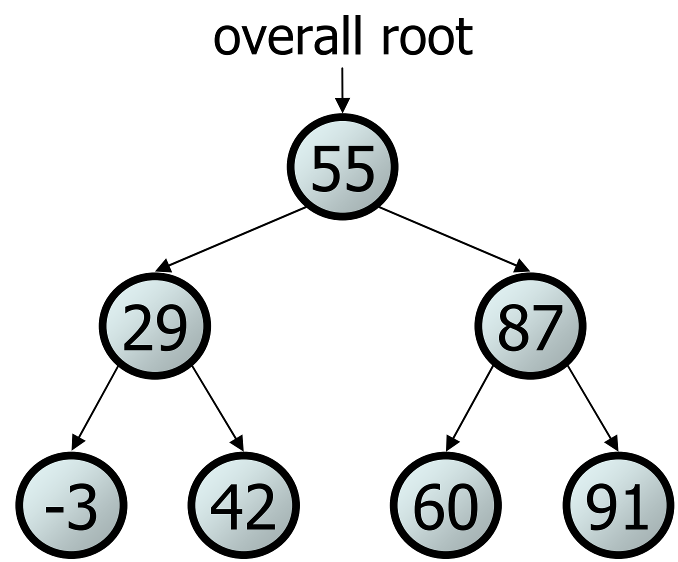
Binary search trees

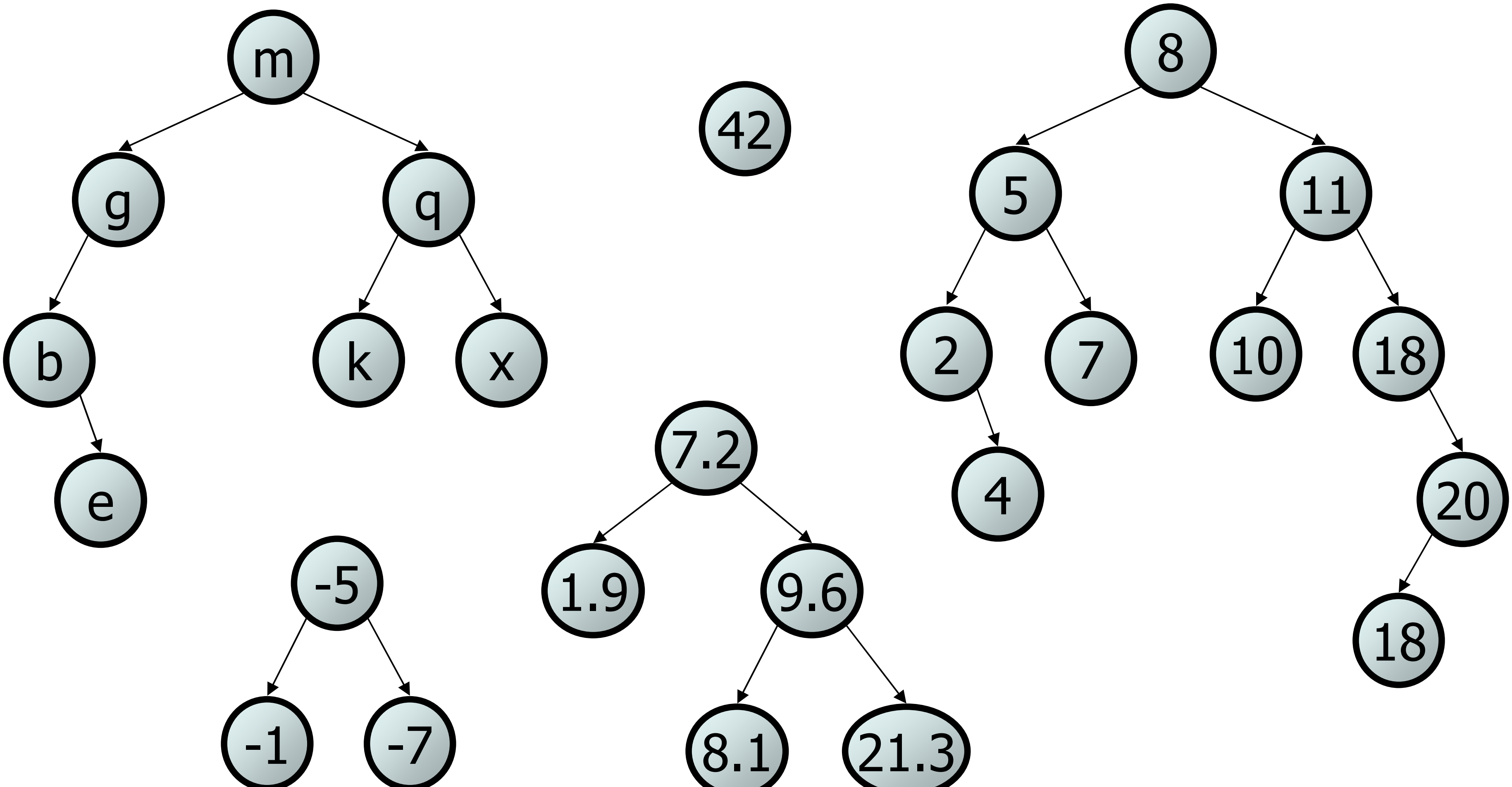
- **binary search tree** ("BST"): a binary tree that is either:
  - empty (`null`), or
  - a root node R such that:
    - every element of R's left subtree contains data "less than" R's data,
    - every element of R's right subtree contains data "greater than" R's,
    - R's left and right subtrees are also binary search trees.

- BSTs store their elements in sorted order, which is helpful for searching/sorting tasks.



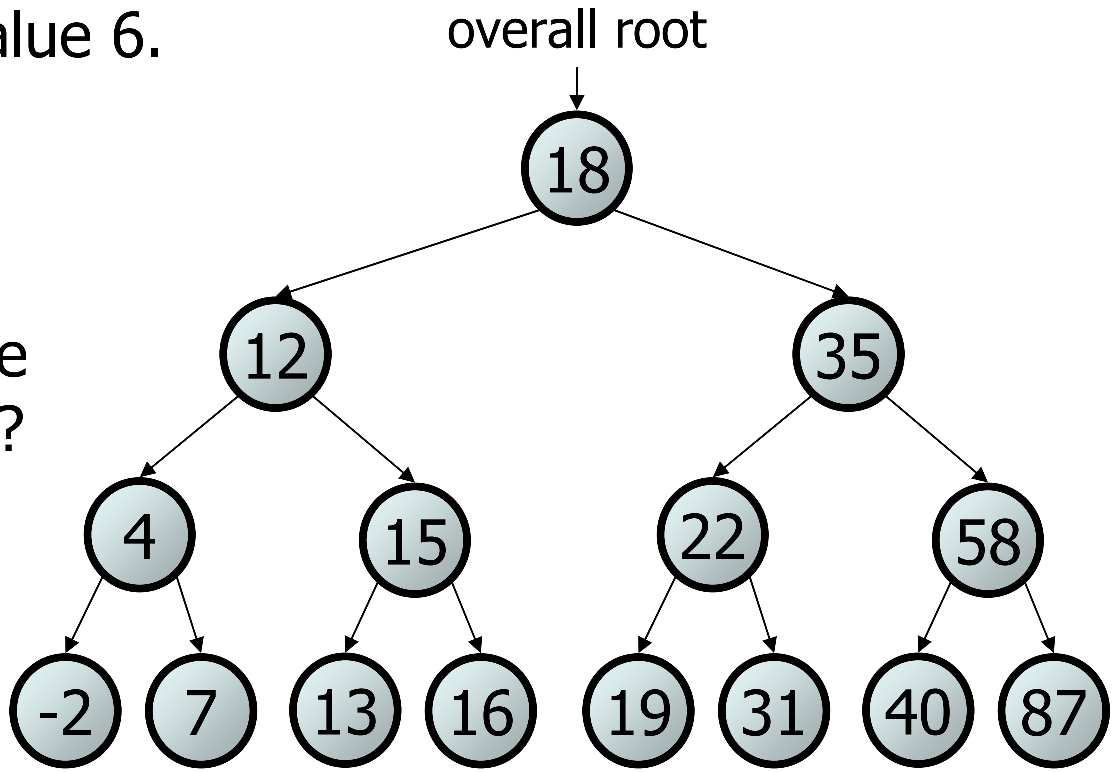
Exercise

- Which of the trees shown are legal binary search trees?



Searching a BST

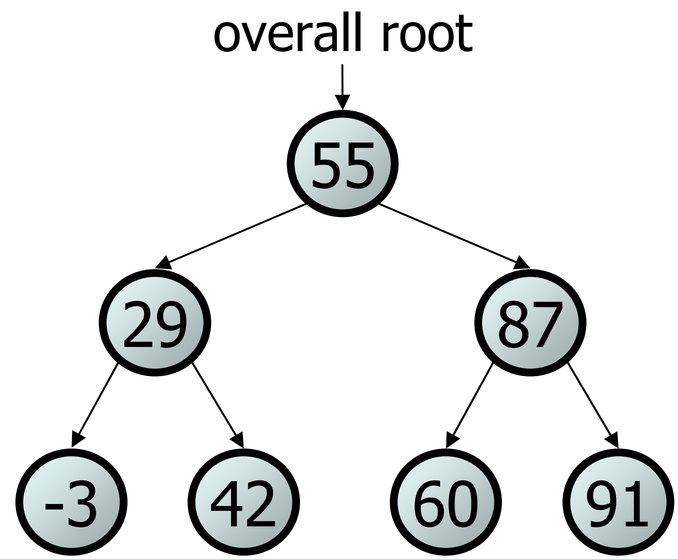
- Describe an algorithm for searching the tree below for the value 31.
- Then search for the value 6.
- What is the maximum number of nodes you would need to examine to perform any search?



Exercise

- Convert the `IntTree` class into a `SearchTree` class.
  - The elements of the tree will constitute a legal binary search tree.

- Add a method `contains` to the `SearchTree` class that searches the tree for a given integer, returning `true` if found.
  - If a `SearchTree` variable `tree` referred to the tree below, the following calls would have these results:
    - `tree.contains(29)` → `true`
    - `tree.contains(55)` → `true`
    - `tree.contains(63)` → `false`
    - `tree.contains(35)` → `false`



Exercise solution

```
// Returns whether this tree contains the given integer.
public boolean contains(int value) {
    return contains(overallRoot, value);
}

private boolean contains(IntTreeNode root, int value) {
    if (root == null) {
        return false;
    } else if (root.data == value) {
        return true;
    } else if (root.data > value) {
        return contains(root.left, value);
    } else {   // root.data < value
        return contains(root.right, value);
    }
}
```

Adding to a BST

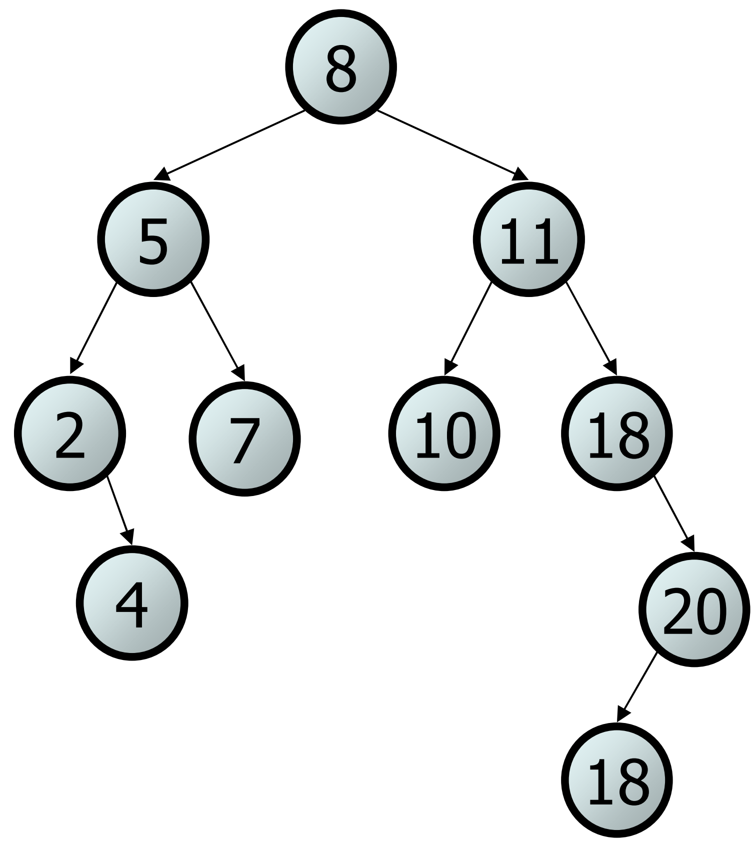
- Suppose we want to add the value 14 to the BST below.
  - Where should the new node be added?

- Where would we add the value 3?

- Where would we add 7?

- If the tree is empty, where should a new value be added?

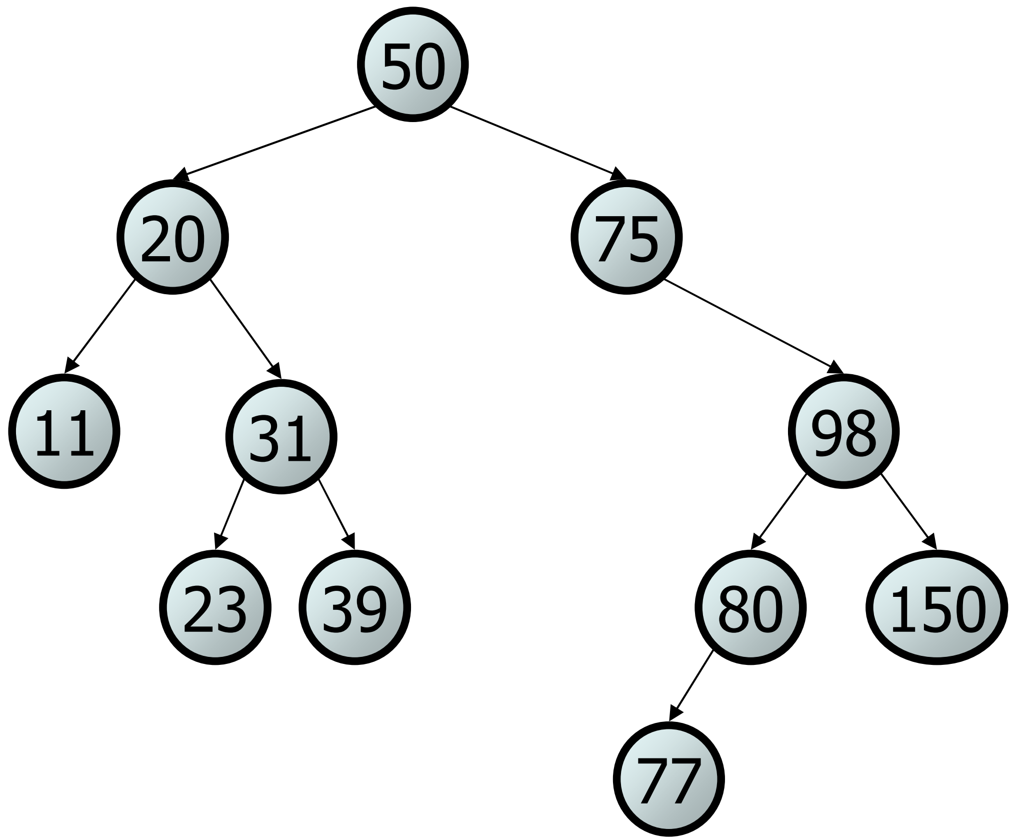
- What is the general algorithm?



Adding exercise

- Draw what a binary search tree would look like if the following values were added to an initially empty tree in this order:

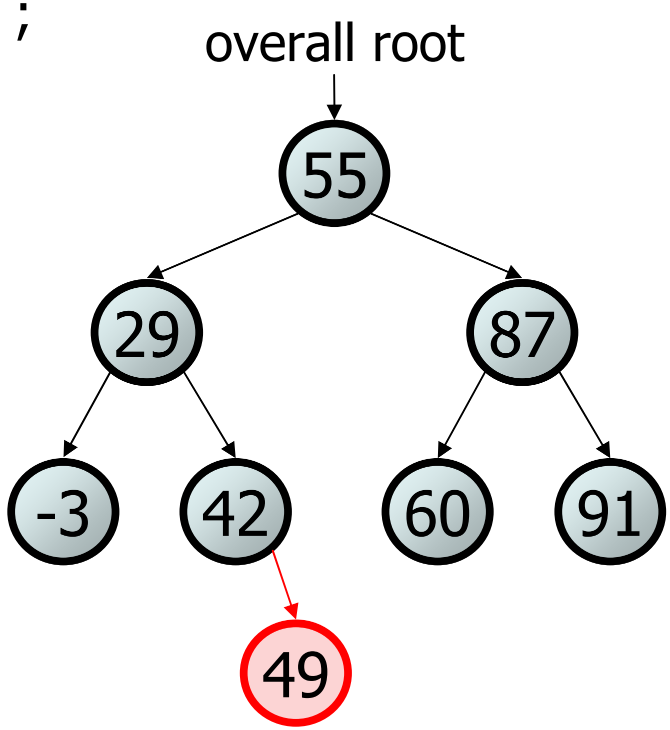
50
20
75
98
80
31
150
39
23
11
77



Exercise

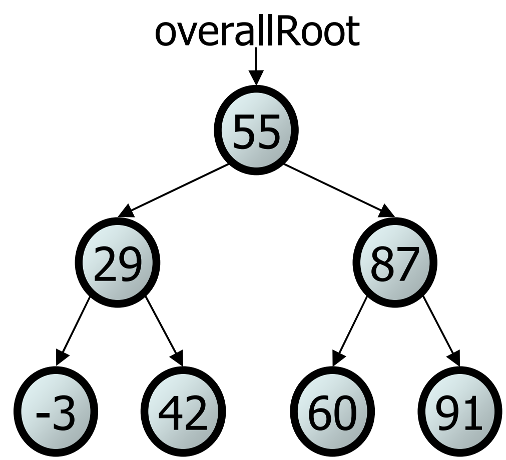
- Add a method `add` to the `SearchTree` class that adds a given integer value to the tree. Assume that the elements of the `SearchTree` constitute a legal binary search tree, and add the new value in the appropriate place to maintain ordering.

  - `tree.add(49);`



An incorrect solution

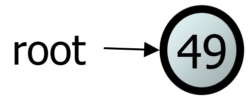
```
// Adds the given value to this BST in sorted order.
public void add(int value) {
    add(overallRoot, value);
}

private void add(IntTreeNode root, int value) {
    if (root == null) {
        root = new IntTreeNode(value);
    } else if (root.data > value) {
        add(root.left, value);
    } else if (root.data < value) {
        add(root.right, value);
    }
    // else root.data == value;
    // a duplicate (don't add)
}
```



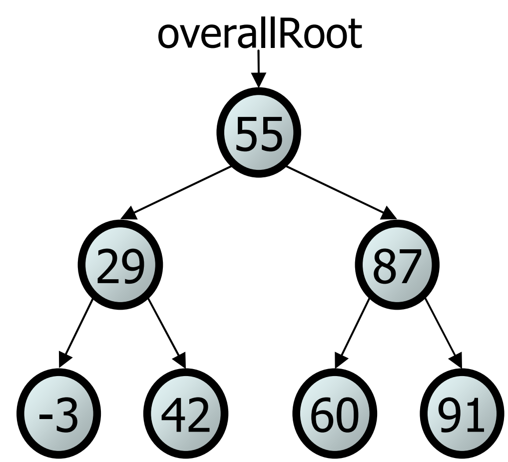
- Why doesn't this solution work?

The problem

- Much like with linked lists, if we just modify what a local variable refers to, it won't change the collection.




```
private void add(IntTreeNode root, int value) {
    if (root == null) {
        root = new IntTreeNode(value);
    }
```



  - In the linked list case, how did we actually modify the list?
    - by changing the `front`
    - by changing a node's `next` field

A poor correct solution

```
// Adds the given value to this BST in sorted order. (bad style)
public void add(int value) {
    if (overallRoot == null) {
        overallRoot = new IntTreeNode(value);
    } else if (overallRoot.data > value) {
        add(overallRoot.left, value);
    } else if (overallRoot.data < value) {
        add(overallRoot.right, value);
    }
    // else overallRoot.data == value; a duplicate (don't add)
}

private void add(IntTreeNode root, int value) {
    if (root.data > value) {
        if (root.left == null) {
            root.left = new IntTreeNode(value);
        } else {
            add(overallRoot.left, value);
        }
    } else if (root.data < value) {
        if (root.right == null) {
            root.right = new IntTreeNode(value);
        } else {
            add(overallRoot.right, value);
        }
    }
    // else root.data == value; a duplicate (don't add)
}
```

x = change(x);

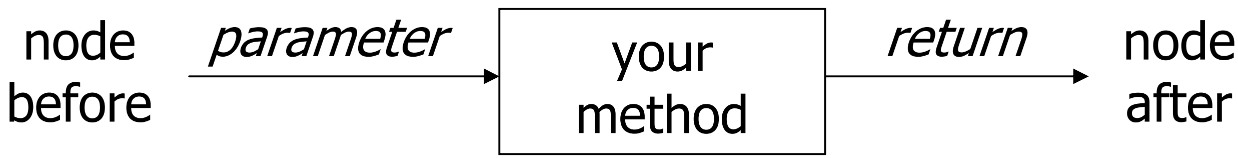
- String methods that modify a string actually return a new one.
  - If we want to modify a string variable, we must re-assign it.

```
String s = "lil bow wow";
s.toUpperCase();
System.out.println(s);    // lil bow wow
s = s.toUpperCase();
System.out.println(s);    // LIL BOW WOW
```

  - We call this general algorithmic pattern **x = change(x);**
  - We will use this approach when writing methods that modify the structure of a binary tree.

Applying x = change(x)

- Methods that modify a tree should have the following pattern:
  - input (parameter): old state of the node
  - output (return): new state of the node

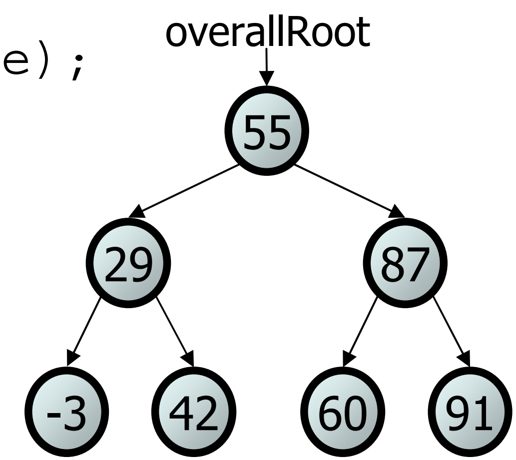


- In order to actually change the tree, you must reassign:

```
root = change(root, parameters);
root.left = change(root.left, parameters);
root.right = change(root.right, parameters);
```

A correct solution

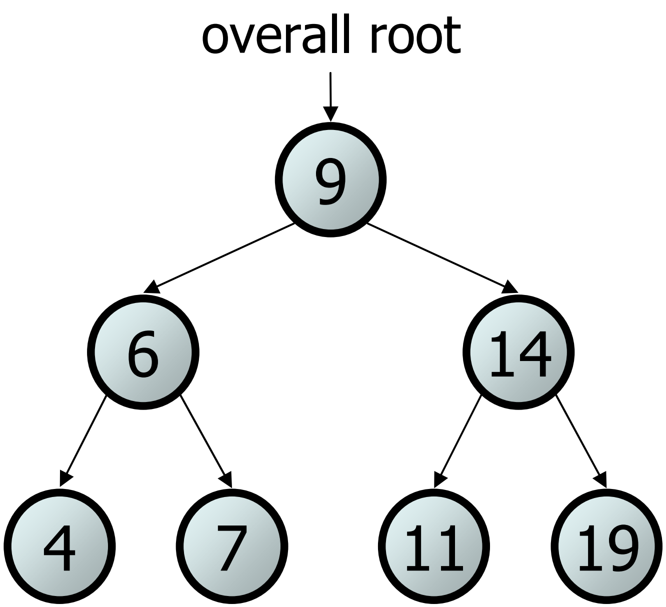
```
// Adds the given value to this BST in sorted order.
public void add(int value) {
    overallRoot = add(overallRoot, value);
}

private IntTreeNode add(IntTreeNode root, int value) {
    if (root == null) {
        root = new IntTreeNode(value);
    } else if (root.data > value) {
        root.left = add(root.left, value);
    } else if (root.data < value) {
        root.right = add(root.right, value);
    } // else a duplicate

    return root;
}
```

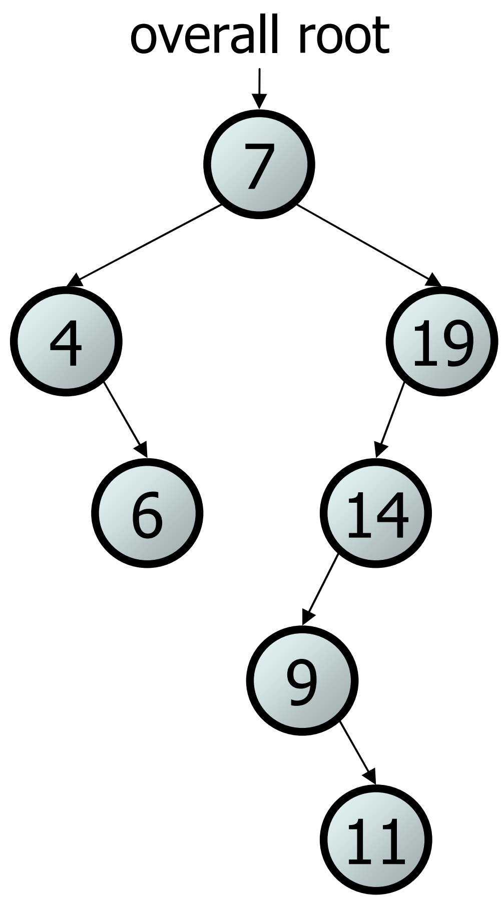


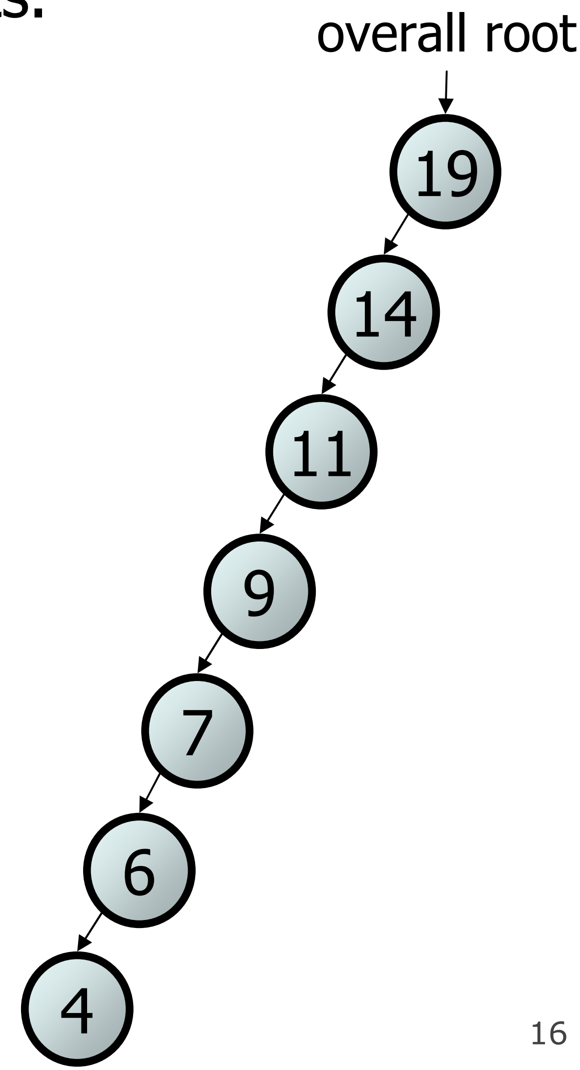
- Think about the case when `root` is a leaf...

Searching BSTs

- The BSTs below contain the same elements.
  - What orders are "better" for searching?



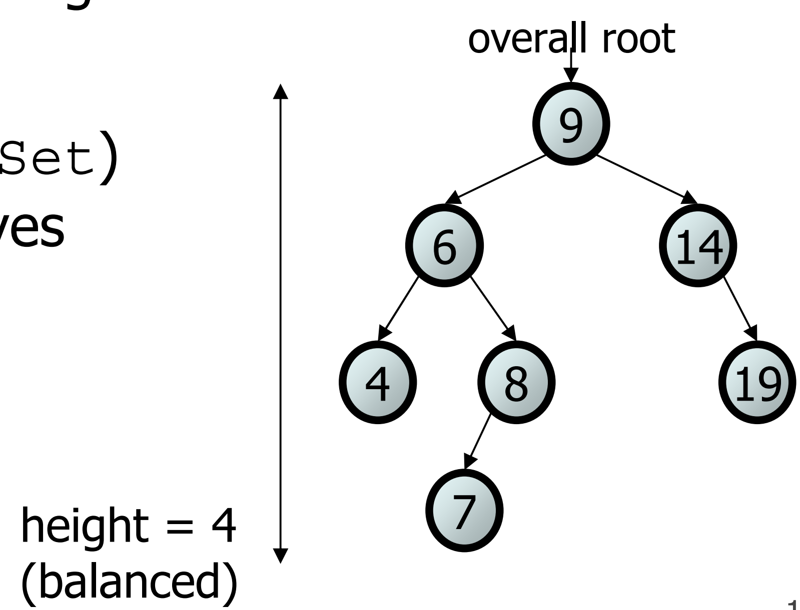




Trees and balance

- **balanced tree**: One whose subtrees differ in height by at most 1 and are themselves balanced.
  - A balanced tree of N nodes has a height of ~ $\log_2 N$.
  - A very unbalanced tree can have a height close to N.

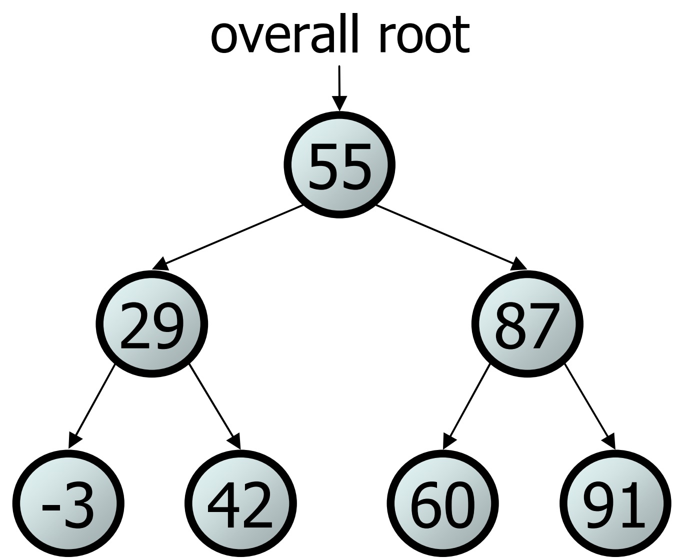
  - The runtime of adding to / searching a BST is closely related to height.
  - Some tree collections (e.g. `TreeSet`) contain code to balance themselves as new nodes are added.



Exercise

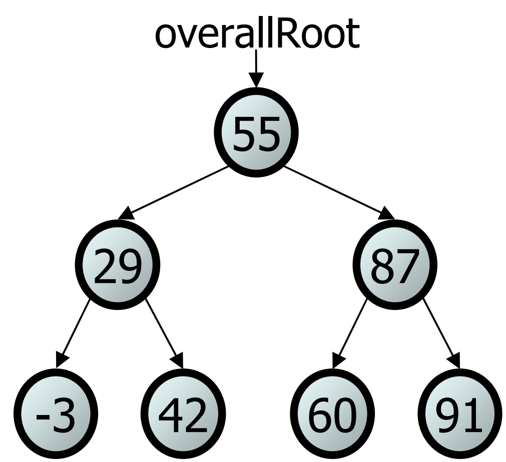
- Add a method `getMin` to the `IntTree` class that returns the minimum integer value from the tree. Assume that the elements of the `IntTree` constitute a legal binary search tree. Throw a `NoSuchElementException` if the tree is empty.

```
int min = tree.getMin();  // -3
```



Exercise solution

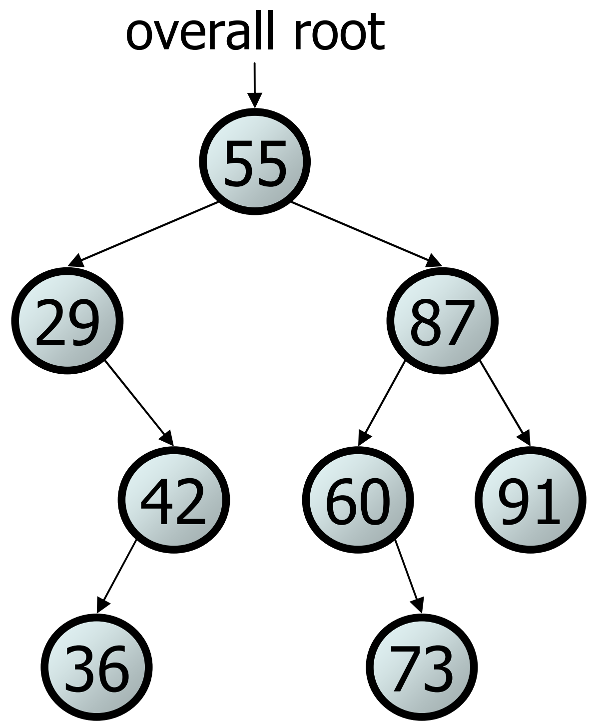
```
// Returns the minimum value from this BST.
// Throws a NoSuchElementException if the tree is empty.
public int getMin() {
    if (overallRoot == null) {
        throw new NoSuchElementException();
    }
    return getMin(overallRoot);
}

private int getMin(IntTreeNode root) {
    if (root.left == null) {
        return root.data;
    } else {
        return getMin(root.left);
    }
}
```



Exercise

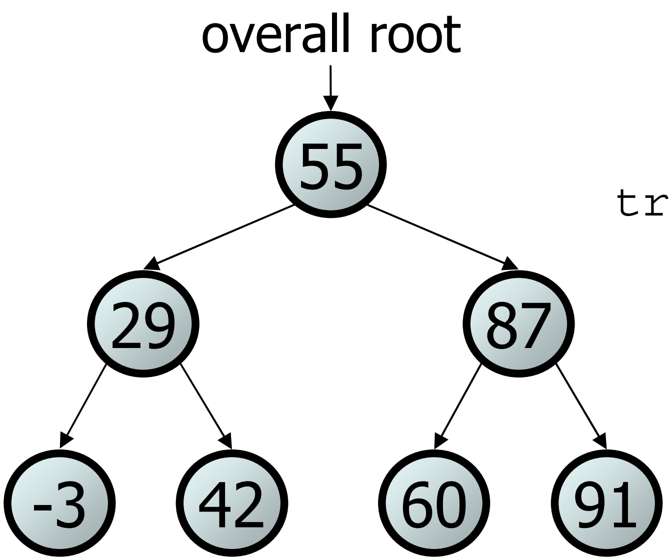
- Add a method `remove` to the `IntTree` class that removes a given integer value from the tree, if present. Assume that the elements of the `IntTree` constitute a legal binary search tree, and remove the value in such a way as to maintain ordering.

  - `tree.remove(73);`
  - `tree.remove(29);`
  - `tree.remove(87);`
  - `tree.remove(55);`

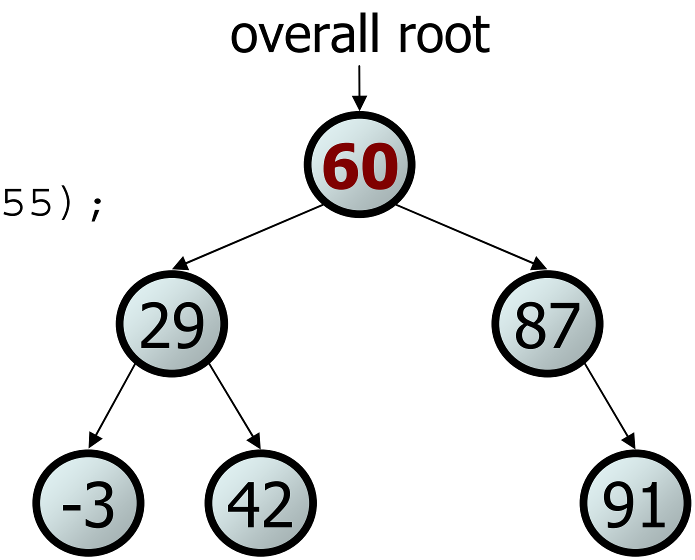


Cases for removal

- Possible states for the node to be removed:
  - a leaf: replace with null
  - a node with a left child only: replace with left child
  - a node with a right child only: replace with right child
  - a node with both children: replace with min value from right



`tree.remove(55);`



Exercise solution

```
// Removes the given value from this BST, if it exists.
public void remove(int value) {
    overallRoot = remove(overallRoot, value);
}

private IntTreeNode remove(IntTreeNode root, int value) {
    if (root == null) {
        return null;
    } else if (root.data > value) {
        root.left = remove(root.left, value);
    } else if (root.data < value) {
        root.right = remove(root.right, value);
    } else {  // root.data == value; remove this node
        if (root.right == null) {
            return root.left;    // no R child; replace w/ L
        } else if (root.left == null) {
            return root.right;   // no L child; replace w/ R
        } else {
            // both children; replace w/ min from R
            root.data = getMin(root.right);
            root.right = remove(root.right, root.data);
        }
    }
    return root;
}
```