CSE 143
Lecture 17

Recursive Backtracking

reading: notes on course web site

Exercise: Permutations

- Write a method `permute` that accepts a string as a parameter and outputs all possible rearrangements of the letters in that string. The arrangements may be output in any order.

    - Example:
    `permute("MARTY")`
    outputs the following sequence of lines:

MARTY	MYRAT	ATYMR	RTMAY	TARMY	YMTAR
MARYT	MYRTA	ATYRM	RTMYA	TARYM	YMTRA
MATRY	MYTAR	AYMRT	RTAMY	TAYMR	YAMRT
MATYR	MYTRA	AYMTR	RTAYM	TAYRM	YAMTR
MAYRT	AMRTY	AYRMT	RTYMA	TRMAY	YARMT
MAYTR	AMRYT	AYRTM	RTYAM	TRMYA	YARTM
MRATY	AMTRY	AYTMR	RYMAT	TRAMY	YATMR
MRAYT	AMTYR	AYTRM	RYMTA	TRAYM	YATRM
MRTAY	AMYRT	RMATY	RYAMT	TRYMA	YRMAT
MRTYA	AMYTR	RMAYT	RYATM	TRYAM	YRMTA
MRYAT	ARMTY	RMTAY	RYTMA	TYMAR	YRAMT
MRYTA	ARMYT	RMTYA	RYTAM	TYMRA	YRATM
MTARY	ARTMY	RMYAT	TMARY	TYAMR	YRTMA
MTAYR	ARTYM	RMYTA	TMAYR	TYARM	YRTAM
MTRAY	ARYMT	RAMTY	TMRAY	TYRMA	YTMAR
MTRYA	ARYTM	RAMYT	TMRYA	TYRAM	YTMRA
MTYAR	ATMRY	RATMY	TMYAR	YMART	YTAMR
MTYRA	ATMYR	RATYM	TMYRA	YMATR	YTARM
MYART	ATRMY	RAYMT	TAMRY	YMRAT	YTRMA
MYATR	ATRYM	RAYTM	TAMYR	YMRTA	YTRAM

Examining the problem

- Think of each permutation as a set of choices or **decisions**:
  - Which character do I want to place first?
  - Which character do I want to place second?
  - ...

  - **solution space**: set of all possible sets of decisions to explore

- We want to generate all possible sequences of decisions.
      for (each possible first letter):
          for (each possible second letter):
              for (each possible third letter):
                  ...
                      print!

  - This is called a **depth-first search**

Decision trees
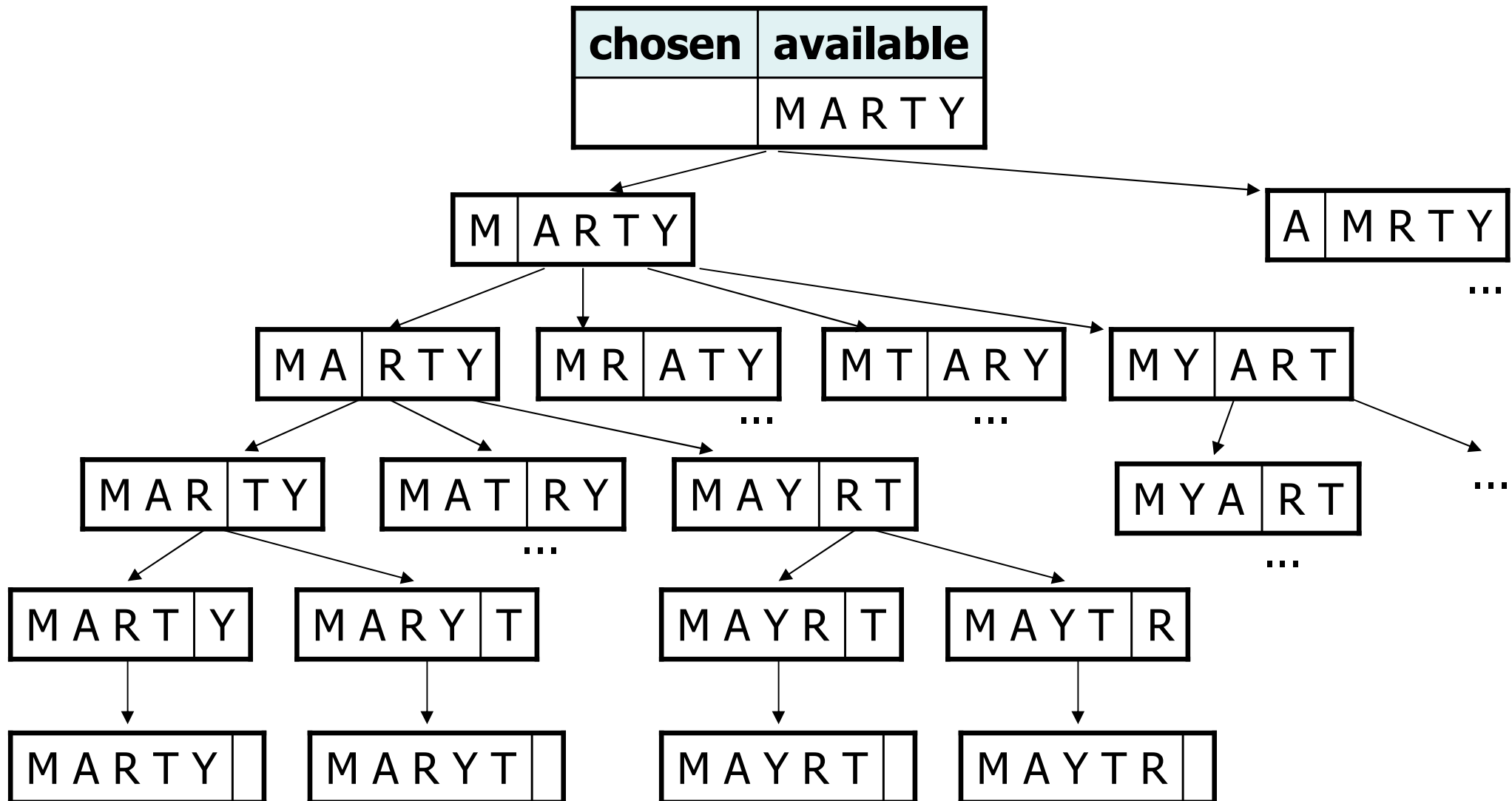
Backtracking

- **backtracking**: A general algorithm for finding solution(s) to a computational problem by trying partial solutions and then abandoning them ("backtracking") if they are not suitable.

  - a "brute force" algorithmic technique (tries all paths; not clever)
  - often (but not always) implemented recursively

  Applications:
  - producing all permutations of a set of values
  - parsing languages
  - games: anagrams, crosswords, word jumbles, 8 queens
  - combinatorics and logic programming

Backtracking algorithms

A general pseudo-code algorithm for backtracking problems:

explore(**choices**):
- if there are no more **choices** to make: stop.

- else:
  - Make a single choice **C** from the set of choices.
    - Remove **C** from the set of **choices**.

  - explore the remaining **choices**.

  - Un-make choice **C**.
    - Backtrack!

Backtracking strategies

- When solving a backtracking problem, ask these questions:
  - What are the "choices" in this problem?
    - What is the "base case"? (How do I know when I'm out of choices?)

  - How do I "make" a choice?
    - Do I need to create additional variables to remember my choices?
    - Do I need to modify the values of existing variables?

  - How do I explore the rest of the choices?
    - Do I need to remove the made choice from the list of choices?

  - Once I'm done exploring the rest, what should I do?

  - How do I "un-make" a choice?

Permutations revisited

- Write a method `permute` that accepts a string as a parameter and outputs all possible rearrangements of the letters in that string. The arrangements may be output in any order.

    – Example:
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    sequence of lines:

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MATYR	MYTRA	AYMTR	RTAYM	TAYRM	YAMTR
MAYRT	AMRTY	AYRMT	RTYMA	TRMAY	YARMT
MAYTR	AMRYT	AYRTM	RTYAM	TRMYA	YARTM
MRATY	AMTRY	AYTMR	RYMAT	TRAMY	YATMR
MRAYT	AMTYR	AYTRM	RYMTA	TRAYM	YATRM
MRTAY	AMYRT	RMATY	RYAMT	TRYMA	YRMAT
MRTYA	AMYTR	RMAYT	RYATM	TRYAM	YRMTA
MRYAT	ARMTY	RMTAY	RYTMA	TYMAR	YRAMT
MRYTA	ARMYT	RMTYA	RYTAM	TYMRA	YRATM
MTARY	ARTMY	RMYAT	TMARY	TYAMR	YRTMA
MTAYR	ARTYM	RMYTA	TMAYR	TYARM	YRTAM
MTRAY	ARYMT	RAMTY	TMRAY	TYRMA	YTMAR
MTRYA	ARYTM	RAMYT	TMRYA	TYRAM	YTMRA
MTYAR	ATMRY	RATMY	TMYAR	YMART	YTAMR
MTYRA	ATMYR	RATYM	TMYRA	YMATR	YTARM
MYART	ATRMY	RAYMT	TAMRY	YMRAT	YTRMA
MYATR	ATRYM	RAYTM	TAMYR	YMRTA	YTRAM

Exercise solution

```java
// Outputs all permutations of the given string.
public static void permute(String s) {
    permute(s, "");
}

private static void permute(String s, String chosen) {
    if (s.length() == 0) {
        // base case: no choices left to be made
        System.out.println(chosen);
    } else {
        // recursive case: choose each possible next letter
        for (int i = 0; i < s.length(); i++) {
            char c = s.charAt(i);                      // choose
            s = s.substring(0, i) + s.substring(i + 1);
            chosen += c;

            permute(s, chosen);                        // explore

            s = s.substring(0, i) + c + s.substring(i + 1);
            chosen = chosen.substring(0, chosen.length() - 1);
        }                                              // un-choose
    }
}
```

Exercise solution 2

```java
// Outputs all permutations of the given string.
public static void permute(String s) {
    permute(s, "");
}

private static void permute(String s, String chosen) {
    if (s.length() == 0) {
        // base case: no choices left to be made
        System.out.println(chosen);
    } else {
        // recursive case: choose each possible next letter
        for (int i = 0; i < s.length(); i++) {
            String ch = s.substring(i, i + 1);   // choose

            String rest = s.substring(0, i) +    // remove
                          s.substring(i + 1);

            permute(rest, chosen + ch);        // explore
        }
    }        // (don't need to "un-choose" because
}            //  we used temp variables)
```

Exercise: Combinations

- Write a method `combinations` that accepts a string s and an integer k as parameters and outputs all possible k-letter words that can be formed from unique letters in that string. The arrangements may be output in any order.

  - Example:
    `combinations("GOOGLE", 3)`
    outputs the sequence of
    lines at right.

  - To simplify the problem, you may assume that the string s contains at least k unique characters.

EGL	LEG
EGO	LEO
ELG	LGE
ELO	LGO
EOG	LOE
EOL	LOG
GEL	OEG
GEO	OEL
GLE	OGE
GLO	OGL
GOE	OLE
GOL	OLG

Initial attempt

```java
public static void combinations(String s, int length) {
    combinations(s, "", length);
}

private static void combinations(String s, String chosen, int length) {
    if (length == 0) {
        System.out.println(chosen);     // base case: no choices left
    } else {
        for (int i = 0; i < s.length(); i++) {
            String ch = s.substring(i, i + 1);
            if (!chosen.contains(ch)) {
                String rest = s.substring(0, i) + s.substring(i + 1);
                combinations(rest, chosen + ch, length - 1);
            }
        }
    }
}
```

– Problem: Prints same string multiple times.

Exercise solution

```java
public static void combinations(String s, int length) {
    Set<String> all = new TreeSet<String>();
    combinations(s, "", all, length);
    for (String comb : all) {
        System.out.println(comb);
    }
}

private static void combinations(String s, String chosen,
                                 Set<String> all, int length) {
    if (length == 0) {
        all.add(chosen);              // base case: no choices left
    } else {
        for (int i = 0; i < s.length(); i++) {
            String ch = s.substring(i, i + 1);
            if (!chosen.contains(ch)) {
                String rest = s.substring(0, i) + s.substring(i + 1);
                combinations(rest, chosen + ch, all, length - 1);
            }
        }
    }
}
```