CSE 143
Lecture 18

More Recursive Backtracking

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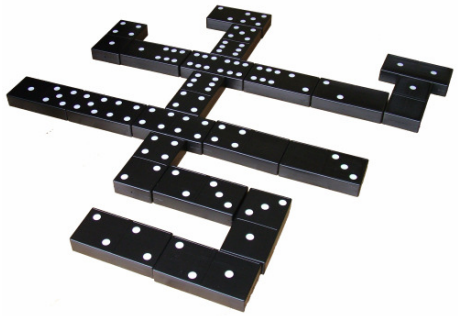
Exercise: Dominoes

- The game of dominoes is played with small black tiles, each having 2 numbers of dots from 0-6. Players line up tiles to match dots.



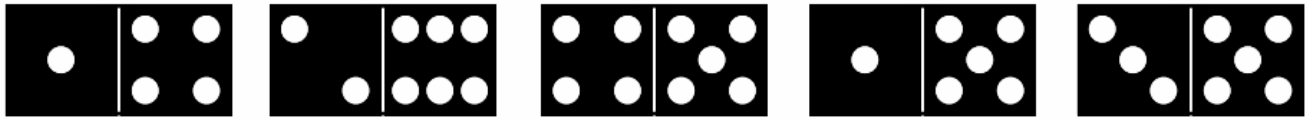
- Given a class `Domino` with the following public methods:

```
int first()                 // first dots value
int second()                // second dots value
void flip()                 // inverts 1st/2nd
boolean contains(int n)     // true if 1st/2nd == n
String toString()           // e.g. "(3|5)"
```

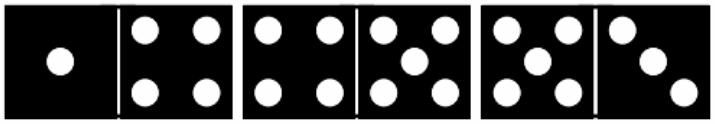
- Write a method `hasChain` that takes a `List` of dominoes and a starting/ending dot value, and returns whether the dominoes can be made into a chain that starts/ends with those values.

Domino chains

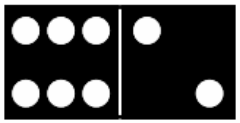
- Suppose we have the following dominoes:



- We can link them into a chain from 1 to 3 as follows:
  - Notice that the 3|5 domino had to be flipped.



- We can "link" one domino into a "chain" from 6 to 2 as follows:



Exercise client code

```
import java.util.*;   // for ArrayList

public class SolveDominoes {
    public static void main(String[] args) {
        // [(1|4), (2|6), (4|5), (1|5), (3|5)]
        List<Domino> dominoes = new ArrayList<Domino>();
        dominoes.add(new Domino(1, 4));
        dominoes.add(new Domino(2, 6));
        dominoes.add(new Domino(4, 5));
        dominoes.add(new Domino(1, 5));
        dominoes.add(new Domino(3, 5));
        System.out.println(hasChain(dominoes, 5, 5));   // true
        System.out.println(hasChain(dominoes, 1, 5));   // true
        System.out.println(hasChain(dominoes, 1, 3));   // true
        System.out.println(hasChain(dominoes, 1, 6));   // false
        System.out.println(hasChain(dominoes, 1, 2));   // false
    }

    public static boolean hasChain(List<Domino> dominoes,
                                   int start, int end) {
        ...
    }
}
```

Exercise solution

```
public boolean hasChain(List<Domino> dominoes, int start, int end) {
    if (start == end) {
        for (Domino d : dominoes) {
            if (d.contains(start)) { return true; }
        }
        return false;                                   // base case
    } else {
        for (int i = 0; i < dominoes.size(); i++) {
            Domino d = dominoes.remove(i);    // choose
            if (d.first() == start) {         // explore
                if (hasChain(dominoes, d.second(), end)) {
                    return true;
                }
            } else if (d.second() == start) {
                if (hasChain(dominoes, d.first(), end)) {
                    return true;
                }
            }
            dominoes.add(i, d);                         // un-choose
        }
        return false;
    }
}
```

Exercise: Print chain

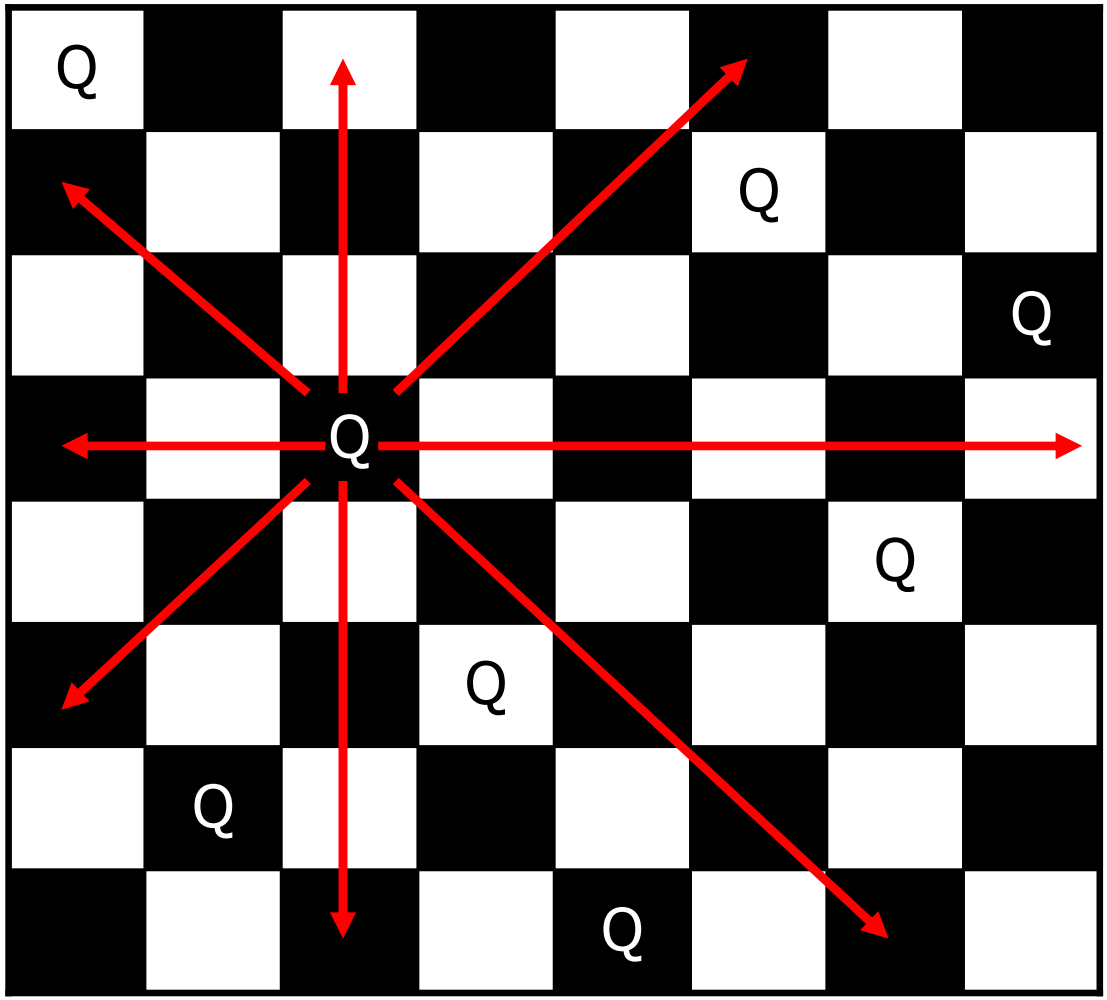
- Write a variation of your `hasChain` method that also prints the chain of dominoes that it finds, if any.

```
hasChain(dominoes, 1, 3);
```

```
[(1|4), (4|5), (5|3)]
```

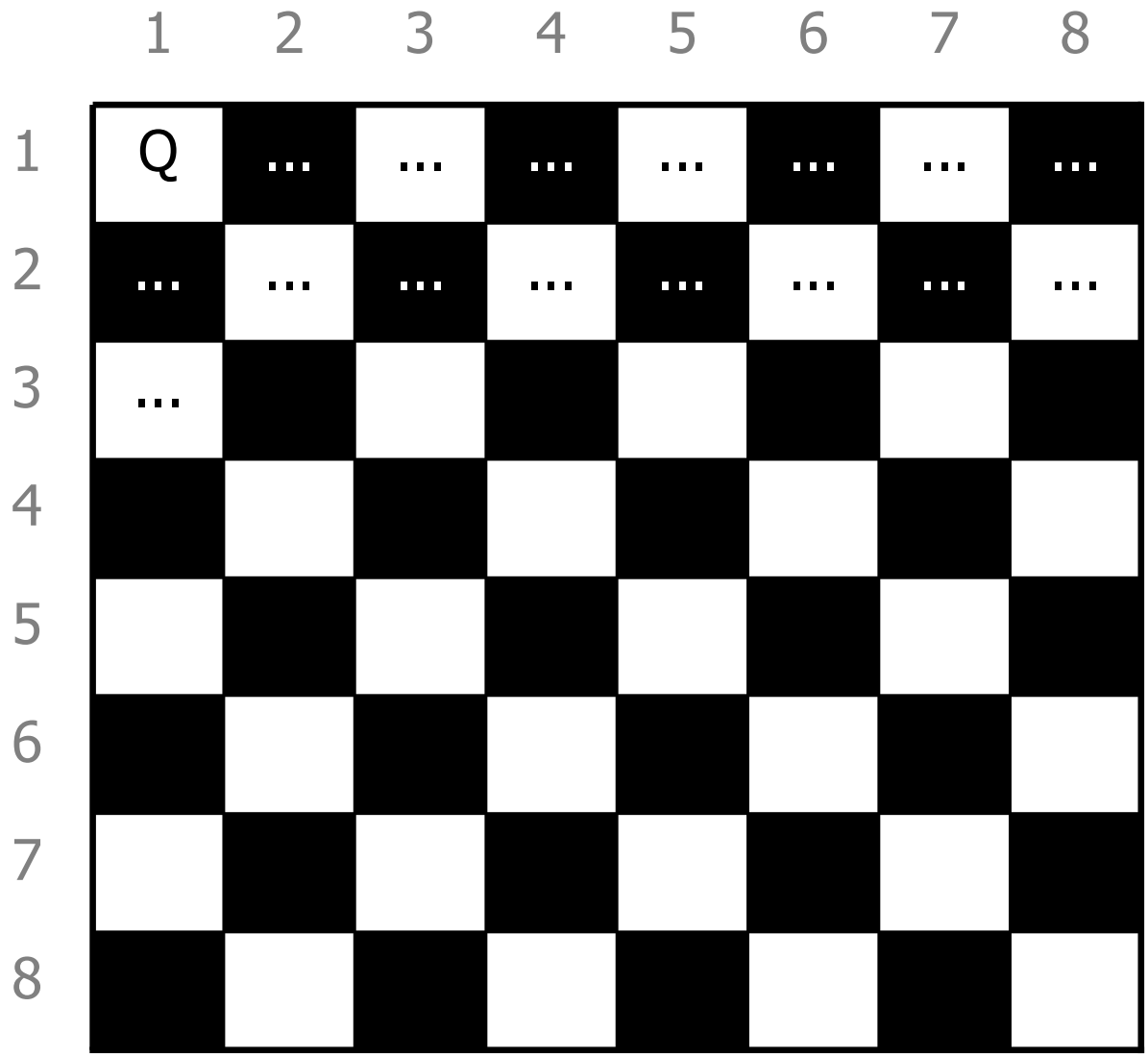
The "8 Queens" problem

- Consider the problem of trying to place 8 queens on a chess board such that no queen can attack another queen.
  - What are the "choices"?
  - How do we "make" or "un-make" a choice?
  - How do we know when to stop?



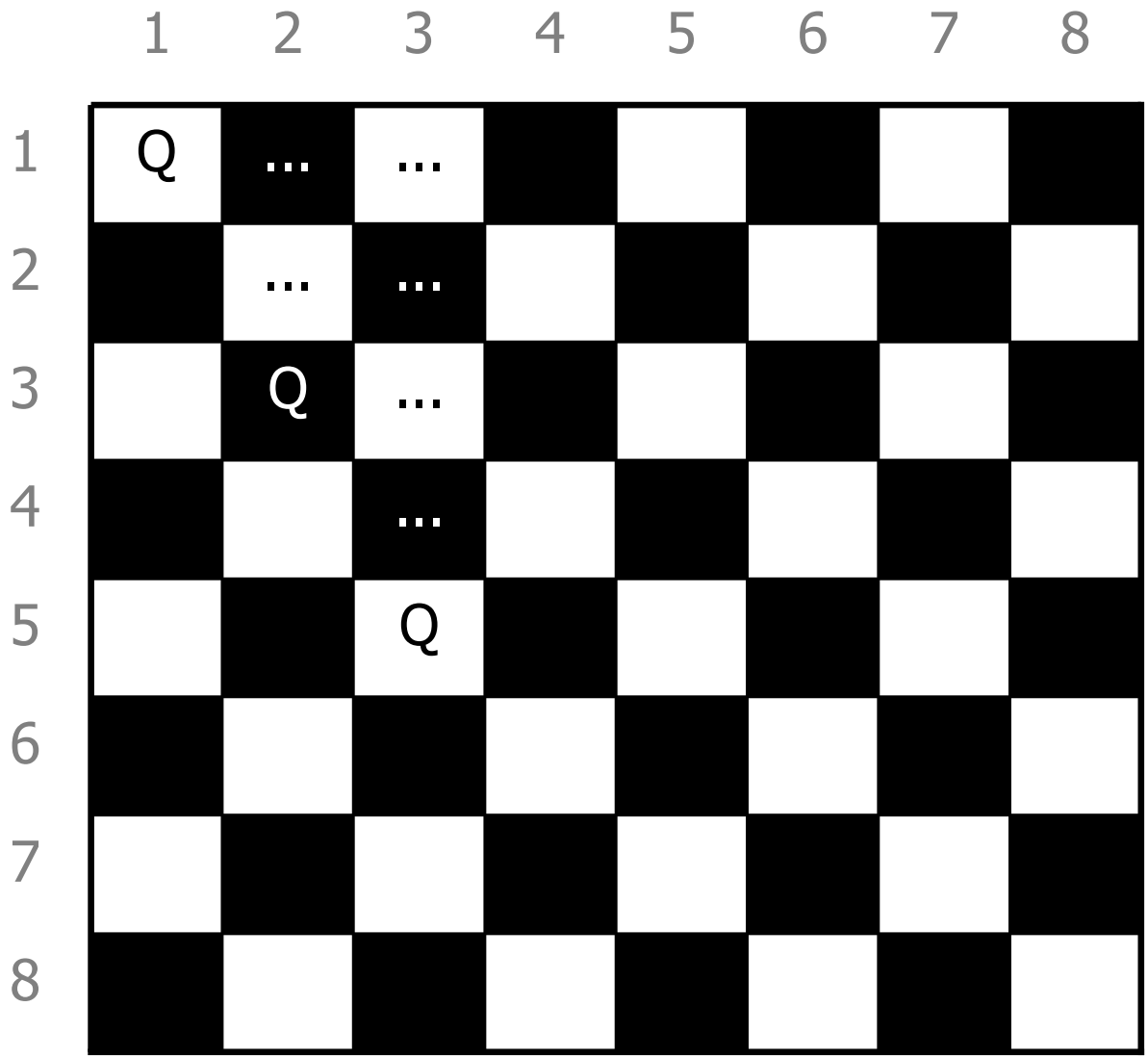
Naive algorithm

- for (each square on board):
  - Place a queen there.
  - Try to place the rest of the queens.
  - Un-place the queen.
  - How large is the solution space for this algorithm?
    - 64 * 63 * 62 * ...



Better algorithm idea

- Observation: In a working solution, exactly 1 queen must appear in each row and in each column.
  - Redefine a "choice" to be valid placement of a queen in a particular column.
  - How large is the solution space now?
    - 8 * 8 * 8 * ...



Exercise

- Suppose we have a `Board` class with the following methods:

Method/Constructor	Description
`public` **`Board`**`(int size)`	construct empty board
`public boolean` **`isSafe`**`(int row, int column)`	`true` if queen can be safely placed here
`public void` **`place`**`(int row, int column)`	place queen here
`public void` **`remove`**`(int row, int column)`	remove queen from here
`public String` **`toString`**`()`	text display of board

- Write a method `solveQueens` that accepts a `Board` as a parameter and tries to place 8 queens on it safely.
  - Your method should stop exploring if it finds a solution.

Exercise solution

```
// Searches for a solution to the 8 queens problem
// with this board, reporting the first result found.
public static void solveQueens(Board board) {
    if (solveQueens(board, 1)) {
        System.out.println("One solution is as follows:");
        System.out.println(board);
    } else {
        System.out.println("No solution found.");
    }
}

...
```

Exercise solution, cont'd.

```
// Recursively searches for a solution to 8 queens on this
// board, starting with the given column, returning true if a
// solution is found and storing that solution in the board.
// PRE: queens have been safely placed in columns 1 to (col-1)
public static boolean solveQueens(Board board, int col) {
    if (col > board.size()) {
        return true;   // base case: all columns are placed
    } else {
        // recursive case: place a queen in this column
        for (int row = 1; row <= board.size(); row++) {
            if (board.isSafe(row, col)) {
                board.place(row, col);              // choose
                if (explore(board, col + 1)) {      // explore
                    return true;   // solution found
                }
                b.remove(row, col);                 // un-choose
            }
        }
        return false;   // no solution found
    }
}
```