CSE 143
Lecture 20

Binary Search Trees

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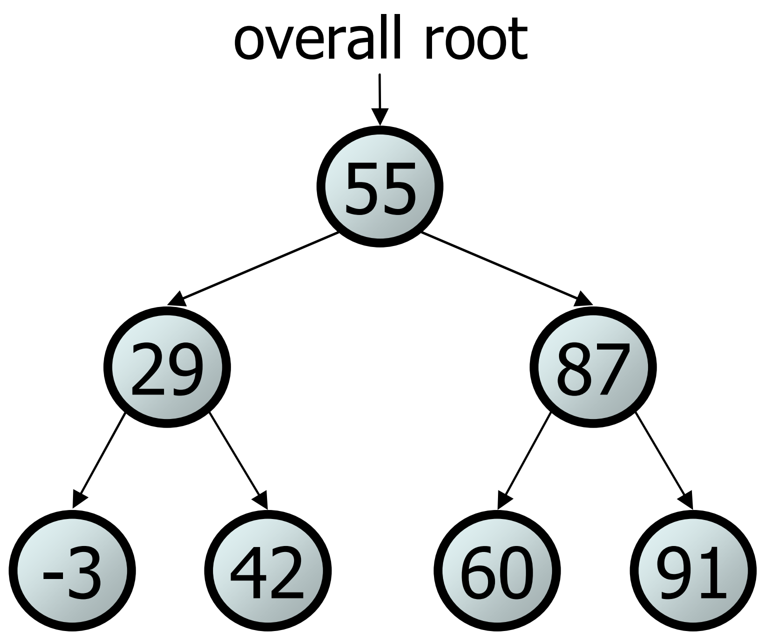
Binary search trees

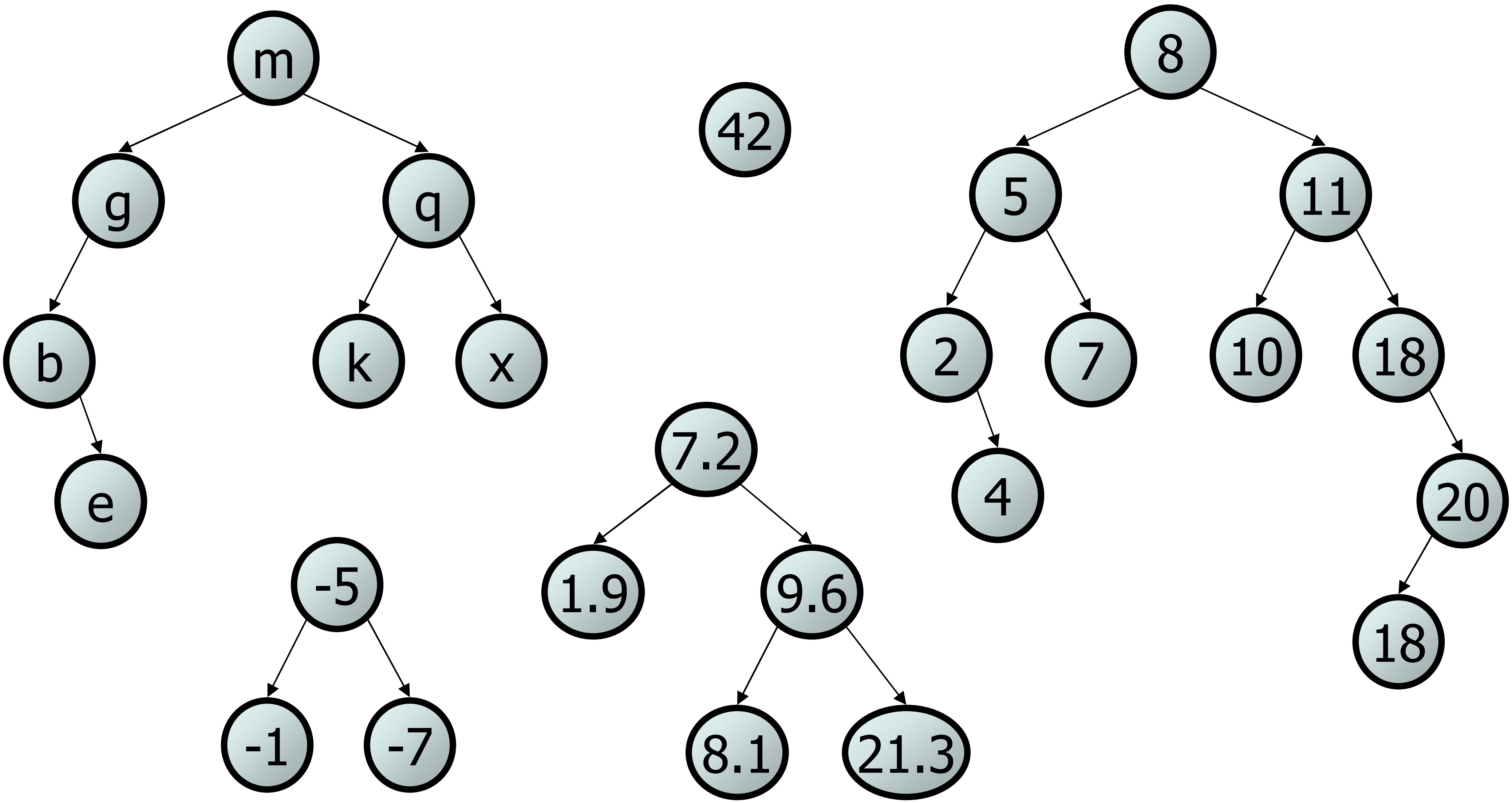
- **binary search tree** ("BST"): a binary tree that is either:
  - empty (`null`), or
  - a root node R such that:
    - every element of R's left subtree contains data "less than" R's data,
    - every element of R's right subtree contains data "greater than" R's,
    - R's left and right subtrees are also binary search trees.

- BSTs store their elements in sorted order, which is helpful for searching/sorting tasks.



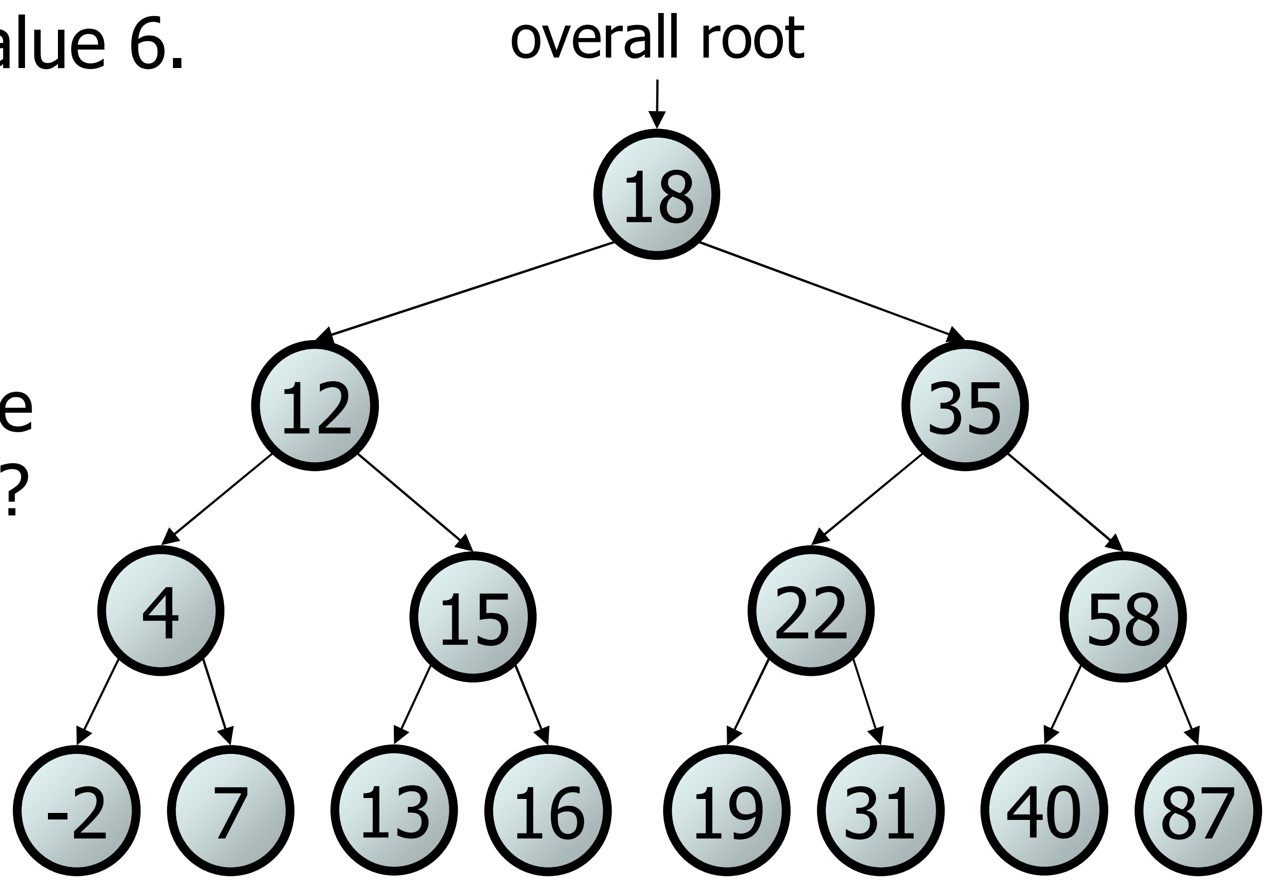
Exercise

- Which of the trees shown are legal binary search trees?



Searching a BST

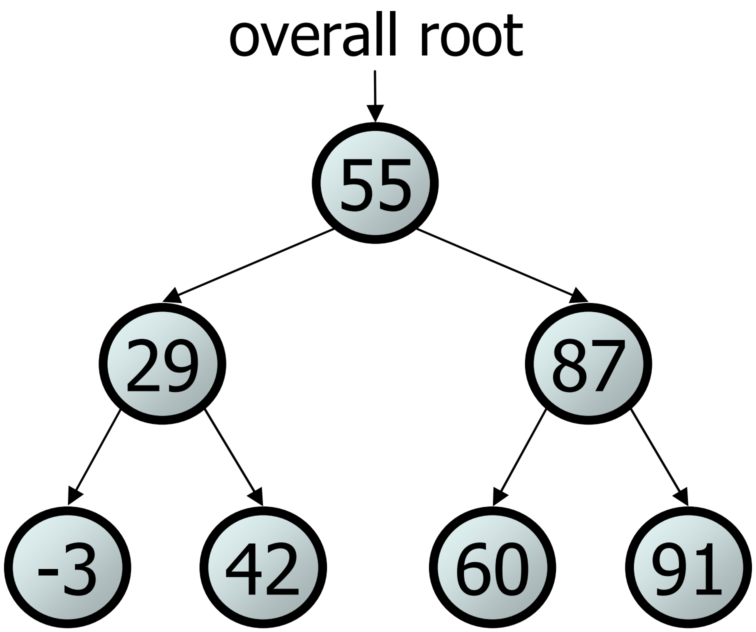
- Describe an algorithm for searching the tree below for the value 31.
- Then search for the value 6.
- What is the maximum number of nodes you would need to examine to perform any search?



Exercise

- Convert the `IntTree` class into a `SearchTree` class.
  - The elements of the tree will constitute a legal binary search tree.

- Add a method `contains` to the `SearchTree` class that searches the tree for a given integer, returning `true` if found.
  - If a `SearchTree` variable `tree` referred to the tree below, the following calls would have these results:
    - `tree.contains(29)` → `true`
    - `tree.contains(55)` → `true`
    - `tree.contains(63)` → `false`
    - `tree.contains(35)` → `false`



Exercise solution

```
// Returns whether this tree contains the given integer.
public boolean contains(int value) {
    return contains(overallRoot, value);
}

private boolean contains(IntTreeNode node, int value) {
    if (node == null) {
        return false;
    } else if (node.data == value) {
        return true;
    } else if (node.data > value) {
        return contains(node.left, value);
    } else {   // root.data < value
        return contains(node.right, value);
    }
}
```

Adding to a BST

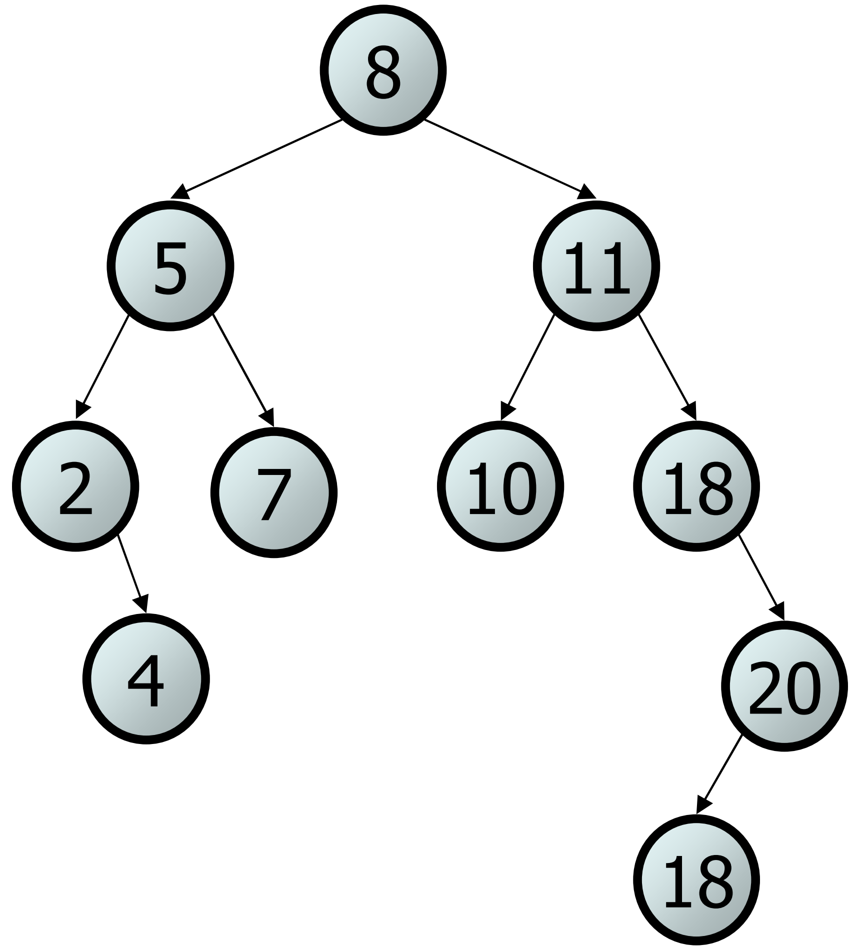
- Suppose we want to add the value 14 to the BST below.
  - Where should the new node be added?

- Where would we add the value 3?

- Where would we add 7?

- If the tree is empty, where should a new value be added?

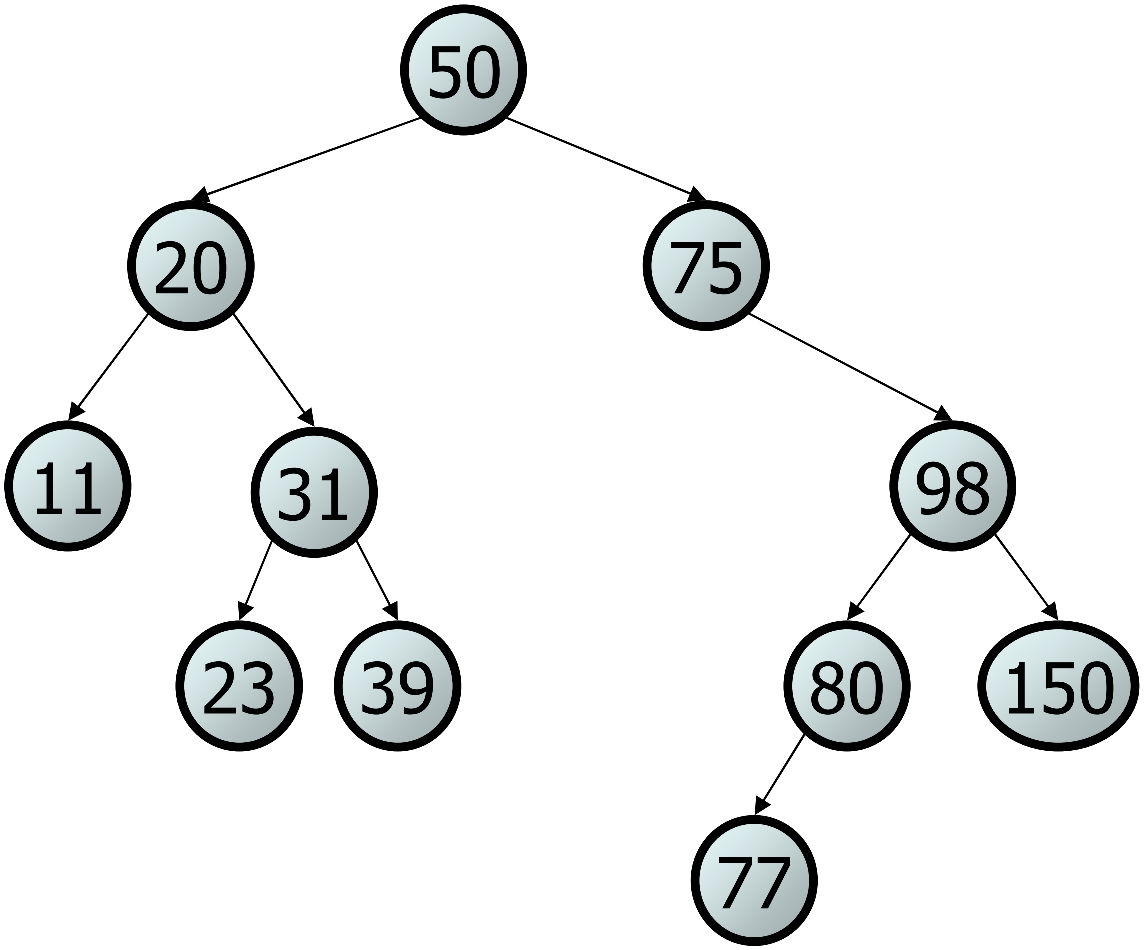
- What is the general algorithm?



Adding exercise

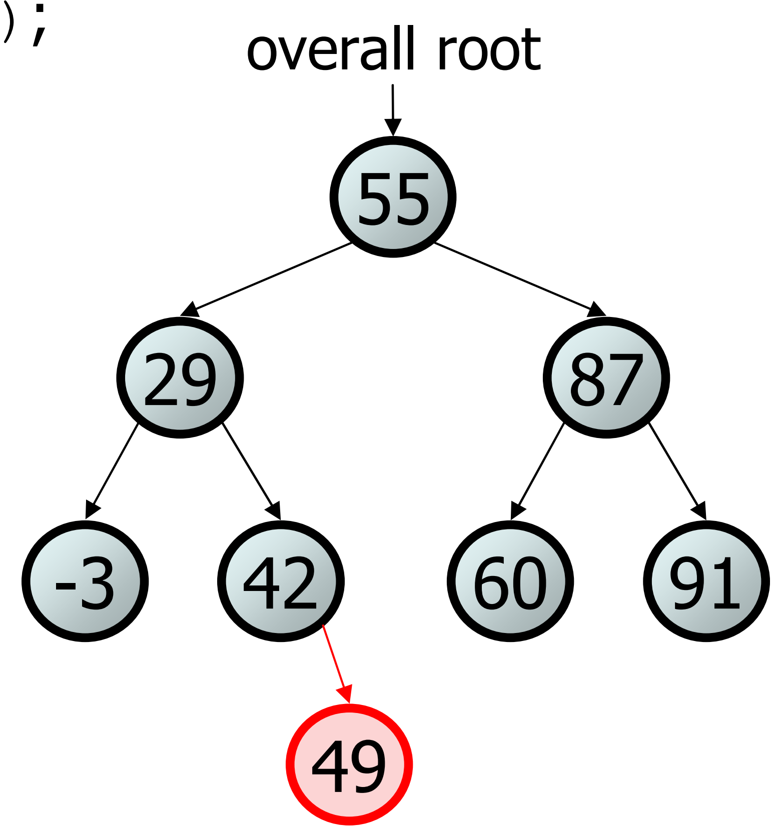
- Draw what a binary search tree would look like if the following values were added to an initially empty tree in this order:

50
20
75
98
80
31
150
39
23
11
77



Exercise

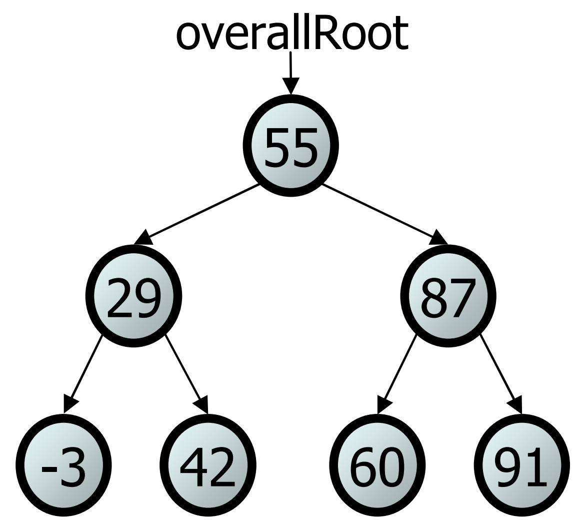
- Add a method `add` to the `SearchTree` class that adds a given integer value to the tree. Assume that the elements of the `SearchTree` constitute a legal binary search tree, and add the new value in the appropriate place to maintain ordering.
  - `tree.add(49);`



An incorrect solution

```
// Adds the given value to this BST in sorted order.
public void add(int value) {
    add(overallRoot, value);
}

private void add(IntTreeNode node, int value) {
    if (node == null) {
        node = new IntTreeNode(value);
    } else if (node.data > value) {
        add(node.left, value);
    } else if (node.data < value) {
        add(node.right, value);
    }
    // else node.data == value, so
    // it's a duplicate (don't add)
}
```



- Why doesn't this solution work?

The x = change(x) pattern

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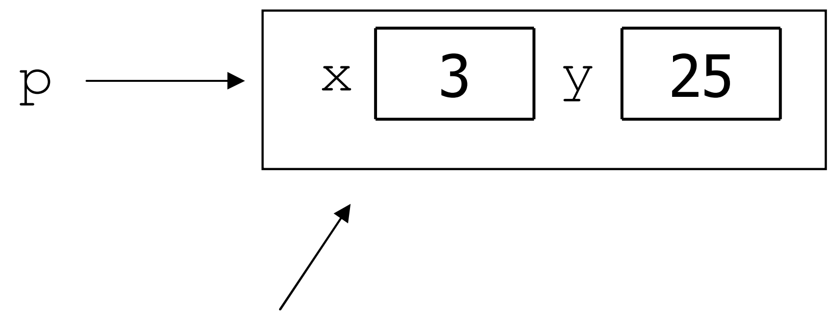
A tangent: Change a point

- What is the state of the object referred to by `p` after this code?

```
public static void main(String[] args) {
    Point p = new Point(3, 25);
    change(p);
    System.out.println(p);
}

public static void change(Point thePoint) {
    thePoint.x = 99;
    thePoint.y = -1;
}

// answer: (99, -1)
```



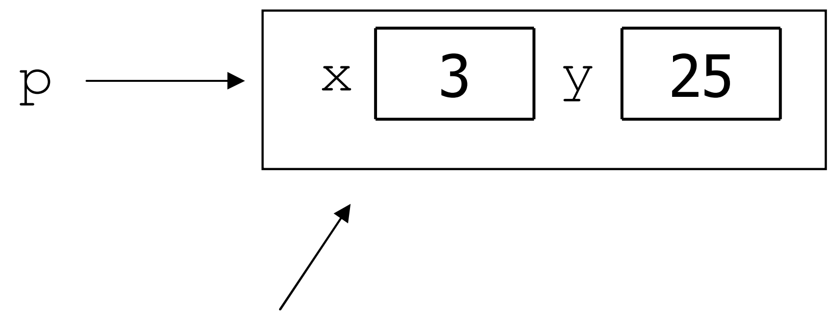
Change point, version 2

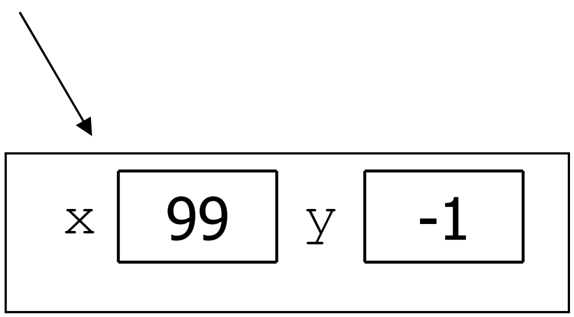
- What is the state of the object referred to by `p` after this code?

```
public static void main(String[] args) {
    Point p = new Point(3, 25);
    change(p);
    System.out.println(p);
}

public static void change(Point thePoint) {
    thePoint = new Point(99, -1);
}

// answer: (3, 25)
```





Changing references

- If a method *dereferences a variable* (with `.`) and modifies the object it refers to, that change will be seen by the caller.

```
public static void change(Point thePoint) {
    thePoint.x = 99;              // affects p
    thePoint.setY(-12345);        // affects p
```

- If a method *reassigns a variable to refer to a new object*, that change will *not* affect the variable passed in by the caller.

```
public static void change(Point thePoint) {
    thePoint = new Point(99, -1);  // p unchanged
    thePoint = null;               // p unchanged
```

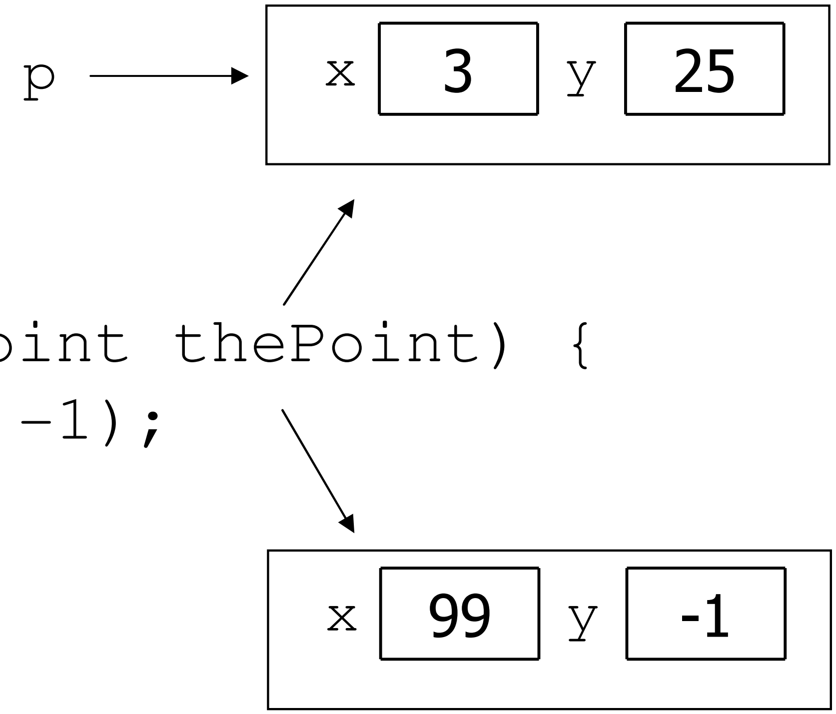
Change point, version 3

- What is the state of the object referred to by `p` after this code?

```
public static void main(String[] args) {
    Point p = new Point(3, 25);
    change(p);
    System.out.println(p);
}

public static Point change(Point thePoint) {
    thePoint = new Point(99, -1);
    return thePoint;
}

// answer: (3, 25)
```



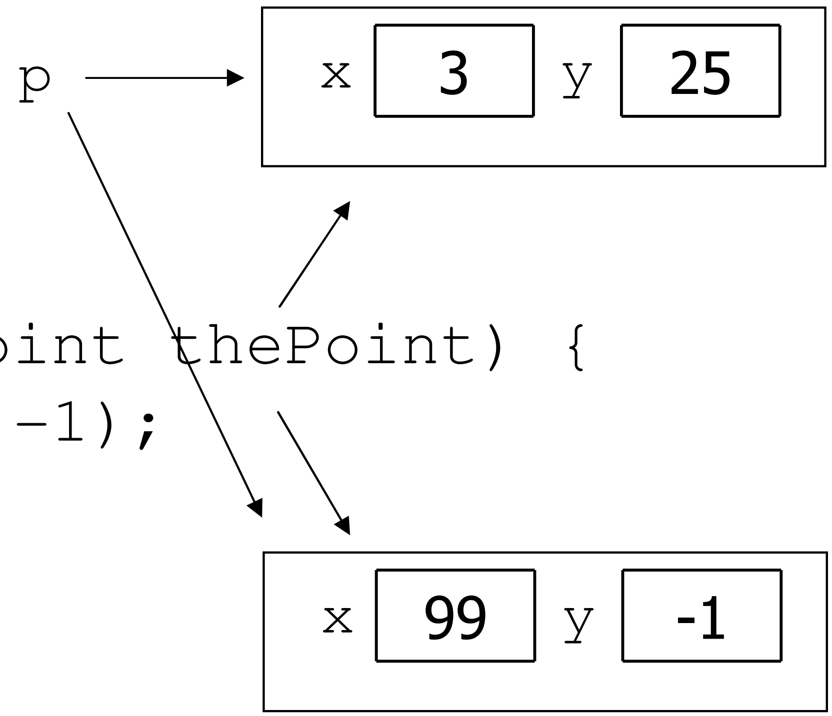
Change point, version 4

- What is the state of the object referred to by `p` after this code?

```
public static void main(String[] args) {
    Point p = new Point(3, 25);
    p = change(p);
    System.out.println(p);
}

public static Point change(Point thePoint) {
    thePoint = new Point(99, -1);
    return thePoint;
}

// answer: (99, -1)
```



x = change(x);

- If you want to write a method that can change the object that a variable refers to, you must do three things:
  1. **pass** in the original state of the object to the method
  2. **return** the new (possibly changed) object from the method
  3. **re-assign** the caller's variable to store the returned result

```
    p = change(p);    // in main

public static Point change(Point thePoint) {
    thePoint = new Point(99, -1);
    return thePoint;
```

- We call this general algorithmic pattern **x = change(x);**

x = change(x) and strings

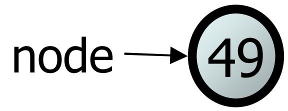
- String methods that modify a string actually return a new one.
  - If we want to modify a string variable, we must re-assign it.

```
String s = "lil bow wow";
s.toUpperCase();
System.out.println(s);    // lil bow wow
s = s.toUpperCase();
System.out.println(s);    // LIL BOW WOW
```

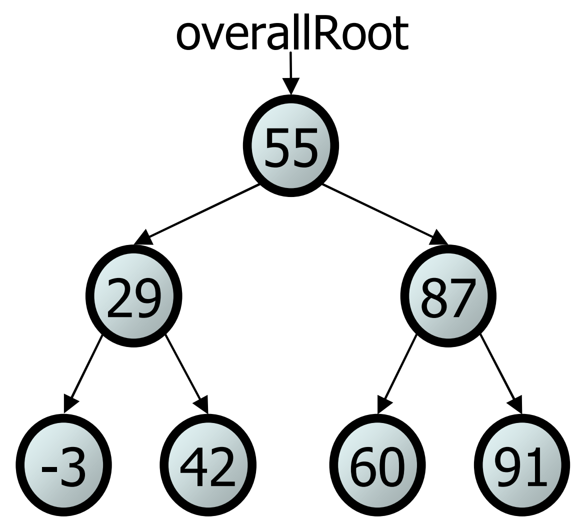
- We use x = change(x) in methods that modify a binary tree.
  - We will **pass** in a node as a parameter and **return** a node result.
  - The node passed in must be **re-assigned** via x = change(x).

The problem

- Much like with linked lists, if we just modify what a local variable refers to, it won't change the collection.



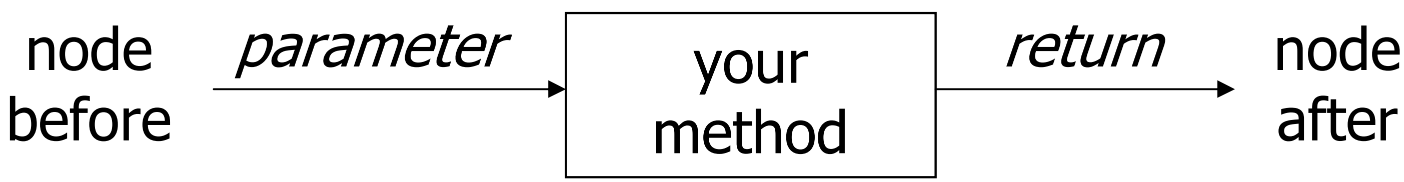

```
private void add(IntTreeNode node, int value) {
    if (node == null) {
        node = new IntTreeNode(value);
    }
```



  – In the linked list case, how did we actually modify the list?
    - by changing the `front`
    - by changing a node's `next` field

Applying x = change(x)

- Methods that modify a tree should have the following pattern:
  - input (parameter): old state of the node
  - output (return): new state of the node



- In order to actually change the tree, you must reassign:

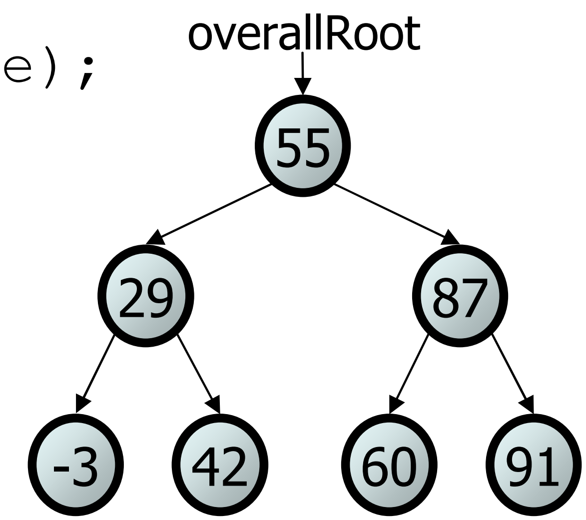
```
node        = change(node, parameters);
node.left   = change(node.left, parameters);
node.right  = change(node.right, parameters);
overallRoot = change(overallRoot, parameters);
```

A correct solution

```
// Adds the given value to this BST in sorted order.
public void add(int value) {
    overallRoot = add(overallRoot, value);
}

private IntTreeNode add(IntTreeNode node, int value) {
    if (node == null) {
        node = new IntTreeNode(value);
    } else if (node.data > value) {
        node.left = add(node.left, value);
    } else if (node.data < value) {
        node.right = add(node.right, value);
    } // else a duplicate

    return node;
}
```

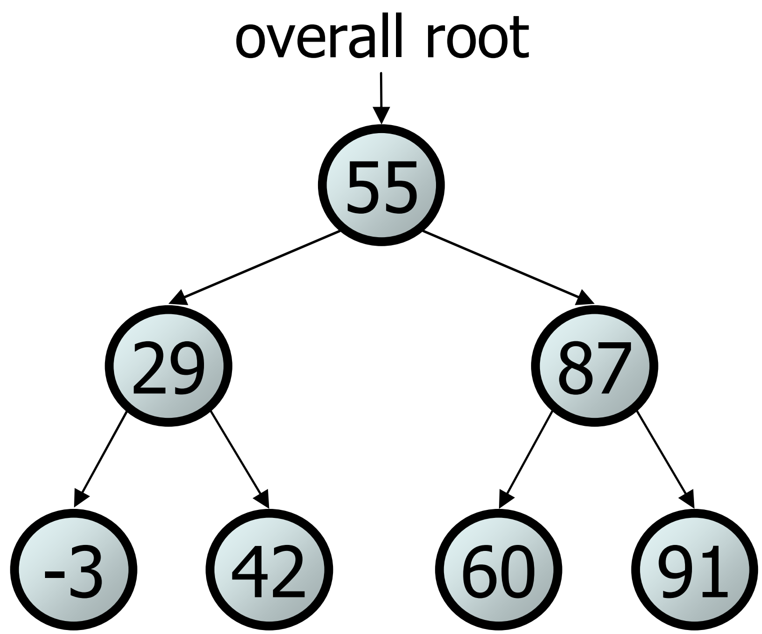


- Think about the case when `node` is a leaf...

Exercise

- Add a method `getMin` to the `IntTree` class that returns the minimum integer value from the tree. Assume that the elements of the `IntTree` constitute a legal binary search tree. Throw a `NoSuchElementException` if the tree is empty.

```
int min = tree.getMin();  // -3
```



Exercise solution

```
// Returns the minimum value from this BST.
// Throws a NoSuchElementException if the tree is empty.
public int getMin() {
    if (overallRoot == null) {
        throw new NoSuchElementException();
    }
    return getMin(overallRoot);
}

private int getMin(IntTreeNode root) {
    if (root.left == null) {
        return root.data;
    } else {
        return getMin(root.left);
    }
}
```